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EEG/MEG Error Bounds for a Dynamic Dipole Source with a Realistic Head Model

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Abstract: This work presents the background and derivation of Cramér-Rao bounds on the errors of estimating the parameters (moment and location) of a dynamic current dipole source using data from electro- and magneto-encephalography. A realistic head model, based on knowledge of surfaces separating tissues of different conductivities, is used.

Keywords: Cramér-Rao Bounds, Electro- and Magneto-Encephalography, Nonlinear Discrete Systems, Equivalent Current Dipole

1. Introduction

We analyze the fundamental limitations in estimating the position, orientation and intensity of dynamic brain sources with data from electro- and magneto-encephalography (EEG/MEG). We derive the Cramér-Rao lower bound (CRB) on the covariance of the estimated parameters of a dynamic dipole source. Our results extend previous work on parameter estimation of a fixed brain source through computing the CRB for nonlinear dynamical source models [1]. We perform computer simulations for representative sources, using data of an actual EEG/MEG system and the realistic head model used in [2].

An equivalent current dipole (ECD) is a model representing the focus of neural current sources in the cerebral cortex associated with neural activity in response to for example, sensory, motor or cognitive stimuli, epilleptic foci, etc. In this case, the aim of EEG/MEG is to determine the three locations and three dipolar momentum parameters of the ECD model. The ECD may be stationary during the measurement procedure or more generally, it can be moving and varying during the measurement stage, depending on the particular brain functions being studied. There exist algorithms that estimate the instantaneous parameters of a moving ECD as a function of time for each sample separately, as if the source was stationary. It is conjectured here that the parameters' evolution is described by the trajectory of a dynamic model given by a set of known difference equations. Under this assumption, the above algorithms do not use the information contained in the evolution model. The approach presented in [3] is based on a deterministic description of dipole trajectories. We introduce here the use of a linear dynamic stochastic model to describe these trajectories.

In [2] a CRB on the errors of estimating the parameters of a stationary ECD source when using data from EEG, MEG, or the combined EEG/MEG modality has been derived. The Cramér-Rao bound of the estimation error has the form,

$$E\{(g(y)-\vartheta)(g(y)-\vartheta)^T\} \geq \mathfrak{J}^{-1} \quad (1)$$

where ϑ is the parameter vector, y is the vector of measured data, $g(y)$ is any unbiased estimate of ϑ, and $\mathfrak{J}$ is the Fisher information matrix (FIM)[6].

We use a realistic head model consisting on separate layers of tissues of different conductivity. The surfaces separating the layers are assumed to be known from previous data obtained from magnetic resonance or X-ray tomographic studies. A mesh of triangles is used to form a net that covers each surface. The following measurement equation was obtained in [2] applying the boundary element method (BEM) to solve the forward problem with this realistic multilayer head model, assuming isotropic conductivity for each layer,

$$y = m(\vartheta)+e \quad (2)$$

where $m(\cdot)$ is the signal part and e includes measurement noise and modeling errors.

We apply the CRB derived for a discrete-time nonlinear filtering problem in [5]. This lower bound is applicable in particular to multidimensional dynamic systems with additive Gaussian noise having the form,

$$\vartheta_{t+1} = f_t(\vartheta_t)+w_t \quad (3)$$

$$y_t = m_t(\vartheta_t)+e_t \quad (4)$$

where ϑ_t is the parameter state at time t, $\{y_t\}$ is the measurement process, $f_t(\cdot)$ and $m(\cdot)$ are (generally) nonlinear functions, and $\{w_t\}$ and e_t are independent Gaussian processes with zero mean and invertible covariance matrices Q_t and R_t, respectively.

We compute the Fisher information matrix for estimating the ECD parameters ϑ_t, denoted J_t. The matrix J_t^{-1} provides the CRB on the mean-square error of estimating ϑ_t. The matrix J_t can be recursively computed in t.

We demonstrate the applicability of this procedure, computing the CRB for simulated brain source trajectories generated by a linear noisy ECD model, and using real data for the multilayer head model. Our method was devised to assess the quality in tracking moving sources. It is still unknown how accurately this stochastic dynamic model may depict a real neurophysiological situation.

2. Models for Source, Head and Measurements

In this study we consider a dipole whose parameters are the position $\boldsymbol{p}$ and its moment $\boldsymbol{q}$. This model has six independent parameters: three location components of $\boldsymbol{p}$ in Cartesian coordinates and the three components of the dipolar moment $\boldsymbol{q}$.

A dynamic dipole model is like a single fixed dipole but varying its six independent parameters with time. This assumption may be useful in situations where neural activity propagates, as in the case of an epileptogenic source [3] or during auditory evoked response (AER) experiments [4]. In practice the dynamic assumption means that we get more than one time-slice of data while the measurement process is carried out. For the dynamic dipole and for $t = 1,2,...,$ the parameter vector is written $\vartheta_t = [\boldsymbol{q}_t^T, \boldsymbol{p}_t^T]^T = [\vartheta_1(t),...\vartheta_6(t)]^T$.

A realistic model considers the head as separate layers of tissues of different conductivities and arbitrary shape. Geometrically the head is considered a volume consisting of layers of homogeneous and isotropic volume conductors separated by closed surfaces.

A mesh of triangles is used to form a net that covers each surface. The total number of tessellation triangles is denoted m_T. The tessellation is necessary to apply the BEM for instance, to solve the integral equations (see [2]).

In EEG, electrical potentials at multiple locations on the scalp are measured using a cap with electrodes. The MEG system consists of a helmet with an array of SQUIDs and pick-up coils arranged to measure components of the magnetic field outside the head. In the combined modality the measurements are taken simultaneously from the EEG and MEG channels. We assume that the EEG instrument has m_E electrodes and that the MEG instrument has m_B magnetic sensors.

Let $\boldsymbol{e}_E$ be the vector of total noise affecting the potential measurements and $\boldsymbol{e}_B$ be the total noise affecting the magnetic fields measurements. For the combined modality, we assume that the noise $\boldsymbol{e} = [\boldsymbol{e}_B^T, \boldsymbol{e}_E^T]^T$ is zero-mean Gaussian noise with known covariance R.

The solution with BEM and a center of gravity technique of the discrete combined measurement equations for the tessellated head model is

$$y = \begin{bmatrix} \boldsymbol{b}_0 \\ 0 \end{bmatrix} + \begin{bmatrix} A \\ H_E \end{bmatrix} (D+H)^{\#} \Phi_0 + \boldsymbol{e} = m(\vartheta_t) + \boldsymbol{e} \quad (5)$$

The upper part of $\boldsymbol{y}$ is composed by the projections of the magnetic field along the pick-up coils of the MEG helmet, and the lower part by the potentials measured by the EEG electrodes.

The only functional dependence on the dipole parameters are in $\boldsymbol{b}_0$ and ϕ_{0_i}, through well known relationships. ϕ_{0_i} represents the electrical potential at electrode i assuming the source is immersed in an infinite homogeneous medium of conductivity $1\Omega^{-1}m^{-1}$. If $\boldsymbol{B}_{0_i}$ represents the magnetic field assuming similar conditions, then $\boldsymbol{b}_{0_i}$ is its projection along the *i-th* pick-up coil.

The matrices A, H_E, D and H depend upon the shape of the layers, their conductivities and the sensor configuration. Their complete definition can be seen in [2].

3. FIMs for a Dynamic Dipole Source

Assume no a priori information of the parameters to be estimated is available, except that they are not random variables. From (5) we can derive a single snapshot FIM (SS-FIM) (see [2])

$$\mathfrak{J}_t(\vartheta_t) = \nabla_{\vartheta_t} m^T(\vartheta_t) R^{-1} (\nabla_{\vartheta_t} m^T(\vartheta_t))^T \quad (6)$$

where $\nabla_X = \left[\frac{\partial}{\partial x_1}, ..., \frac{\partial}{\partial x_r}\right]^T$.

Consider now that the dipoles are known to move according to a linear multidimensional dynamic system with additive Gaussian noise, $\vartheta_{t+1} = A_t\vartheta_t + w_t$. We can compute a single snapshot FIM including this a priori information (SS-FIM-AP). For any $t>0$, $p_{\vartheta_t,y_t}(\vartheta_t, y_t) = p_{y_t/\vartheta_t}(y_t/\vartheta_t)p_{\vartheta_t}(\vartheta_t)$. A FIM $\tilde{J}_t$, can be written [6] which can be decomposed as $\tilde{J}_t = J_t^D + J_t^P$. The first term J_t^D represents the information obtained from the measured data, and J_t^P represents the a priori information. For our system we can recursively compute SS-FIM-AP as

$$\begin{aligned} P_0 &= E\vartheta_0\vartheta_0^T \\ P_{t+1} &= AP_tA^T + Q_t \\ J_{t+1} &= \mathfrak{J}_{t+1} + P_{t+1}^{-1} \end{aligned} \quad (7)$$

Actually, for a sequence of snapshots, the interest is on the sequence $\{J_t\}$ of FIMs for estimating state vectors $\{\vartheta_t\}$. It has been proved in [5] that the sequence $\{J_t\}$ obeys a recursion for the nonlinear filtering problem. Let Δ_z^x denote the operator $\nabla_z\nabla_x^T$, and $p_t = p(\Theta_t, Y_t)$, with $\Theta_t = (\vartheta_0,...\vartheta_t)$ and $Y_t = (y_0,...,y_t)$ for an arbitrary t. Then

$$J_{t+1}\, D_t^{22} - D_t^{21}\,(J_t + D_t^{11})^{-1}\, D_t^{12} \quad (8)$$

$$J_0 = E\,\{-\Delta_{\vartheta_0}^{\vartheta_0} \log p\,(\vartheta_0)\} \quad (9)$$

where

$$D_t^{11} = E\{-\Delta_{\vartheta_t}^{\vartheta_t} \log p\,(\vartheta_{t+1}/\vartheta_t)\} \quad (10)$$

$$D_t^{12} = E\{-\Delta_{\vartheta_t}^{\vartheta_{t+1}} \log p\,(\vartheta_{t+1}/\vartheta_t)\} \quad (11)$$

$$D_t^{21} = (D_t^{12})^T \quad (12)$$

$$D_t^{22} = E\{-\Delta_{\vartheta_{t+1}}^{\vartheta_{t+1}} \log p\,(\vartheta_{t+1}/\vartheta_t)\} + E\,\{-\Delta_{\vartheta_{t+1}}^{\vartheta_{t+1}} \log p\,(\vartheta_{t+1}/\vartheta_t)\} \quad (13)$$

For our linear dynamic system the recursion becomes

$$\begin{aligned} J_0 &= E\,\{-\Delta_{\vartheta_0}^{\vartheta_0} \log p\,(\vartheta_0)\} \\ J_{t+1} &= \mathfrak{J}_{t+1}\,(\vartheta_{t+1}) + Q_t^{-1} - (Q_t^{-1})^T A_t \\ &(J_t + A_t^T Q_t^{-1} A_t)^{-1} A_t^T Q_t^{-1} \end{aligned} \quad (14)$$

J_t will be called multiple snapshot FIM (MS-FIM). Observe that it includes the SS-FIM of the estimate vector at time t. If $m(\vartheta_t)$ was a linear function, the solution of the problem would become the well known Kalman filter.

$\tilde{J}_t$ includes the information from both the present measured data and the embedded dynamic evolution. J_t includes the information of $\tilde{J}_t$ and additionally the information from past measured data (beginning at $t = 1$). In other words, under the assumption of a linear stochastic dynamic model, $\tilde{J}_t$ represents the information given by only one snapshot at time t, and J_t represents the cumulative information from the previous t snapshots.

4. Simulation Experiments

In this section we present the results of simulation experiments for a single dynamic source with geometrical data from a real head. The MEG instrument modeled in this study is a helmet of $m_B = 160$ magnetic sensors by Biomagnetic Technologies Inc. (San Diego CA, USA). The EEG instrument is modeled as a cap with $m_E = 37$ electrodes. We consider a real head with three surfaces with 652 triangles in each, i. e., a total of 1956 tessellation triangles.

We consider typical values for the instrument noises, $\sigma_B = 3.5\cdot 10^{-14}T$ and $\sigma_E = 4\cdot 10^{-7}V$, to be used in $R = E\{ee^T\} = \text{diag}([\sigma_B^2 I_{m_B}\ \sigma_E^2 I_{m_E}])$.

For our example we take a trajectory motivated by [1, 4] in their studies of AER. The particular region of interest (ROI) is placed in the temporal lobe of the right cerebral hemisphere (see Fig. 1). We consider the initial point and 5 trajectory points with $\| \mathbf{p} \| \in$ ROI. These points come from a dynamic source with $A_t = 0.9 \cdot I_6$, and variances $\sigma_q^2 = 7 \cdot 10^{-18}\,(A \cdot m)^2$, $\sigma_p^2 = 7 \cdot 10^{-5}\,(m)^2$, $\sigma_{qo}^2 = 1 \cdot 10^{-18}\,(A \cdot m)^2$, $\sigma_{qo}^2 = 1 \cdot 10^{-5}\,(m)^2$, to be used in $Q = E\,(ww^T) = \text{diag}\,([\sigma_q^2 I_{m3}\, \sigma_p^2 I_{m3}])$ and $P_0 = E\,(\vartheta_0 \vartheta_0^T) = \text{diag}\,(\sigma_{qo}^2 I_{m3}\, \sigma_{po}^2 I_{m3}])$. We compute J_t for $t \in (1,...,5)$ using (14) for the given numerical values.

$C_t = J_t^{-1}$ can be compared with different references according to the a priori information we assume. The alternatives we propose are:

1. We do not have any a priori information (actually we surely know that the ECD is within the head and more precisely within the brain). In this case $C_{REF1} = \mathfrak{J}_t^{-1}$ any $t > 0$.
2. The parameters corresponding to any single snapshot are assumed to lie inside a ROI. This ROI is represented as the large oval inside the head in Fig. 2. In this case, the reference is computed according to the worst $\tilde{J}_t$. The reference C_{ROI} (for any $t > 0$) is equal to that $\tilde{J}_t^{-1}$ with the largest determinant in the interval $1 \le t \le 5$.
3. The dynamic evolution of the components is known but it is not used to improve the confidence of the measurements by tracking them. In Fig. 2, the dispersions for particular snapshots are represented by ovals around the particular points of the trajectory. In this case we compare C_t with $\tilde{J}_t^{-1}$ for any t > 0. We denote $\tilde{C}_t = \tilde{J}_t^{-1}$.

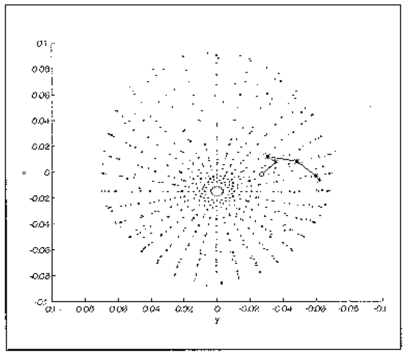

Fig. 1 Trajectory points on the brain (upper view).

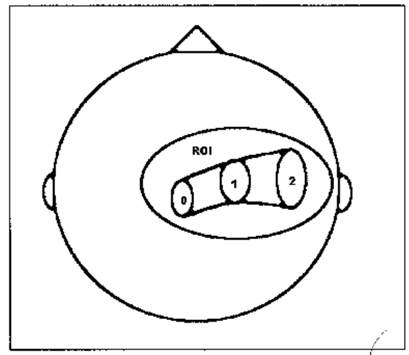

Fig. 2 Schematic of the regions of interest, the trajectory and its uncertainty.

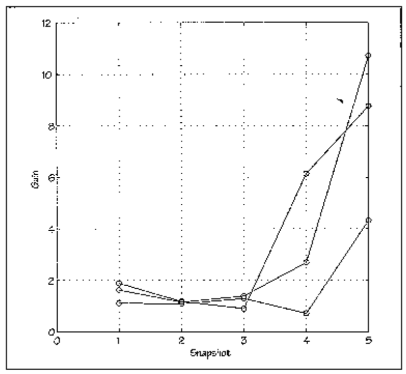

Fig. 3 C_t vs. C_{ROI}.

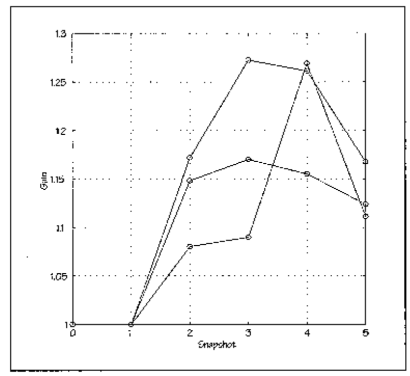

Fig. 4 C_t vs. $\tilde{C}_t$.

Results for the last two alternatives are presented in Fig. 3 and Fig. 4. In those figures we plot the gain G_t, defined as the pointwise ratio between the diagonal values of the corresponding CRB matrices

$$G_t = \text{diag}\,(\text{J}_{\text{REF}i}{}^{-1} ./ J_t^{-1}) \qquad (15)$$

In the figures, we only show the $\mathbf{p}$ components of G_t. Both, Fig. 3 and Fig. 4 make sense for values of $t \ge 1$.

Note that in Fig. 3, $C_{\text{ROI}} = \tilde{C}_2$. For the highest values of t we observe large gains. This is the kind of improvement one expects from considering all the previous information. But this increase is also due to the fact that the ECD is approaching more external regions where the E/MEG have better performance.

Figure 4 shows the gain when comparing C_t with $\tilde{C}_t$. We observe that the gain tends to decay for the last two snapshots. This could be due to the fact that the improvement coming from the tracking is not as large as the improvement coming from the fact that the ECD is approaching more external regions, i. e., the increment in $\mathfrak{J}_t$ improves both C_t and $\tilde{C}_t$.

5. Conclusions

Based on the results presented in [5] a Cramér-Rao error bound has been derived for the error in estimating the parameters of a dynamic dipole source with E/MEG measurements, assuming the realistic head model presented in [2]. As an example, it has been shown that the E/MEG error bounds for the estimation of the parameters of a multiple component brain source are reduced assuming a dynamic dipole model instead of just an arrangement of individual dipoles.

Our procedure is illustrated with an example patterned after some AER studies. As it might be expected, the additional information provided by the dynamic evolution model, contributed to lowering the CRB. In this way there is potential for improving current static, single snapshot based estimates of the source parameters.

However, in a more realistic context, it must be noticed that some of the parameters of the dynamic model are unknown and should be simultaneously estimated with the dipole parameters. This would increase the CRB, but this is still the subject of current investigations.

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