Passive Localization of Near-Field Sources by Path Following

David Starer and Arye Nehorai

***Abstract*—A new algorithm for passively estimating the ranges and bearings of multiple narrow-band sources using a uniform linear sensor array is presented. The algorithm is computationally efficient and converges globally. It minimizes the MUSIC cost function subject to geometrical constraints imposed by the curvature of the received wavefronts. The estimation problem is reduced to one of solving a set of two coupled 2-D polynomial equations. The proposed algorithm solves this nonlinear problem using a modification of the path-following (or homotopy) method. For an array having m sensors, the algorithm reduces the global 2-D search over range and bearing to $2(m-1)$ independent 1-D searches. This imparts a high degree of parallelism that can be exploited to obtain source location estimates very efficiently.**

I. Introduction

Various algorithms have been proposed for the task of passive source localization using sensor arrays, with some of the most notable recent advances having been made in the area of eigendecomposition-based methods. However, the majority of such algorithms have been restricted to localization of sources in the far-field and were only locally convergent. In this correspondence (see also [7] for more details), we present a new method of overcoming both of these limitations simultaneously using an eigendecomposition-based approach which is applicable with uniform linear arrays (ULA's).

Many multiple-source localization techniques make use of the plane-wave (or far-field) assumption in which the sources are assumed to be sufficiently distant that the waves emanating from them are essentially planar when they reach the array. If this assumption is justified, each source location is parameterized by bearing only. However, practical situations often arise where the sources are close to the array, and thus the waves impinging on it cannot be assumed to be planar. In such cases, the source locations cannot be parameterized in terms of their bearings only; the parameter vector must be extended to include the source ranges. The violation of the plane-wave assumption, and the increased dimension of the parameter vector, necessitates the use of more sophisticated localization algorithms than those applicable under the plane-wave assumption.

A conventional approach to estimating both ranges and bearings is to use active methods in which range can be determined from the time of flight of a radar or sonar pulse reflected from an object of interest. An obvious disadvantage of this approach is that the object may be able to detect that its position is being measured, and it may be able to use countermeasures to disrupt the measurement. In contrast, if the object of interest is radiating energy (i.e., if it is a source), then passive methods can measure the source's position using only the energy radiated. Such passive methods are, therefore, less susceptible to countermeasures. However, when using a single array, these methods usually base their range estimates on the curvature of the wavefronts emitted by the source. In the past, high-resolution range estimation using wavefront curvature has been a difficult problem.

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In this correspondence, we derive a new form of the MUSIC cost function [6] for passive estimation of ranges as well as bearings. The cost function utilizes the wavefront curvature and is valid for ULA's in the Fresnel region with sensors in the same plane as the sources. We propose minimizing this cost function using a modification of the path-following (or homotopy) method [3], which is globally convergent.

Unlike conventional methods, such as grid search algorithms, for global passive near-field source localization that require an exhaustive 2-D search, the algorithm presented here needs to solve only $2(m-1)$ independent 1-D function minimization problems, where m is the number of sensors in the array. The algorithm can, therefore, be implemented in parallel using $2(m-1)$ separate processors. This, potentially, yields a large increase in speed, which is limited only by the time taken to perform a single 1-D search.

A. Problem Formulation

The MUSIC algorithm [1] for locating n narrow-band radiating sources using N data snapshots gathered from an array of m sensors consists offinding the n parameter vectors θ that minimize the cost function

$$F(\theta) = a^H(\theta) W a(\theta) \tag{1.1}$$

where the superscript H denotes complex conjugate transposition and $W = \hat{G}\hat{G}^H$. The matrix $\hat{G}$ is the matrix of eigenvectors associated with the $m-n$ smallest eigenvalues of the data sample covariance matrix $\hat{R}$, where

$$\hat{R} = \frac{1}{N}\sum_{t=1}^{N} y(t)y^H(t). \tag{1.2}$$

The vector $y(t) \in \mathbb{C}^{m\times 1}$ is the data vector gathered from the array at snapshot t. The kth entry of the steering vector $a(\theta)$ is given by

$$a_k(\theta) = \exp\{-i\omega_0\tau_k(\theta)\} \tag{1.3}$$

where the ω_0 is the known center frequency of the narrow-band signals.

Assume the use of an arbitrary coordinate system and a uniform linear array with kth sensor placed a distance kd_0 from the origin. For a signal emanating from location θ, with θ parameterized in terms of a range r and a bearing ϕ, the difference $\tau_k(\theta)$ in time between the signal arriving at a sensor k and the signal arriving at the origin is

$$\tau_k(\theta) = \frac{r - l_k}{c_0} = \frac{2\pi}{\lambda_0\omega_0}(r - l_k) \tag{1.4}$$

where c_0 is the speed of propagation of the waves, and λ_0 is their wavelength. The distance l_k from location θ to the kth sensor is

$$l_k = r\sqrt{1 + \left(\frac{kd_0}{r}\right)^2 - 2\left(\frac{kd_0}{r}\right)\sin\phi}. \tag{1.5}$$

In the near-field case, MUSIC amounts to a 2-D search for the n bearings between $-\pi/2$ and $\pi/2$ and n ranges between zero and infinity which minimize $F(\theta)$ in (1.1). The usual plane-wave (or far-field) approximation, which is valid for sources with very large ranges, is obtained by retaining only terms up to the first power of

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(kd_0/r_j) in the binomial expansion of (1.5). In near-field cases, this approximation may not be valid, and a more accurate approximation is required.

Such an approximation is obtained by retaining terms up to the second power of (kd_0/r) in the binomial expansion of (1.5). This yields the so-called Fresnel approximation

$$l_k = r\left[1 - \left(\frac{kd_0}{r}\right)\sin\phi + \frac{1}{2}\left(\frac{kd_0}{r}\right)^2\cos^2\phi\right]. \quad (1.6)$$

The range error introduced by this approximation has been studied by numerical simulation in [8] where it was shown that for ranges greater than ten times the array length, the error introduced is less than 0.5%.

With the use of (1.6) and (1.4), the steering vector elements (1.3) can be written approximately as

$$\tilde{a}_k(\theta) = \mu^{k^2}\lambda^k \quad (1.7)$$

where

$$\lambda = \exp\{i\alpha_\lambda\}; \qquad \mu = \exp\{i\alpha_\mu\} \quad (1.8a)$$

and

$$\alpha_\lambda = -2\pi\frac{d_0}{\lambda_0}\sin\phi;$$
$$\alpha_\mu = \pi\frac{d_0^2}{\lambda_0 r}\cos^2\phi. \quad (1.8b)$$

Using the Fresnel region steering vector elements defined by (1.7), the MUSIC cost function (1.1) can be converted into the following 2-D polynomial:

$$\tilde{F}(\lambda,\mu) = \sum_{j=0}^{m-1}\sum_{k=0}^{m-1} w_{jk}\lambda^{k-j}\mu^{k^2-j^2} \quad (1.9a)$$

where w_{jk} is the $j+1, k+1$ entry of $W \triangleq \hat{G}\hat{G}^H$, that is

$$w_{jk} = [\hat{G}\hat{G}^H]_{j+1,k+1} \quad j,\quad k = 0,\cdots,m-1. \quad (1.9b)$$

The cost function to be minimized for near-field source localization is now the 2-D polynomial (1.9a). Once the n unit-modulus values of λ and μ are found which minimize (1.9a), the ranges and bearings can be calculated directly using (1.8a) and (1.8b). Note that these values of λ and μ are not the roots of $\tilde{F}(\lambda,\mu)$ in (1.9a) because they are points constrained to lie on the unit circle, whereas the "roots" of $\tilde{F}(\lambda,\mu)$ are not isolated points but rather form a manifold in a 4-D space.

Remark: The conventional far-field Root-MUSIC cost function [1] is a special case of our more general expression (1.9a). To see this, observe that for the far-field approximation where $\mu = 1$, (1.9a) reduces to the usual 1-D polynomial form of Root-MUSIC:

$$\tilde{F}(\lambda,1) = \sum_{j=0}^{m-1}\sum_{k=0}^{m-1} w_{jk}\lambda^{k-j} = \sum_{k=-m}^{m} w_k'\lambda^k \quad (1.10)$$

where the coefficients w_k' are the sums along the diagonals of $\hat{G}\hat{G}^H$. □

II. Global Minimization of the Cost Function

In this section, we derive a simple, computationally efficient, globally convergent algorithm for minimization of $\tilde{F}(\lambda,\mu)$. Differentiating (1.9a) with respect to α_λ and α_μ and setting these gradients equal to zero gives

$$p(\alpha_\lambda|\alpha_\mu) \triangleq \frac{\partial\tilde{F}(\lambda,\mu)}{\partial\alpha_\lambda} = i\sum_{j=0}^{m-1}\sum_{k=0}^{m-1}\cdot(k-j)w_{jk}\lambda^{k-j}\mu^{k^2-j^2} = 0, \quad (2.1a)$$

$$q(\alpha_\mu|\alpha_\lambda) \triangleq \frac{\partial\tilde{F}(\lambda,\mu)}{\partial\alpha_\mu} = i\sum_{j=0}^{m-1}\sum_{k=0}^{m-1}\cdot(k^2-j^2)w_{jk}\lambda^{k-j}\mu^{k^2-j^2} = 0. \quad (2.1b)$$

Solving this system of polynomial equations for values of λ and μ on the unit circle enables values of bearing ϕ and range r to be calculated directly from (1.8a) and (1.8b).

Observe that α_λ is obtained by solving (2.1a) for given α_μ, and that α_μ is obtained by solving (2.1b) for given α_λ. For each α_μ, there exist $2(m-1)$ roots λ of $p(\alpha_\lambda|\alpha_\mu)$. Hence, the roots of $p(\alpha_\lambda|\alpha_\mu)$ describe a set of $2(m-1)$ curves of α_λ as a function of α_μ. Similarly, the roots of $q(\alpha_\mu|\alpha_\lambda)$ describe a set of $2(m-1)^2$ curves of α_μ as a function of α_λ. The curves satisfying $p(\alpha_\lambda|\alpha_\mu) = 0$ will be termed λ-paths, and the curves satisfying $q(\alpha_\mu|\alpha_\lambda) = 0$ will be termed μ-paths.

The intersections of the λ-paths and μ-paths represent the points where both gradient polynomials are zero simultaneously and therefore represent local turning points (maxima, minima, or saddle points) of the cost function. The local turning points which correspond to minima of $\tilde{F}(\lambda,\mu)$ represent the candidate estimates of source ranges and bearings. The algorithm to be described locates *all* such turning points and then selects only those n absolute minimum points as the desired source locations.

The algorithm follows the λ-paths as α_μ is incremented in small steps from zero to $\alpha_{\mu_{\max}}$. Instead of following both the λ-paths and the μ-paths, the algorithm saves computations by following only the λ-paths and by detecting any intersections with μ-paths. It follows the λ-paths rather than the μ-paths because there are fewer λ-paths than μ-paths. Each λ-path can be tracked by its own independent root-tracking processor. All $2(m-1)$ such processors can operate in parallel. The steps of the overall near-field source localization algorithm are given in Table 1, and in more detail in [7]. In outline, it proceeds as follows:

Initialization: The search is started from $\alpha_\mu = 0$ (i.e. $\mu = 1$). From (1.8b), this is equivalent to imposing the far-field approximation. Under this condition, $p(\alpha_\lambda|0)$ is a 1-D polynomial whose roots can be found using standard factorization methods. Thus, factorization of $p(\alpha_\lambda|0)$ provides the starting points on the λ-paths.

Path Following: The λ-paths are followed as a function of α_μ using a predictor-corrector method in which the positions of the paths at the next discrete step of α_μ are predicted using an Euler predictor, and the predicted values are then corrected using a Newton corrector. Fig. 1 illustrates this idea.

Detecting Intersections: In order to detect intersections with μ-paths, $q(\alpha_\mu|\alpha_\lambda)$ is evaluated at each step along the λ-paths. An intersection of a λ-path and a μ-path is detected whenever the sign of q changes. Specifically, a local minimum is detected when $q(\alpha_\mu^{(l)}|\alpha_\lambda^{(l)}) \leq 0$ and $q(\alpha_\mu^{(l+1)}|\alpha_\lambda^{(l+1)}) \geq 0$ where the superscript in parenthesis denotes the step number along the λ-paths.

Local Minimization: Whenever a local minimum is detected, the midpoint between two steps straddling the local minimum is used as the initial parameter vector in an efficient, gradient-based local search to find the exact candidate source location.

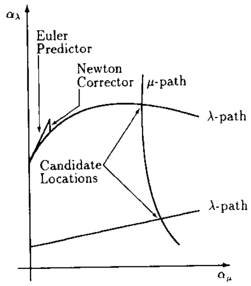

Fig. 1. Near-field source localization by path following.

Path Termination: The path-following algorithm is terminated when α_μ reaches an appropriately chosen upper limit. Since $\alpha_\mu = \pi d_0^2 \cos^2\phi/\lambda_0 r$, increasing α_μ slowly from zero corresponds to searching for sources starting at infinite range and then moving closer to the array. A suitable minimum range beyond which searching is meaningless could (for example) be chosen to be a small range of π wavelengths. For sources close to broadside ($\phi \approx 0$), this corresponds to a maximum value of 0.25 for α_μ.

Global Minima Selection: After all local minima have been located and the corresponding values of $\tilde{F}(\lambda, \mu)$ evaluated, the n values of α_λ and α_μ which minimize the cost function are chosen as the source locations. The corresponding bearings and ranges are computed using (1.8b).

A. Computations for Path-Following Method

At each step of the path-following algorithm, each processor finds the value of α_λ which satisfies

$$p(\alpha_\lambda|\alpha_\mu) = 0 \tag{2.2}$$

using Euler prediction followed by Newton Correction.

The Euler method predicts $\alpha_\lambda(\alpha_\mu + \Delta\alpha_\mu)$ using the recursion

$$\alpha_\lambda(\alpha_\mu + \Delta\alpha_\mu) = \alpha_\lambda(\alpha_\mu) + \frac{d\alpha_\lambda}{d\alpha_\mu}\Delta\alpha_\mu. \tag{2.3}$$

In order to find the term $d\alpha_\lambda/d\alpha_\mu$ in (2.3), regard $p(\alpha_\lambda|\alpha_\mu)$ as a function of α_λ, and regard α_λ, in turn, as a function of α_μ. Then differentiate (2.2) with respect to α_μ to give

$$\frac{dp(\alpha_\lambda|\alpha_\mu)}{d\alpha_\mu} = \frac{\partial p(\alpha_\lambda|\alpha_\mu)}{\partial\alpha_\lambda}\frac{d\alpha_\lambda}{d\alpha_\mu} + \frac{\partial p(\alpha_\lambda|\alpha_\mu)}{\partial\alpha_\mu} = 0. \tag{2.4}$$

Thus, the gradient to be used in (2.3) is

$$\frac{d\alpha_\lambda}{d\alpha_\mu} = -\frac{\partial p(\alpha_\lambda|\alpha_\mu)}{\partial\alpha_\mu}\bigg/\frac{\partial p(\alpha_\lambda|\alpha_\mu)}{\partial\alpha_\lambda}. \tag{2.5}$$

Discretization errors inevitably occur using the Euler prediction, so a correction step based on Newton's method is used to place the estimate of α_λ back on the λ-path. For given α_μ, the value of α_λ satisfying (2.2) is corrected using the Newton recursion

$$\alpha_\lambda^{(k+1)} = \alpha_\lambda^{(k)} - \left[\frac{\partial p(\alpha_\lambda|\alpha_\mu)}{\partial\alpha_\lambda}\right]^{-1} p(\alpha_\lambda|\alpha_\mu). \tag{2.6}$$

Further details regarding this Newton correction are given in [7]. Note that this correction is a 1-D update (i.e., the update involves scalars only). Many stopping criteria can be used to decide when to terminate the Newton correction (see e.g., [2]). However, in practice, termination after one or two interactions has been found to be sufficient.

In order to ensure that no intersection points are overlooked, (e.g., by stepping over more than one intersection) the step size in moving from $\alpha_\mu = 0$ to $\alpha_\mu = 0.25$ should be sufficiently small. In practice, the algorithm appears to be robust, and ten equally spaced steps have proved to give good results for the cases considered in Section III. Alternatively, it is possible to detect intersection points using the Argument theorem of complex analysis.

Once an intersection of λ- and μ-paths corresponding to a local minimum is detected, the algorithm performs a local search to find the exact intersection or parameter vector. Any gradient-based method, such as Newton's method for example, can be used to find the parameter vector $\theta = [\alpha_\lambda, \alpha_\mu]^T$, which minimizes $\tilde{F}(\lambda, \mu)$ locally. Further details regarding the Newton algorithm for local minimization appear in [7].

Observe that *a priori* knowledge concerning the sources' ranges can be exploited by varying α_μ between restricted limits corresponding to the known minimum and maximum ranges.

The total computational burden imposed by the algorithm is approximately $2n_s m^3$ flops, where n_s is the number of steps taken along each path. However, if, as recommended, $2(m-1)$ parallel processors are used, with each processor dedicated to following a single path, then each processor is required to perform only approximately $n_s m^2$ flops. The only other known globally convergent algorithm for this application is a multidimensional grid search. For a grid search to be effective in minimizing multimodal cost functions and to achieve maximum accuracy, the grid size must be chosen to be less than the square root of the Cramér–Rao lower bound, and this is proportional to N^{-1}. The cost function must be evaluated at each grid point. Thus, the number of grid points is proportional to N^2, and can become very large. In contrast, it has been shown by numerical examples [7] that the mean-square error of the algorithm described in this correspondence is close to the Cramér–Rao lower bound and, as explained above, is independent of N. The number of computations required by this new algorithm is, therefore, far less than the number required by a grid search.

For any given α_μ, it is possible for (2.1a) to have roots λ which are not on the unit circle. In practice, this situation has been found to occur extremely rarely. However, if it is desired to accommodate this unlikely possibility, then λ could be parameterized in terms of a radius and an angle, and the paths of these parameters could be followed as α_μ is incremented.

III. Simulation Example

To demonstrate its operation, the algorithm was used to estimate the locations of two sources using a uniform linear array consisting of five identical sensors spaced half a wavelength apart. The 3dB beamwidth of this array was calculated to be 20.5° using equation (IIB-57) in [4]. The coordinate system was chosen to have its origin placed at one end of the array, and its x-axis aligned with the array. The sources were located within one beamwidth at bearings of $+5°$ and $+17°$ from the normal to the array at ranges of 20 and 15 wavelengths from the origin respectively. The sources emitted independent random narrow-band complex Gaussian waveforms, and the signal-to-noise ratio was 3dB per source measured at the array. The data sample covariance matrix was computed from 256 snapshots. The path following algorithm of Fig. 2 was implemented with α_μ incremented in ten equal steps from 0 to 0.25.

Fig. 3 shows a typical set of results. The lines in the figure indicate the paths of α_λ satisfying $p(\alpha_\lambda|\alpha_\mu) = 0$ as a function of α_μ (i.e., the λ-paths that were tracked). The stars indicate the set of intersections between λ-paths and μ-paths corresponding to local minima which were detected by monitoring the value of $q(\alpha_\mu|\alpha_\lambda)$ while following the λ-paths.

The set of local minima represents the candidate parameter vectors. For the particular realization shown in Fig. 3, three candidate

- Initialize m, n, $\hat{G}$, $\Delta\alpha_\mu$ and $\alpha_{\mu_{max}}$;
- Compute initial values α_{λ_k}, $k = 1, \ldots, 2(m-1)$, that satisfy $p(\alpha_{\lambda_k} | \alpha_\mu = 0) = 0$;
- Evaluate and store $q(\alpha_\mu = 0 | \alpha_{\lambda_k})$, $k = 1, \ldots, 2(m-1)$;
- For $k := 1$ to $2(m-1)$ do {follow $2(m-1)$ λ-paths in parallel}
  begin
  - ∘ $\alpha_\mu := 0$;
  - ∘ while $\alpha_\mu < \alpha_{\mu_{max}}$ do
    begin
    - · Predict $\alpha_{\lambda_k}(\alpha_\mu + \Delta\alpha_\mu)$ using the Euler predictor (2.3);
    - · Correct $\alpha_{\lambda_k}(\alpha_\mu + \Delta\alpha_\mu)$ using the Newton corrector (2.6);
    - · Evaluate and store $q(\alpha_\mu + \Delta\alpha_\mu | \alpha_{\lambda_k})$;
    - · If $q(\alpha_\mu + \Delta\alpha_\mu | \alpha_{\lambda_k})) \geq 0$ and $q(\alpha_\mu | \alpha_{\lambda_k}) \leq 0$ do
      begin
      $\theta := \frac{1}{2}\left\{[\alpha_{\lambda_k}(\alpha_\mu + \Delta\alpha_\mu),\ \alpha_\mu + \Delta\alpha_\mu]^T + [\alpha_{\lambda_k}(\alpha_\mu),\ \alpha_\mu]^T\right\}$;
      Minimize $\tilde{F}(\lambda, \mu)$ locally using a gradient-based algorithm;
      Store the minimizing θ and $\tilde{F}(\lambda, \mu)$
      end;
    - · $\alpha_\mu := \alpha_\mu + \Delta\alpha_\mu$;

    end;

  end;
- Choose n distinct values θ which globally minimize $\tilde{F}(\lambda, \mu)$;
- Compute n bearings and ranges using (1.8b).

Fig. 2. Path-folowing algorithm for near-field source localization.

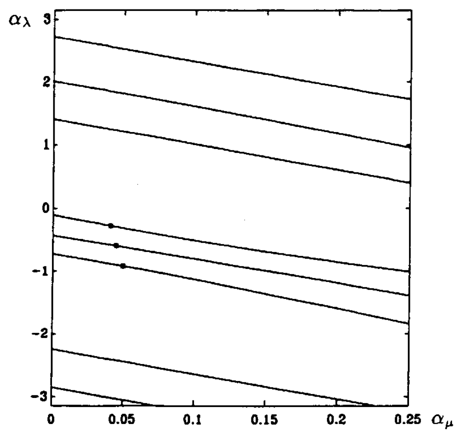

Fig. 3. Near-field source localization map for the simulation example.

TABLE I
NEAR-FIELD CANDIDATE LOCATION PARAMETERS FOR THE SIMULATION EXAMPLE

α_λ	Bearing	α_μ	Range	$\tilde{F}(\lambda, \mu)$
−0.275684	5.03	0.038799	20.1	0.000115
−0.915290	16.9	0.048441	14.8	0.000097
−0.594345	10.9	0.043353	17.4	0.031960

parameter vectors were located and their values are tabulated in Table I. It is clear that the $n = 2$ candidates minimizing the cost function $\tilde{F}(\alpha, \mu)$ are the first two in Table I. These have bearings of $5.03°$ and $16.9°$ and ranges of 20.1 and 14.8 wavelengths respectively which compare well with the true location parameters given above.

IV. CONCLUSION

A new algorithm has been presented for passive localization of near-field sources using uniform linear arrays. Using the Fresnel approximation, the MUSIC cost function has been shown to reduce to a 2-D polynomial. It was shown that the popular Root-MUSIC cost function [1] is a special case of the cost function derived in this paper. Minimization of the cost function in the near-field case was shown to be obtained by finding the roots of a set of two 2-D gradient polynomials. An algorithm based on path following was proposed for finding the roots of this set of polynomials. The algorithm is globally convergent and is computationally efficient. It makes use of the basic ideas of path following, but requires only $2(m-1)$ paths to be followed. Furthermore, each path can be tracked by its own independent processor, and all $2(m-1)$ such processors can operate in parallel to give a large increase in speed.

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Improving Resolution for Array Processing by New Array Geometry and Spatial Filter

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***Abstract*— A new array processing method based on a new array geometry and a spatial filter to improve the resolution is presented. The array aperture can be increased by the new array geometry. The spatial frequency aliasing can be removed and the effective signal-to-noise (SNR) can be raised by the spatial filter. Simulation results are presented to illustrate that the performance obtained by the new method is much better than that obtained by the existing methods.**

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