From the textbook, “Probability and Random Processes for Electrical and Computer Engineers” by John A. Gubner, first edition, do the following 8 problems from the problem section for Chapter 1 starting on page 48 (each problem is worth 5 points):

1. Problems 17, 31, 38, 43, 44, 56, 68 and 69

Also, do the following problem worth 10 points:

2. Recall that the \emph{limes superior} and \emph{limes inferior} of a sequence of sets are defined as

\[ \limsup A_n = \bigcap_{i=1}^{\infty} \left( \bigcup_{j=i}^{\infty} A_j \right) \quad \text{and} \quad \liminf A_n = \bigcup_{i=1}^{\infty} \left( \bigcap_{j=i}^{\infty} A_j \right). \]

Equivalently,

\[ \limsup A_n = \bigcap_{i=1}^{\infty} B_i \quad \text{where} \quad B_i = \bigcup_{j=i}^{\infty} A_j \]

and

\[ \liminf A_n = \bigcup_{i=1}^{\infty} C_i \quad \text{where} \quad C_i = \bigcap_{j=i}^{\infty} A_j. \]

Explain in detail that the following characterizations are equivalent to these definitions:

\[ \limsup A_n = \{ x \in \mathbb{R} : x \in A_n \text{ for infinitely many sets } A_n \} \]
\[ \liminf A_n = \{ x \in \mathbb{R} : x \in A_n \text{ for all but finitely many sets } A_n \}. \]

(Note: “all but finitely many sets” means with the possible exception of a finite number of sets.)

For \( n \in \mathbb{N} = \{1, 2, \ldots\} \) define the following sets:

\[ A_{2n} = \left\{(x, y) \in \mathbb{R}^2 : \left(x - \frac{1}{n}\right)^2 + y^2 \leq 1 \right\} \]

and

\[ A_{2n-1} = \left\{(x, y) \in \mathbb{R}^2 : \left(x + \frac{1}{n}\right)^2 + y^2 \leq 1 \right\}. \]

Determine \( \limsup A_n \) and \( \liminf A_n \) for this example.