

ESE 326 - Probability and Statistics for Engineering - Fall 2007

Final Exam

18 December 2007

Name: _____

The exam consists of 8 problems, each worth 5 points for a total of 40 points. Explain all your reasoning and be precise in your formulations. No credit will be given for unsubstantiated answers.

Statistical tables with the numerical values for the Binomial, standard normal, T , and χ^2 -distributions are attached at the end of the test. You will not need other tables.

1. Suppose that 5% of all males are color blind and that 0.25% of females are color blind. Assuming that there are an equal number of males and females, what is the probability that a randomly chosen color blind person is male?

2. A controlled satellite is known to have an orbit which at any given time has an error from its reference trajectory which is described by three independent and normally distributed (Gaussian) random variables X_i , $i = 1, 2, 3$, with mean 0 and variance 1. Let R denote the distance of the satellite to its reference trajectory,

$$R = \sqrt{X_1^2 + X_2^2 + X_3^2}.$$

Explain that R^2 has a χ^2 -distribution with 3 degrees of freedom. (You may freely use results derived in class and need not prove these, but explain your reasoning.) Find the distance R which has the property that the satellite will be within distance R of its reference trajectory for 90% of all times.

3. The probability density function of jointly continuous random variables X and Y is given by

$$f(x, y) = \begin{cases} 2(x + y) & \text{for } 0 \leq y \leq x \leq 1, \\ 0 & \text{otherwise} \end{cases}.$$

Find $P(X + Y \leq 1)$.

4. Metal conduits or hollow pipes are used in electrical wiring. In testing 1-inch pipes, these data were obtained on the outside diameter (in inches) of the pipe:

1.281	1.293	1.287	1.286
1.288	1.293	1.291	1.295
1.292	1.291	1.290	1.296
1.289	1.289	1.286	1.291
1.291	1.288	1.289	1.286

- (a) Find unbiased estimates for the mean and variance of the outside diameter of the pipe based on this sample.
- (b) Assume that sampling is from a normal distribution. Give a 95% confidence interval on the mean outside diameter of pipes of this type.
- (c) The makers of this pipe claim that the mean outside diameter is 1.29 inches. Does your confidence interval lead you to suspect this figure? Explain.

5. Quality and reliability are becoming important aspects of computer hardware and software. Past experience shows that the probability of failure during the first 1000 hours of operation for 16-kbit dynamic RAMs produced in the United states is 0.2. It is hoped that new technology and stricter quality control have reduced this failure rate. To verify this contention, 20 systems will be monitored for 1000 hours and the number of failures will be recorded.
- (a) Set up the appropriate null and alternative hypotheses to test this assertion and explain the practical consequences of making a type I and type II error.
 - (b) If H_0 is true and $p = 0.2$, what is the expected number of failures during the first 1000 hours in the 20 trials?
 - (c) Let us agree to reject H_0 in favor of H_1 if the observed number of failures, X , is at most 1. What is the type I error α for this test?
 - (d) Suppose that it is essential that the test be able to distinguish between a failure rate of 0.2 and a failure rate of 0.1. Find the probability that the test as designed will be able to do so. That is, find the probability β of a type II error if indeed $p = 0.1$.

6. The following data represent the fuel gas temperature in degrees Fahrenheit (x) and unit heat rate in Btus per kilowatt hour (y) for a combustion engine to be used in coal gasification.

x	100	150	200	250	300	350	400	450	500
y	99.1	98.5	98.2	98.0	97.8	97.6	97.5	97.0	96.8

Estimate the regression line $\hat{\mu}_{Y|x} = \beta_0 + \beta_1 x$.

7. For the data set given in problem 6, test the null hypothesis $H_0 : \beta_1 = 0$ versus the alternative $H_1 : \beta_1 \neq 0$ at the $\alpha = 5\%$ level of significance.

8. For the data set given in problem 6, calculate 95% confidence intervals for the parameters β_0 and β_1 .