SSM 553 - Nonlinear Systems - Spring 2009

Homework Set 1 due: Tuesday, February 3

1. For the 2-dimensional system

$$\dot{x} = -y + x\left(\mu - x^2 - y^2\right)$$
$$\dot{y} = x + y\left(\mu - x^2 - y^2\right)$$

draw the phase portraits for the qualitatively different cases depending on the parameter $\mu \in \mathbb{R}$. Show that the stability of the origin changes as μ crosses 0. What else happens? Hint: Change the system to polar coordinates.

2. From the textbook, Nonlinear Systems by Hassan Khalil, third edition, do Problem 2.5 in Chapter 2, pg. 78

3. (!!) Let

$$h(t,a) = \frac{1 + 2at^2}{1 + t + a^3t^3} \qquad \text{and} \qquad a = a(u,v) = \frac{v^2}{u^2 + v^2}.$$

Show that the equilibrium $(u,v) = (0,0)$ of the system

$$\dot{u} = \frac{h_t}{h}u, \quad \dot{v} = \frac{h_t}{h}v,$$

is attractive, but not stable.
Hint: Change the system into polar coordinates (r,θ); show that

$$\dot{r} = \frac{h_t}{h}r, \quad \dot{\theta} \equiv 0;$$

for the initial conditions $\theta_0^n = \frac{1}{n}$ and $r_0^n = \sin(\frac{1}{n})$ at initial time $t_0 = 0$ evaluate the solutions at $t_n = \frac{1}{\sin(\frac{1}{n})^3}$. What happens in the limit $n \to \infty$?