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dimensional input, $v(t)$ is the disturbance noise, and $H(q^{-1})$ denotes the system transfer function matrix. The elements of $H(q^{-1})$ are rational functions of the unit delay operation q^{-1} , i.e., $q^{-1}u(t) = u(t-1)$, etc. Available multivariable identification algorithms are usually designed only for left matrix fraction description (LMFD) models, i.e., for

$$y(t) = A^{-1}(q^{-1})B(q^{-1})u(t) + v(t) \quad (2)$$

where $A(q^{-1})$ and $B(q^{-1})$ are $p \times p$ and $p \times m$ polynomial matrices, respectively, see, e.g., [1]-[6].

However, in many practical situations in control, plant monitoring, radar, and others, the model (2) may not describe directly the desired physical mechanism inside the system. Consider, for instance, a plant whose inputs undergo local processes of autoregressive (AR) type and then they are superimposed by a global moving average (MA) process (typically, weighted delay or multipath in a medium). This description corresponds to the right MFD (RMFD) model

$$y(t) = C(q^{-1})D^{-1}(q^{-1})u(t) + v(t) \quad (3)$$

where the polynomial matrix $C(q^{-1})$ is $p \times m$ and $D(q^{-1})$ is an $m \times m$ diagonal matrix. The input-output relationship of this model, which shall be called the DRMFD for the diagonal denominator RMFD, is illustrated in Fig. 1.

In this paper we shall derive recursive identification algorithms for the DRMFD model (3) shown in Fig. 1. The algorithms will be of prediction error (PE) type developed by Ljung [7], [8], and of output error model reference (MR) designed by Landau [3], [9]. They will be particularly advantageous in situations where the DRMFD of the true system is such that the elemental subsystems $c_i^j(q^{-1})/d_j(q^{-1})$ are minimal. (The superscript o denotes the true system.)

We should remark that an alternative way to estimate the DRMFD model (3) would be to first apply LMFD algorithms and then apply transformations from LMFD to RMFD forms. However, this approach would suffer from two main disadvantages, especially when the true system has a minimal DRMFD in the above sense. The first disadvantage is too high a load of computations, due to the overparametrization in the model as well as the need of transformation; the second disadvantage is inaccuracy due to the use of an inappropriate model.

Another approach for estimating the model (3) using available algorithms would be to apply the instrumental variable (IV) type algorithm of [10] designed for the multiinput single-output case of the model (3) by estimating the corresponding transfer function entry by entry. This method cannot be applied to all inputs and outputs together. For the multioutput case, the same denominators associated with the inputs would have to be separately estimated for each output as different polynomials. In general, the ineffectiveness of available algorithms for the DRMFD systems is due to the fact that they are not constrained to the special form of (3).

The main difficulty in deriving algorithms for the RMFO model (3) stems from the fact that one cannot eliminate the inverse $D^{-1}(q^{-1})$ by premultiplying (3) by $D(q^{-1})$.

To demonstrate the usefulness of the model (3), we mention two practical examples of this form. The first one is the DRMFD model of the power system considered by Sinha and Caines [11]. The other example is the multivariable rational model of signals received in distributed array of sensors from multiple distant sources of energy waves; see [12]-[14]. This scenario often occurs in radar, seismology, and astronomy.

The convergence of the RMFD output error MR algorithm below is analyzed in [13] using the ordinary differential equation (ODE) method of Ljung [15]. The results give the positive realness condition for its convergence to the true RMFD of the system. This condition generalizes the scalar case in [13]. The RMFD PE scheme converges to a local minimum of the criterion w.p.1 by the general results of the PE family of algorithms [7].

II. THE RMFD ALGORITHMS

The Basic Assumptions

- 1) The true system transfer function $H^o(q^{-1})$ is strictly proper, stable, and with monic denominators.
- 2) The stochastic processes $u(t)$ and $v(t)$

Recursive Identification Algorithms for Right Matrix Fraction Description Models

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Abstract—Multivariable identification algorithms are usually designed only for left matrix fraction description (LMFD) models. In this paper we consider recursive identification algorithms for right matrix fraction description (RMFD) models with diagonal denominator matrices. The algorithms are of prediction error (PE) and model reference (MR) type. Results from simulations illustrate the performance of the algorithms.

I. INTRODUCTION

Over the last few years, various algorithms have been suggested for on-line identification of multivariable system transfer functions. A general multivariable linear finite order discrete time system can be modeled by the input-output relationship

$$y(t) = H(q^{-1})u(t) + v(t) \quad (1)$$

where $y(t)$ is the p -dimensional output vector at time t , $u(t)$ the m -

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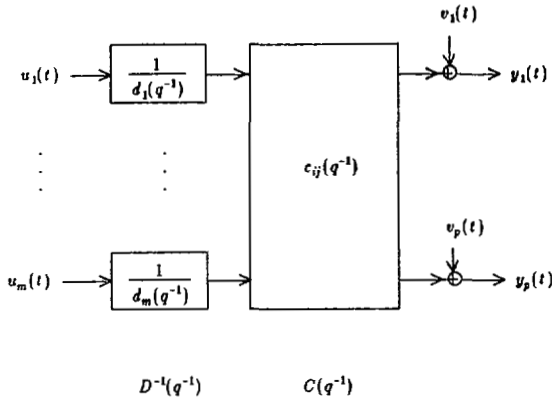


Fig. 1. Schematic description of the DRMFD model.

are stationary with rational spectral densities and such that all their moments exist (note, they can also have deterministic components). 3) The system operates in open loop and/or the noise $v(t)$ is white. 4) The input $u(t)$ is uncorrelated with $v(t)$. 5) The different inputs are uncorrelated. 6) The order of the algorithm is not smaller than the true system order. These assumptions were used in the convergence analysis of [13].

Notice that we have not included an elemental minimality assumption on the DRMFD of the true system. The reason for this is that when this description is nonminimal, the algorithms below can still yield an estimated transfer function which gives the true input-output relations for all nonminimal subsystems; see [13].

The Prediction Signal

To derive the regression expression for the predicted output vector, note from (3) that

$$y_i(t) = \sum_{j=1}^m \frac{c_{ij}(q^{-1})}{d_j(q^{-1})} u_j(t) + v_i(t) \quad 1 \leq i \leq p \tag{4}$$

so that with the assumption of uncorrelated inputs we may write

$$\hat{y}_i(t, \theta) = \sum_{j=1}^m \hat{y}_{ij}(t, \theta) \tag{5}$$

where θ denotes the unknown system parameter vector and

$$\hat{y}_{ij}(t, \theta) = \frac{c_{ij}(q^{-1})}{d_j(q^{-1})} u_j(t). \tag{6}$$

For notational convenience, we shall restrict ourselves to the special case where the orders of the polynomials $c_{ij}(q^{-1})$ are all equal to n_c , and the orders of $d_j(q^{-1})$ equal to n_d . Equation (6) can be rewritten equivalently as

$$\hat{y}_{ij}(t, \theta) = \phi_{ij}^T(t, \theta) \theta_{ij} \tag{7}$$

where

$$\phi_{ij}(t, \theta) = [-\hat{y}_{ij}(t-1, \theta), \dots, -\hat{y}_{ij}(t-n_d, \theta), u_j(t-1), \dots, u_j(t-n_c)]^T \tag{8a}$$

$$\begin{aligned} \theta_{ij} &= [d_{j1}, \dots, d_{jn_d}, c_{ij1}, \dots, c_{ijn_c}]^T \\ &= [(\theta_j^d)^T, (\theta_j^c)^T]^T. \end{aligned} \tag{8b}$$

Observe that the first part of θ_{ij} is independent of i . Introduce also the notation

$$\hat{y}_i(t, \theta) = [\hat{y}_{i1}(t, \theta), \dots, \hat{y}_{im}(t, \theta)]^T, \tag{9a}$$

$$\phi_{ji}(t, \theta) = [\hat{y}_j^T(t-1, \theta), \dots, \hat{y}_j^T(t-n_d, \theta)]^T, \tag{9b}$$

$$u(t) = [u_1(t), \dots, u_m(t)]^T, \tag{9c}$$

$$\phi_u(t) = [u^T(t-1), \dots, u^T(t-n_c)]^T. \tag{9d}$$

Then, by grouping expressions (7) for all inputs, we obtain

$$\hat{y}_i(t, \theta) = [-\phi_{ji}^T(t, \theta), \phi_u^T(t)] [(\theta^d)^T, (\theta_j^c)^T]^T \tag{10}$$

where

$$\theta^d = [d_{11}, \dots, d_{m1}, d_{12}, \dots, d_{mn_d}]^T \tag{11a}$$

$$\theta_j^c = [c_{j11}, \dots, c_{jm1}, c_{j12}, \dots, c_{jmn_c}]^T. \tag{11b}$$

The terms d_{jk} above refer to the coefficient of q^{-k} in $d_j(q^{-1})$, and similarly for c_{ijk} in $c_{ij}(q^{-1})$. Now stacking (10) for all outputs, we get the multivariable difference equation

$$\hat{y}(t, \theta) = \phi^T(t, \theta) \theta \tag{12}$$

where¹

$$\phi(t, \theta) = \begin{bmatrix} -\phi_{j1}^T(t, \theta) & \phi_u^T(t) & \dots & 0 \\ -\phi_{jp}^T(t, \theta) & 0 & \dots & \phi_u^T(t) \end{bmatrix}^T \quad (n_d + p \cdot n_c) \cdot m \times p \tag{13a}$$

$$\theta = [(\theta^d)^T; (\theta_1^c)^T, \dots, (\theta_p^c)^T]^T \quad (n_d + p \cdot n_c) \cdot m \times 1. \tag{13b}$$

The special structure of $\phi(t, \theta)$ and θ reflects the particular form of the DRMFD model (3).

The Gradient Matrix

For the derivation of the recursive PE algorithm [7], [8] we need also the gradient matrix

$$\psi(t, \theta) = \frac{d\hat{y}^T(t, \theta)}{d\theta}. \tag{14}$$

From (4) it is easy to check that the entries of this matrix are given here by

$$\frac{\partial \hat{y}_i(t, \theta)}{\partial d_{jk}} = -\frac{c_{ij}(q^{-1})}{d_j^2(q^{-1})} u_j(t-k) = -\frac{1}{d_j(q^{-1})} \hat{y}_{ij}(t-k, \theta) \tag{15a}$$

$$\frac{\partial \hat{y}_i(t, \theta)}{\partial c_{ijk}} = \begin{cases} \frac{1}{d_j(q^{-1})} u_j(t-k) & l=i \\ 0 & l \neq i \end{cases} \tag{15b}$$

This implies that $\psi(t, \theta)$ has here the same form as $\phi(t, \theta)$, except for the prefiltering of $\hat{y}_{ij}(t, \theta)$ and $u_j(t)$ by $1/d_j(q^{-1})$.

The Recursive PE Algorithm

The general recursions of the Gauss-Newton type PE algorithm for quadratic criterion [7], [8] are

$$\epsilon(t) = y(t) - \hat{y}(t) \tag{16a}$$

$$\hat{\Lambda}(t) = \hat{\Lambda}(t-1) + \gamma(t) [\epsilon(t) \epsilon^T(t) - \hat{\Lambda}(t-1)] \tag{16b}$$

$$S(t) = \psi^T(t) P(t-1) \psi(t) + \lambda(t) \hat{\Lambda}(t) \tag{16c}$$

$$L(t) = P(t-1) \psi(t) S^{-1}(t) \tag{16d}$$

$$\hat{\theta}(t) = [\hat{\theta}(t-1) + L(t) \epsilon(t)]_{D_M} \tag{16e}$$

$$P(t) = [P(t-1) - L(t) S^{-1}(t) L^T(t)] / \lambda(t) \tag{16f}$$

$$\text{update } \phi(t) \rightarrow \phi(t+1) \tag{16g}$$

$$\text{update } \psi(t) \rightarrow \psi(t+1) \tag{16h}$$

$$\hat{y}(t+1) = \phi^T(t+1) \hat{\theta}(t). \tag{16i}$$

The weighting sequence $\lambda(t)$ is updated in the stationary case by $\lambda(t) = \lambda_0 \lambda(t-1) + (1 - \lambda_0)$ where typically $\lambda_0 = 0.99$ and $\lambda(0) = 0.95$. The variable $\gamma(t)$ is updated by $\gamma(t) = \gamma(t-1) / [\gamma(t-1) + \lambda(t)]$. The

¹ Notice that $\phi_{ij}(t, \theta)$ and θ_{ij} defined earlier are not the ij th entries of $\phi(t, \theta)$ and θ .

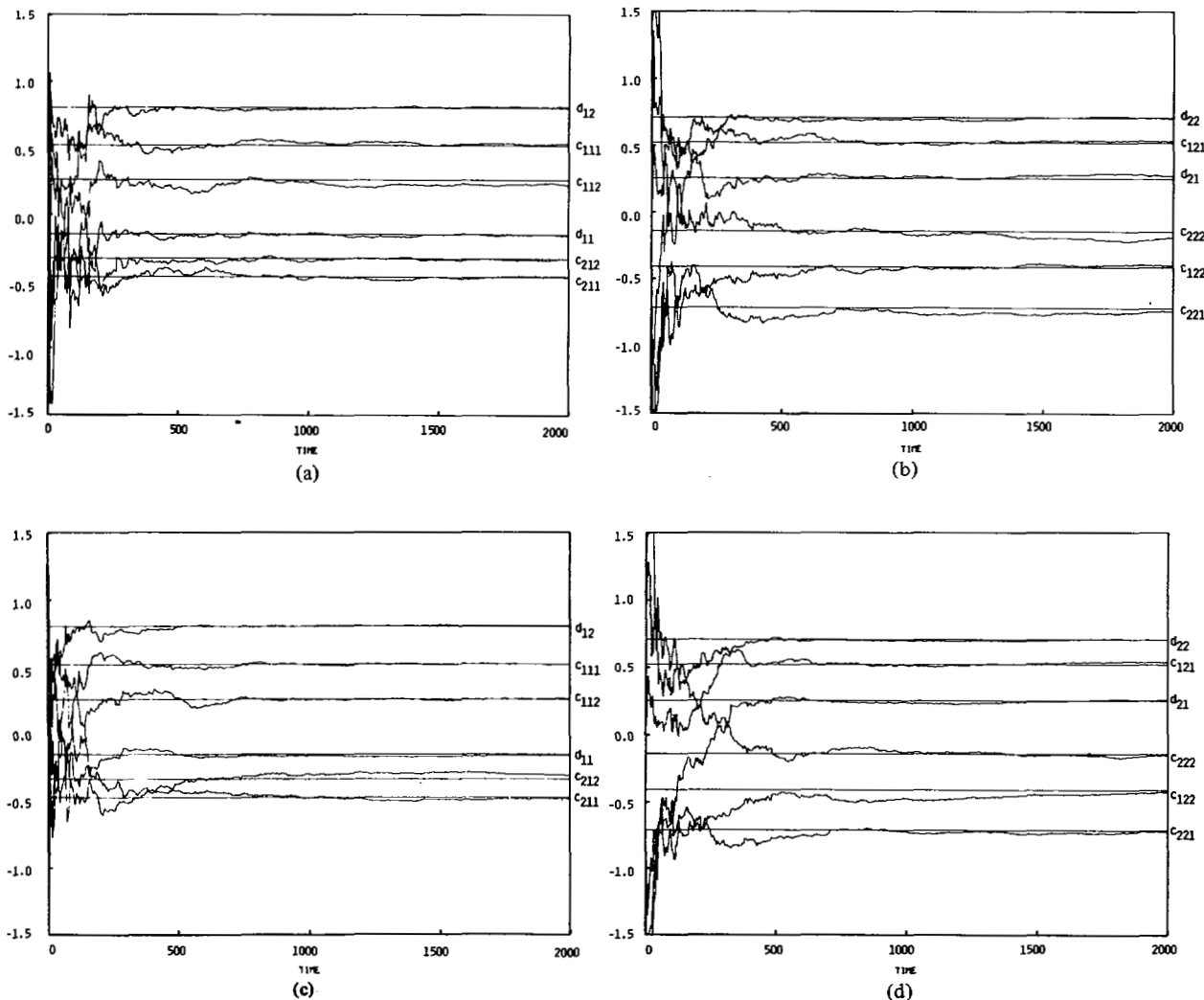


Fig. 2. Parameter estimates with their true values for the 2×2 second-order system of Section III using the recursive RMFD algorithms. $SNR(i, j) = 0$ dB. (a) MR, coefficients corresponding to input 1. (b) MR, coefficients corresponding to input 2. (c) PE, coefficients corresponding to input 1. (d) PE, coefficients corresponding to input 2.

above recursions are applicable to our case with the distinction that $\phi(t)$ and $\psi(t)$ have the special forms described in (6-15) for $\phi(t, \theta)$ and $\psi(t, \theta)$ with θ replaced by $\hat{\theta}(t)$; see also [13].

Output Error MR Algorithm

The output error MR algorithm is based on the assumption of linearity of the regression equation (12) in the model θ , which implies that it can be found by replacing $\psi(t)$ with $\phi(t)$ above. Prefiltering of the output error MR algorithm can be performed to avoid the positive realness condition required for the convergence, as was suggested by Landau [3], [17]; see also [16] and [18]. For the various possible prefilterings in our case, see [13].

III. SIMULATION EXAMPLES

The system to be identified is as in (3) with two-inputs, two-outputs, and second-order polynomials. The input signal $u(t)$ and the noise $v(t)$ are white, normally distributed, with zero mean, and identity covariance matrix. (Note that the above algorithms are equally applicable for inputs and noise of ARMA type, with or without deterministic components.) $c_{ij}^o(q^{-1})$ are scaled to give the desired signal-to-noise ratio $SNR(i, j) = 0$ dB between the power of the i th noiseless subsystem output and the noise power at the i th output. The true system polynomials are:

$$d_1^o(q^{-1}) = 1 - 0.15q^{-1} + 0.80q^{-2}$$

$$d_2^o(q^{-1}) = 1 + 0.25q^{-1} + 0.70q^{-2}$$

$$c_{11}^o(q^{-1}) = 0.52q^{-1} + 0.26q^{-2}$$

$$c_{12}^o(q^{-1}) = 0.52q^{-1} - 0.41q^{-2}$$

$$c_{21}^o(q^{-1}) = -0.47q^{-1} - 0.33q^{-2}$$

$$c_{22}^o(q^{-1}) = -0.71q^{-1} - 0.14q^{-2}$$

We use the methods of Section II with the assumption of known orders. Initializations used are $P(0) = 100I_2$, $\hat{\lambda}(0) = I_2$, and $\hat{\theta}(0) = 0$. From the convergence analysis of [13] we expect the algorithms to converge asymptotically w.p.1 to the true system parameters.

Fig. 2 illustrates the results of the RMFD algorithms for the above system. Fig. 2(a) and (b) shows the estimated coefficients computed by the RMFD MR algorithm (without prefiltering) as a function of time, for the first and second input, respectively. The straight lines denote the true values. The separation into different input parameters was done for convenience of presentation. Fig. 2(c) and (d) shows the analogous results from the RMFD PE algorithm.

The simulation results show in general that the estimates approach their true value after several hundred time samples. In [13] we present a first-

order example. The convergence rate was usually faster for first-order systems than for second order; see also [19] for similar results in the scalar case. Note also that there is no significant difference in the convergence rate and accuracy between the denominator and the numerator estimates. The convergence rate can be improved by using a recursive-iterative algorithm; see, e.g., [5], [10].

IV. CONCLUSION

We have presented recursive identification algorithms of PE and output error MR type for the RMFD model (3), which is different from LMFD models commonly used in system identification.

Some possible extensions for more general modeling and IV schemes are discussed in [13]. Inclusion estimation of the noise model can improve the accuracy of the results but would require more computations and decrease the robustness of the algorithm with respect to the noise properties. Other modifications to unknown inputs are particularly useful for on-line localization and identification of multiple distant radiating sources from signals received in a distributed array of passive radar sensors. This modification can be done by using past errors as estimates of the inputs. The close relation between the RMFD model and the controller canonical form (see, e.g., [20]) suggests that more research on the recursive RMFD identification will be useful for adaptive control.

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Model Updating Improves MRAC Performance

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Abstract—The performance of an MRAC design can be improved by using the model-updating concept, which places the state of the system into the reference model at regular time intervals. This paper discusses criteria for determining this updating and a proof is given to show that model updating does not influence the stability properties of an MRAC design. A typical example indicates an improvement in the control performance obtained by the application of model updating.

INTRODUCTION

Model-reference adaptive control (MRAC) is a well-established design method that has demonstrated its capabilities in many interesting applications; for example [1]–[3]. Asymptotic stability can be proved for linear systems. Even for nonlinear systems, stability and a good performance can be obtained. Model updating can improve the performance of an MRAC design, in particular for systems whose structure does not match the structure of their reference model.

Suppose that the state y of the system differs from the state x of the reference model. Then, via an adjustment of the controller parameters, MRAC will force the system to follow the reference model. If the structure of the system and that of the reference model differ, there is no unique parameter set for the controller that is able to realize a zero error between the state y of the system and the state x of the reference model. Consequently, oscillations of y about x can be expected, as illustrated in Fig. 1.

The philosophy of model updating is to reduce these oscillations. Model updating replaces the state of the reference model by the actual state of the system at some points in time. Ultimately, there is no difference between y and x , and thus there are no parameter adjustments. New reference trajectories are calculated, starting in the actual state of the system. Therefore, model updating avoids unnecessary control efforts and anticipates disturbances better.

In this paper we will discuss criteria for determining an appropriate point in time in which to apply model updating and we will prove that model updating does not influence the asymptotic stability of an MRAC design. Results of a feasibility study, concerning a three-axes slew for satellites, illustrate the improvements of model updating over a standard MRAC design.

UPDATE CRITERIA

Landau [1] has proposed an update at each sample time of MRAC for discrete systems. In that case, we have the series-parallel structure of MRAC. Continuous systems require a different approach. In [4] we used a fixed value for the update interval. Every T seconds the state of the system was introduced into the reference model. The choice of this fixed interval turned out to be critical. Now we will propose a criterion based on the Lyapunov function V . In an MRAC design this function consists of an expression dealing with the error $e(e = x - y)$ and an expression dealing with the difference p between the parameters of the reference model and those of the system, so

$$V(e, p) = e'Pe + V(p).$$

If we use appropriate adaptive laws, we obtain [1]

$$\dot{V}(e) = -e'Qe$$

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