Prospects for quantitative computed tomography imaging in the presence of foreign metal bodies using statistical image reconstruction

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X-ray computed tomography (CT) images of patients bearing metal intracavitary applicators or other metal foreign objects exhibit severe artifacts including streaks and aliasing. We have systematically evaluated via computer simulations the impact of scattered radiation, the polyenergetic spectrum, and measurement noise on the performance of three reconstruction algorithms: conventional filtered backprojection (FBP), deterministic iterative deblurring, and a new iterative algorithm, alternating minimization (AM), based on a CT detector model that includes noise, scatter, and polyenergetic spectra. Contrary to the dominant view of the literature, FBP streaking artifacts are due mostly to mismatches between FBP's simplified model of CT detector response and the physical process of signal acquisition. Artifacts on AM images are significantly mitigated as this algorithm substantially reduces detector-model mismatches. However, metal artifacts are reduced to acceptable levels only when prior knowledge of the metal object in the patient, including its pose, shape, and attenuation map, are used to constrain AM's iterations. AM image reconstruction, in combination with object-constrained CT to estimate the pose of metal objects in the patient, is a promising approach for effectively mitigating metal artifacts and making quantitative estimation of tissue attenuation coefficients a clinical possibility. © 2002 American Association of Physicists in Medicine. [DOI: 10.1118/1.1509443]

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I. INTRODUCTION

Metallic and other dense objects are often placed in the human body, including implants and applicators used in brachytherapy, cochlear simulators and receivers,1 implantable orthopedic appliances, surgical clips and staples, and dental restorations. These high-atomic number, high-density objects attenuate x rays in the diagnostic energy range much more strongly than soft tissue or bone so that far fewer photons traversing these objects reach the detectors. This creates strong artifacts (see Fig. 1) in the form of dark and bright streaks spread across the entire image reconstructed with conventional filtered backprojection (FBP) in x-ray computed tomography (CT) images. These artifacts can mask soft tissue structures not just in the immediate vicinity of the dense object, but also throughout the entire image, rendering it of limited use.

In brachytherapy, the severe streaking artifacts arising from commercial intracavitary applicators make accurate segmentation of bladder and rectal walls, uterus, cervix, and small bowel difficult if not impossible.2,3 These appliances have transverse-plane dimensions as large as 3 cm, have stainless steel walls of 0.5 to 1.0 mm thickness, and often contain thicker stainless steel or brass components. Many vaginal colpostats contain 3 to 5 mm thick tungsten-alloy shields intended to reduce bladder and rectal doses. This dose sparing4 is considered essential by many practitioners to maintain normal tissue injury at acceptable levels. Attempts to circumvent metal artifacts on CT images include the use of plastic unshielded applicators5 and use of bone-imaging window and level settings,2 which diminish visualization of soft-tissue organ boundaries. Also, custom-made Fletcher–Suit applicators with afterloading tungsten shields can be fabricated.6 However, limiting local shielding to afterload-
able configurations, solely to obtain good image quality, severely compromises the utility of internal shielding as an optimization strategy. New low-energy\textsuperscript{7–9} brachytherapy sources, such as Am-241, Yb-169, and Sm-145 have been investigated, in part, because of the ease with which sensitive tissues can be shielded by means of flexible high-atomic number foils. Thus, therapeutic outcome may be improved by customizing shield design for each patient so as to achieve an optimal dose distribution.\textsuperscript{10} Metal artifacts also hinder CT imaging for other types of brachytherapy procedures, including template-guided interstitial implants using metal needles,\textsuperscript{11} permanent prostate implants using metallic I-125 or Pd-103 seeds,\textsuperscript{12} and head and neck implants in the presence of dental prostheses. Clearly, successful mitigation of severe streaking artifacts through improved image reconstruction or processing techniques would be of significant value, not only for image-based brachytherapy planning, but for many other applications of CT imaging as well.

Fig. 1. Image resulting from application of our FBP algorithm to measured sinogram data extracted from a Siemens Somatom Plus 4 scanner. The outer Lucite annulus was omitted from our simulations. The simulation phantom (Ref. 42) contains steel, brass, Teflon, and Al rods.
Most published analyses of FBP metal artifacts assume that such artifacts are due to missing data, i.e., gaps in the transmission sinogram arising from near-complete attenuation of the x-ray beam by foreign metal bodies present in the patient. According to this view, metal artifact mitigation involves identifying regions of the sinogram exhibiting high attenuation followed by processing to approximately replace the lost information. Information replacement strategies include “rubout,” linear interpolation, wavelet interpolation, and adaptive filtering. Such techniques can reduce streaking, but are less effective for complex objects in anatomy and do not ensure quantitative values from replaced sinogram regions. As pointed out by Medoff, successful reconstruction of incomplete data requires a priori knowledge to deal with “null spaces” in the sinogram.

Iterative reconstruction algorithms constitute another strategy for mitigating metal artifacts. Such algorithms are able to incorporate prior knowledge, including the geometry, composition, and location of the metal objects in the patient, as constraints on the reconstructed images. Using a deterministic iterative deblurring technique developed by our group, Wang demonstrated substantial reduction of streaking artifacts in the presence of opaque, convex objects of known location. Our more general approach, called “object-constrained CT (OCCT),” accommodates highly attenuating objects of arbitrary geometry and unknown location by combining estimation of metal object pose with iterative reconstruction of the surrounding tissue attenuation map in a unified framework.

Model-based image reconstruction techniques reject the hypothesis that metal streaking artifacts are solely or even primarily due to missing information. Rather, these methods assume that metal artifacts, indeed all artifacts, arise from discrepancies between the actual CT signal formation process and the model assumed by the reconstruction algorithm. Despite the dominance of the “missing information” view of metal artifacts, a significant literature exists demonstrating that measurement artifacts such as noise, scatter, beam hardening, and high contrast edge effects cause streaks to appear in images reconstructed by FBP. An important early effort to reduce detector model mismatches for transmission CT was the convex algorithm proposed by Lange and Carson. Their iterative expectation-maximization (EM) algorithm was based on the assumption that CT transmission measurement is an inherently stochastic process that is described by the Poisson distribution. More recently, iterative reconstruction algorithms have been proposed that include the scanner x-ray spectrum in their data models and therefore accommodate beam hardening. Our group has developed a novel iterative alternating minimization algorithm, hereafter denoted as AM, which includes background events, e.g., scattered radiation, as well as beam hardening and measurement noise, in its underlying signal formation model.

From the model-based image formation perspective, FBP is a discrete implementation of an analytic model, which assumes that CT transmission measurements are noiseless, and are linear functions of the attenuation line integrals along the corresponding primary photon trajectories through the patient. When scanning subjects comprised of only anatomically native materials under normal conditions, relatively simple corrections to the raw sinogram are sufficient to assure that these assumptions are at least approximately true, resulting in FBP images free of visually obvious artifacts. However, in regions shadowed by highly attenuating objects, noise and nonlinear detector responses due to scatter and spectral hardening dramatically increase, giving rise to pronounced streaking artifacts. The hypothesis of this paper is that image reconstruction algorithms derived from signal acquisition models accounting for these nonlinear effects will yield higher quality images in the presence of metal objects.

In this paper, the performance of three algorithms—conventional FBP, Snyder’s deterministic iterative deblurring (IDB), and O’Sullivan’s model-based iterative algorithm (AM)—is compared when metal objects are present in the scanning subject. Numerical experiments are performed using simulated sinogram data in which the level of noise, spectral polychromaticity and scatter are controlled. Our results clearly demonstrate that metal streaking artifacts on FBP images are dominated by photon scatter, beam hardening, and propagation of noise, arising from low-count sinogram regions, rather than by missing information. In addition, we show that the model-based AM algorithm substantially reduces streaking artifacts, demonstrating the importance of basing image reconstruction upon a realistic model of the CT signal formation process. However, use of prior knowledge of metal geometry to constrain iterative image reconstruction is also shown to be necessary to adequately mitigate streaking artifacts. Finally, we show that AM has the potential to support quantitatively accurate in vivo measurements of tissue attenuation coefficients within patients. This has important implications for radiotherapy treatment planning of low-energy photon brachytherapy and other treatment modalities, which produce dose distributions sensitive to tissue composition heterogeneities.

II. METHODS AND MATERIALS

A. Problem definition

Figure 2 illustrates a typical third-generation CT gantry. The thin fan-shaped x-ray beam traverses the patient and is opposed by a detector array, which measures the photon fluence emerging from the patient. Let X denote the set of discrete spatial positions (pixels) x in the patient supporting the attenuation coefficient-valued image, \( \mu(x,E) \). The data, \( d(y) \), acquired by the detector array are indexed by a vector of discrete values, \( y=(\beta,\gamma) \), where \( \beta \) denotes the source position and \( \gamma \) denotes the detector location in the gantry. The set \( \{d(y) : y \in Y\} \) is called the sinogram. The mean fluence sinogram, \( g(y) \) can be modeled by
where $I_0(y,E)$ is the source intensity (the number of photons of energy $E$ detected by the detector at $y$ in the absence of an attenuating subject); $h(y|x)$ is the scanner’s point-spread function; $\mu(x,E)$ is the linear attenuation coefficient (in units of $\text{mm}^{-1}$) of the patient at position $x$ and energy $E$; and the function $\sigma(y)$ denotes the mean number of background events due to scattered photon fluence and extrafocal radiation associated with the radiation field. $g(y)$ denotes the mean of all possible independent measurements, $d(y)$, which are assumed to be Poisson random variables. If the data are noiseless, then $d(y) = g(y)$ for known $\mu$ and $\sigma$. The point-spread function describes the attenuating effect of voxel $x$ on the photons incident upon the detector-source pair $y$ and includes such effects as finite detector size and voxel discretization. The dimensionless quantity $m(y,E)$ denotes the mean photon attenuation (in units of mean-free path) along the straight-line photon trajectory, $y$.

\begin{equation}
g(y;\mu) = \sum_{E \neq 0} I_0(y,E) e^{-m(y,E)} + \sigma(y),
\end{equation}

where $m(y,E) = \sum_x h(y|x) \mu(x,E)$.

**B. Iterative deblurring algorithm**

Deterministic iterative deblurring, or DB, assumes that the measured data are noiseless, scatter-free, and arise from a monoenergetic source; i.e., $I_0(y,E) = I_0(y) \delta_{E,E_0}$. The CT reconstruction problem is formulated as follows: find the attenuation image, $\mu(x)$, that yields the predicted sinogram, $b(y;\mu) = \sum_{x} h(y|x) \mu(x)$, that most closely approximates its measured counterpart, $a(y) = -\ln(d(y)/I_0(y))$. Iterative deblurring uses Csiszár’s $I$ divergence to quantify the discrepancy between $a(y)$ and $b(y;\mu)$.

\begin{equation}
I(a\|b) = \sum_{y \in Y} a(y) \ln \left( \frac{a(y)}{b(y;\mu)} \right) - \sum_{y \in Y} \left[ a(y) - b(y;\mu) \right].
\end{equation}

Unlike the more common least squares discrepancy measure, Eq. (2) is the only discrepancy measure which satisfies Csiszár’s axioms for rationally choosing the “best” solution to an inverse linear problem when both $a$ and $b$ are nonnegative.\textsuperscript{35} For any positive initial estimate, $\{\hat{\mu}_0(x), x \in X\}$, of the image, the sequence of progressively updated images $\{\hat{\mu}_k(x), x \in X, k=1,2,...\}$,

\begin{equation}
\hat{\mu}_{k+1}(x) = \hat{\mu}_k(x) \frac{1}{H_0(x)} \sum_{y \in Y} \left( \frac{h(y|x)}{\sum_{y' \in X} h(y'|x')} \hat{\mu}_k(x') \right) a(y)
\end{equation}

converges to an image $\{\hat{\mu}(x), x \in X\}$ that minimizes $I(a\|b)$.\textsuperscript{18} Where $H_0(x) = \sum_y h(y|x)$, Equation (3) has the same form as the expectation-maximization algorithm used in emission tomography, which finds the solution that maximizes likelihood of the measured emission rates, assuming Poisson photon emission sources.\textsuperscript{36,37} However, the form of (3) is imposed by the discrepancy measure (2), not by the random processes associated with the CT measurement process.

**Object-constrained CT (OCCT)** refers to the general approach, developed by our group,\textsuperscript{20} for using known attenuation maps of metal applicators and other metal bodies of known geometry to constrain the iterative tissue attenuation map solution. We consider the image to be composed of two components $\mu(x) = \mu_b(x) + \mu_a(x;\theta)$, where $\mu_b(x)$ is the unknown attenuation map of the patient’s body and $\mu_a(x;\theta)$ is the known map of the applicator at the pose, $\theta$ (location and orientation in the patient), which is assumed to be unknown. Let $X_a(\theta)$ denote the subset of $X$ that supports the applicator at pose $\theta$. Then, $\mu_b(x) = 0$ for $x \in X_a(\theta)$, and $\mu_a(x;\theta) = 0$ for $x \in X - X_a(\theta)$. Our initial OCCT implementation sought to identify the body attenuation map, $\mu_b(x)$, and applicator pose, $\theta$, that together minimize the $I$ divergence $I(a\|\Sigma_x h(\cdot|x) \mu(x))$. Briefly, the OCCT method\textsuperscript{20} is initialized by a first guess of the body attenuation, $\hat{\mu}_b^{(0)}(x)$, and an estimated applicator pose, $\hat{\theta}^{(0)}$. Iterations commence from these initial estimates, with the $(k+1)$th estimate obtained from the $k$th estimate, $\hat{\mu}_b^{(k)}(x) = \hat{\mu}_b^{(k+1)}(x) + \dot{\mu}_a(x;\hat{\theta}^{(k)})$, by performing an iterative update, defined by Eq. (3), yielding an intermediate function $\hat{\mu}_b^{(k+1)}(x), x \in X$. This intermediate function is then modified by replacing its values for $x \in X_a(\theta)$ by $\mu_b(x;\theta)$ for a candidate next pose $\theta$, yielding a $\hat{\mu}_b^{(k+1)}(x;\theta) = \hat{\mu}_b^{(k+1)}(x), x \in X - X_a(\theta)$, and $\hat{\mu}_b^{(k+1)}(x;\theta) = \mu_a(x;\theta)$, for $x \in X_a(\theta)$, and $\theta$. A local search is then performed in the neighborhood of the current pose $\hat{\theta}^{(k)}$ to find a $\hat{\theta}^{(k+1)}$ that reduces the $I$ divergence $I(a\|\Sigma_x h(\cdot|x) \hat{\mu}_b^{(k+1)}(x;\theta))$. These steps are then repeated until convergence is achieved. A more complete account, including handling of partially occupied pixels near the applicator boundary, can be found elsewhere.\textsuperscript{20}
C. The alternating minimization algorithm

Alternating minimization, a family of iterative algorithms developed by our group, is able to utilize prior information concerning the geometry of foreign metal bodies, but unlike IDB, is not limited by the simple monoenergetic, noiseless, and scatter-free measurement model of CT x-ray transmission measurement. We assume that the attenuation coefficient of each voxel in the patient can be represented by a linear combination of a small number, \( N \), of constituent substances, e.g., bone and fat equivalent basis vectors,

\[
\mu(x,E) = \sum_{i=1}^{N} \mu_i(E)c_i(x) = \mu(E) \cdot c(x),
\]

(4)

where \( c_i(x) = \rho_i(x)/\rho_i' \) is the partial specific gravity of the \( i \)th constituent of voxel \( x \); and \( \mu_i(E) \) and \( \rho_i' \) are the linear attenuation coefficient at photon energy \( E \) and mass density, respectively, of that constituent in pure form. Substituting Eq. (4) into (1), the mean of all possible measured sinograms becomes

\[
g(y;c) = \sum_{E} I_0(y,E) e^{-\sum_{E} \rho(h(x)|y,E)\mu_i(E)c_i(x)},
\]

(5)

In contrast to (1), the index \( E \) in Eq. (5) ranges over the dummy energy \( E = 0 \), as well as the non-zero discrete energy values of the spectrum, where by definition \( I_0(y,0) = \sigma(y) \) and \( \mu_i(0) = 0 \). The quantity \( I_0(y,E) \) is assumed to be known. We further assume that the measured \( d(y) \) are randomly distributed about the average value, \( g(y) \), according to the Poisson distribution. Thus, the probability of observing a measured sinogram, \( d(y) \), given an image of partial specific gravities, \( c(x) \), is

\[
P(d|c) = \prod_{y} e^{-g(y;c)} \frac{g(y;c)^{d(y)}}{d(y)!}.
\]

(6)

The problem to be solved by AM is to find the vector-valued image, \( \hat{c}(x) \), which maximizes the log-likelihood of (6), \( \log[P(d|c)] \). The AM algorithm is only briefly described here: a more complete description and derivation is presented elsewhere. It is easy to show that \( \hat{c}(x) \) also minimizes the divergence \( I[d(y)||g(y;c)] \). In contrast to the IDB derivation, here \( I[\cdot||\cdot] \) operates on the photon transmission rather than attenuation data space. We now define linear and exponential families of functions, \( \lambda(d) \) and \( \epsilon(I_0, H \cdot \mu) \), respectively,

\[
\lambda(d) = \{ p : p(y,E) \geq 0, \sum_{E \geq 0} p(y,E) = d(y) \},
\]

(7)

\[
\epsilon(I_0, H \cdot \mu) = \{ q : q(y,E) = I_0(y,E) e^{-\sum_{E \geq 0} \rho(h(x)|y,E)\mu_i(E)c_i(x)} \},
\]

(8)

where \( H \) is the matrix with elements \( h(y|x) \). The linear family \( \lambda(d) \) consists of the set of all possible monoenergetic sinograms, parametrized as a function of energy, whose sum over energy gives the measured sinogram, while \( \epsilon(I_0, H \cdot \mu) \) is the set of data means, as a function of energy, generated by the totality of unknown functions, \( c(x) \). Given the definitions (7), the minimization can be recast as follows:

\[
\min_{c} I[d(y)||g(y;c)] = \min_{q} \min_{\lambda(d)} I(p||q).
\]

(9)

Equation (8) leads to an algorithm in which the iterations alternate between estimating \( p \) and \( q \). The zeroth \( (k=0) \) iteration to the solution of (8) begins with assigning an initial value or guess to each \( c_i^{(0)}(x) \). Given the \( k \)th estimate of partial specific gravity, \( c_i^{(k)}(x) \), the \((k+1)\)th estimate is calculated as follows. First, the current estimates of the functions \( q \) and \( p \) from the exponential and linear families are computed:

\[
\hat{q}^{(k)}(y,E) = I_0(y,E) \cdot \exp \left[ - \sum_{x} \sum_{E} \mu_i(E) h(y|x) c_i^{(k)}(x) \right],
\]

(10)

\[
\hat{p}^{(k)}(y,E) = \frac{d(y)}{\sum_{E'} \hat{q}^{(k)}(y,E')},
\]

(11)

Next the back projections \( \hat{b}_i^{(k)} \) and \( \hat{b}_i^{(k)} \) of the current estimates of \( \hat{p} \) and \( \hat{q} \), are calculated

\[
\hat{b}_i^{(k)}(x) = \sum_{E} \sum_{E \neq 0} \mu_i(E) h(y|x) \hat{p}^{(k)}(y,E),
\]

(12)

\[
\hat{b}_i^{(k)}(x) = \sum_{E} \sum_{E \neq 0} \mu_i(E) h(y|x) \hat{q}^{(k)}(y,E).
\]

(13)

Finally, the updated estimate, \( c_i^{(k+1)}(x) \), is calculated iteratively,

\[
c_i^{(k+1)}(x) = c_i^{(k)}(x) - \frac{1}{Z_i(x)} \ln \left( \frac{\hat{b}_i^{(k)}(x)}{\hat{b}_i^{(k)}(x)} \right),
\]

(14)

if \( c_i^{(k+1)}(x)<0 \), then set \( c_i^{(k+1)}(x)=0 \).

The function \( Z_i(x) \) is a precomputed normalization function, which can be freely chosen subject to the constraints reviewed by O’Sullivan et al. in this work,

\[
Z_i(x) = Z_0 = \max_{(y,E)} \sum_{E} \sum_{x} \mu_i(E) h(y|x).
\]

(15)

By virtue of the definition of the dummy energy \( E = 0 \), the estimate for \( q(y,0) \) is always \( \sigma(y) \). Equation (9) becomes

\[
\hat{p}^{(k)}(y,E) = \hat{q}^{(k)}(y,E) \frac{d(y)}{\sum_{E' \neq 0} \hat{q}^{(k)}(y,E') + \sigma(y)},
\]

(16)

demonstrating that \( \hat{p}^{(k)}(y,E) \) is the expected number of counts contributed to \( d(y) \) by photons of energy \( E \) that are not part of the background distribution, \( \sigma(y) \).

At least two other iterative reconstruction algorithms have been proposed for maximizing the likelihood \( \log[P(d|c)] \). Both have been generalized to accommodate the x-ray spectrum in their data models and can be extended to accommodate background events as well. However, AM is unique in that it is based upon an analytic maximization rather than an approximation to the objective function. Thus, it can be proven that the AM iterative sequence converges monotonically. The performance advantages, if any, conferred by this unique mathematical property have not been investigated.
D. Scanning conditions and phantom

The phantom utilized in this study is shown in Fig. 1. As described in more detail elsewhere, it consists of water and Lucite cylinders (total diameter 30.5 cm) giving maximum attenuation and scatter equivalent to abdominal scans. Rods (1.27 cm diameter) of various materials can be inserted into the phantom at accurately defined locations to provide metal artifacts or low contrast objects. For the studies presented here, the four rods were composed of brass, iron, aluminum, and Teflon. From construction drawings, matching synthetic data sets were generated and found to closely match measured data.

To test the reconstruction algorithms, 2D synthetic sinogram data were generated typical of abdominal scanning using a commercial third-generation helical scanner (Siemens Somatom Plus 4, Siemens AG, Erlangen, Germany). This scanner has a fan-beam geometry with a source-to-isocenter distance of 570 mm and a ~52° fan-beam arc of detectors on a radius of 1005 mm. The gantry was assumed to consist of 768 detector samples per gantry position and 1408 gantry positions per revolution. A detector width of 1.2 mm and pixel dimensions of 1×1 mm² were assumed. By reviewing a small number of patients of various sizes scanned with a 120 kVp tube potential, we found that the maximum beam attenuation in the abdomen ranged from 6–8. Modeling representative scan protocols revealed unattenuated flux levels equivalent to several million noise equivalent quanta (NEQ) with an effective energy of ~75 keV. Flux levels dropped to several thousand NEQ in the thickest areas of the abdomen. Primary beam transmission through metal objects was calculated to be less than exp(−15). Since scattered radiation is typically several percent (~4%) in abdomen areas, depending on longitudinal collimation, it dominates the measured signal in the 10–100 NEQ range.

For the polyenergetic spectrum data model, an accelerating potential of 120 kVp and an additional filtration of 2.5 mm Al were assumed. The Birch–Marshall model was used to generate a filtered Bremsstrahlung spectrum (including characteristic x rays) for a tungsten target tube with a target angle of 7°. Based upon an experimental sinogram analysis, a 240 mAs exposure, 3 mm slice thickness at isocenter, and 1 s/gantry rotation results in an incident flux of 1.6×10⁶ photons on each detector in the absence of attenuators. For a typical abdominal scan (30 cm water thickness) with an attenuation of 6 to 8, the scatter contribution represents about 3% of the signal corresponding to 80 counts. This was approximated by setting ρ(y) = 80. For the monoenergetic data model, an energy of 75 keV, an I₀(y) of 1.6×10⁶ photons, and a constant ρ(y) = 80 were assumed.

E. Data model and algorithm implementation

Both the data models and the iterative algorithms are based upon forward- and back-projection operators described elsewhere. The phantom was assumed to consist of discrete pixels, with circular cross sections of the rods approximated by a stair-step boundary. The array elements, h(⋅,⋅), of the point-spread function were precalculated using parametric ray tracing. The fractional contribution of a pixel with its center at (x₁,x₂) and a detector centered at y for a gantry angle, is given by

\[ h(β,γ|x₁,x₂) = (1/Δy) \int_{y-Δy/2}^{y+Δy/2} L(β,γ',x₁,x₂)\,dy', \]

where Δy is the angle subtended by the detector with respect to the source location and L(β,γ,x₁,x₂) is the distance between proximal and distal intersections of the ray path (β,γ) with the pixel (x₁,x₂). In addition to ignoring edge gradient effects, our simulation neglected finite focal spot size and gantry motion blur. Because h(⋅,⋅) is a sparse matrix, the precalculated matrix was saved as an indexed data structure containing only nonzero elements. By restricting the summation indices x and y of the forward and back projection operators to nonzero h(γ|x), the number of arithmetic operations needed to evaluate sums of the form \( \sum_{x,y} h(γ|x) \) can be reduced by 40-fold.

For the polyenergetic spectrum data model, the number of constituents in Eq. (4) was set to one, i.e., \( N=1 \). The unitless partial specific gravity function, \( c(x) \), was computed as the ratio of the attenuation coefficient of the material in the pixel x to that of water, \( μ_{wat}(E=75\,keV) \), both evaluated at 75 keV. This results in an object attenuation function, \( μ(x,E) = μ_{wat}(E)\cdot c(x) \). The high-density water-equivalent phantom thus created will have attenuation values simulating those of the Lucite and metal inserts at the appropriate coordinates (as shown in Fig. 1). To add noise to this model, the data means, including the constant background contribution \( σ \), were passed through a random number generator, which sampled a signal value from the appropriate Poisson distribution. The process created some zero counts in the simulated data and, while this is acceptable to the alternating minimization algorithm, it presents a problem for FBP and IDB algorithms since a negative logarithm results. To avoid ln(0) evaluations, zero transmission values were replaced with ones prior to taking the logarithm. The choice of 1, rather than a smaller value, was made in order to keep the noise variance at a reasonable level, and to avoid atypical conditions for FBP and IDB. In a clinical scanner, photon scattering adds a positive offset to each detector response, on average equivalent to about 100 detected quanta, making the likelihood of encountering zero-count detector readings extremely low.

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Snyder’s OCCT approach can be used to iteratively estimate the pose of metal objects of known geometry for AM as well as IDB. However, for the purposes of this study, the influence of the prior information (attenuation map of the correctly localized object) provided by OCCT on al-
algorithm performance was simulated by setting $\hat{\mu}^{(k+1)}(x; \theta) = \mu_a(x; \theta)$, for the known rod pose and location, $\theta$, whenever $x \in X_a(\theta)$ and using Eqs. (3) or (11) only for $x \notin X_a(\theta)$. Thus, an error-free OCCT localization of the metal rod pose was assumed by our simulations.

The filtered backprojection algorithm (FBP) was implemented for the Siemens fan-beam geometry. The ramp frequency function was modified by a Gaussian filter corresponding to a spatial convolution of the image with a two-dimensional circularly symmetric spatial Gaussian function of the quantity $\sigma_f$ and $\sigma_m$ for the Siemens fan-beam geometry. The ramp frequency function was modified by a Gaussian filter corresponding to a spatial convolution of the image with a two-dimensional circularly symmetric spatial Gaussian function with a FWHM of 1 mm.

**F. Quantification of algorithm performance**

Algorithm performance was assessed by viewing images of the quantity $\mu_{\text{true}}(E = 75 \text{ keV}) \cdot \hat{c}(x)$ with a viewing window of $(0.016, 0.024)$ mm$^{-1}$ ($\pm$20% of the water attenuation). This is equivalent to Hounsfield window and level settings of 400 and 0, respectively. Horizontal profiles, placed 3 mm below the edge of the aluminum rod, were also plotted. Finally, several quantitative measures were evaluated, including mean percent bias, mean percent absolute bias, mean $l$ divergence, and mean squared error:

Mean percent bias = $\text{MPB} = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{\hat{c}(i) - c_{\text{true}}(i)}{c_{\text{true}}(i)} \right) \cdot \hat{c}_{\text{true}}(i)$,

Mean $l$-div = $\frac{1}{N} \sum_{i=1}^{N} \left( \frac{c_{\text{true}}(i)}{\hat{c}(i)} - \frac{c_{\text{true}}(i)}{\hat{c}(i)} + \hat{c}(i) \right)$,

Mean square error = $\text{MSE} = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{\hat{c}(i) - c_{\text{true}}(i)}{c_{\text{true}}(i)} \right)^2$,

Mean percent absolute error = $\text{MPAE} = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{\hat{c}(i) - c_{\text{true}}(i)}{c_{\text{true}}(i)} \right)$,

where $i$ indexes individual pixels, $c_{\text{true}}(i)$ denotes the true image (from which the data model was derived), and $\hat{c}(i)$ denotes the reconstructed image. Each error measure was computed separately over the different regions (see Fig. 1), including the outer Lucite cylindrical shell, the central Lucite cylinder (excluding the area occupied by the metal rods), and the intervening water annulus. Average and percent relative noise were evaluated for the three regions as follows:

Mean noise = $\sqrt{\frac{1}{N} \sum_{i=1}^{N} \left( \hat{c}(i) - \hat{c}_{\text{noisles}}(i) \right)^2}$,

Mean percent noise = $\text{MPN} = 100\% \cdot \frac{\text{Mean noise}}{\text{Mean}(\hat{c}_{\text{noiseless}}(i))}$,

where $\hat{c}(i)$ is the reconstructed image intensity (including the effects of synthetic noise in the modeled data) and $\hat{c}_{\text{noiseless}}(i)$ denotes the corresponding image reconstructed using the same algorithm and data model, but with noise suppressed.

**III. RESULTS**

Figure 3 shows the influence of noise, spectral polychromaticity, and scatter on images reconstructed by FBP. Figure 4 compares the performance of AM and IDB for various monoenergetic sinogram data models while Fig. 5 compares AM and FBP for data models based upon the polynenergetic photon spectrum. Figure 6 shows representative profiles through selected images. Table I presents quantitative image quality metrics, defined by Eqs. (15) and (16), for the inner Lucite cylinder and surrounding water annulus for various combinations of algorithms and sinogram data models.

Under near-ideal conditions (monoenergetic spectrum, no noise, no scatter), Fig. 3(a) demonstrates that FBP performs reasonably well in the presence of metal rods, exhibiting a faint Moiré-type pattern of intensity oscillations, giving rise to MPAEs of 1%–2% (Table 1). The MPR is very low (<0.1%) due to cancellation of negative and positive errors. This artifact, which diminishes as the detector density and number of gantry views increases (data not shown), is the well-characterized aliasing artifact. The MPB is very low (<0.1%) due to cancellation of negative and positive errors. This artifact, which diminishes as the detector density and number of gantry views increases (data not shown), is the well-characterized aliasing artifact. 50 Figure 3(b) demonstrates that measurement noise dramatically exacerbates metal streaking artifacts, producing 8%–31% mean absolute errors, and 15%–64% relative noise. Photon scatter (Fig. 3(c)) produces streaks of similar geometry, but without characteristic shot noise and even larger absolute errors. While streaks associated with beam hardening appear more subtle than monenergetic spectrum noise, the polyenergetic sinogram model with simulated scatter yields mean absolute errors of 16% for water and 43% for the Lucite cylinder. Finally, note that the presence of scatter reduces propagation of noise across the image. When no scatter is present, noise from the photon-poor regions shadowed by the metal rods propagates across the image. Adding scatter increases photon flux in these regions, which reduces noise propagation errors but at the expense of reduced accuracy due to more prominent deterministic streaks.

Our numerical experiments support our view that observed metal artifacts are not due solely or even mostly to “missing” sinogram data, but do indeed arise from system nonlinearities, not included in the simple data acquisition model assumed by FBP. Of these, scatter and noise appear to be the dominant sources of streaking, while beam hardening contributes most to deviation of the mean of the reconstructed $\mu(x)$ from the expected value. It is somewhat surprising that signal noise in the high attenuation regions of the sinogram can dominate metal streaking artifacts, particularly in the absence of scatter.

Figure 4 compares the performance of the IDB and AM iterative reconstruction algorithms for the monoenergetic sinogram model. Comparing columns 1 and 2 of Fig. 4 demonstrates that supplying a priori information regarding the location and attenuation map of the high-density objects in
Fig. 3. Images reconstructed with FBP, based on different simulated sinogram models: (a) monoenergetic, noiseless, scatterless; (b) monoenergetic, noisy \((1.6 \times 10^6 \text{ photons})\), scatterless; (c) monoenergetic, noiseless, with scatter; (d) polyenergetic, noiseless, scatterless; (e) and (f) polyenergetic, scatterless data with noise for (e) \(10^5 \text{ photons}\) and (f) \(1.6 \times 10^6 \text{ photons/detector}\).

Fig. 4. Comparison of IDB and AM algorithms for monoenergetic, scatter-free data model. “Unknown rod attenuation map” means that no prior information regarding composition and location of the high-density rods was supplied. “Known map” means that each updated image is modified to reflect the known rod attenuation values and locations.
the phantom substantially improves the performance of both IDB and AM. Surprisingly, when *a priori* information is not used with otherwise idealized sinogram data, IDB and FBP perform reasonably well, producing relatively subtle artifacts. In contrast, the corresponding AM image [Fig. 4(e)], exhibits significant streaking, resulting in 1% and 5% mean absolute errors in the water annulus and central Lucite cylinder, respectively. Inclusion of noise in the data model dramatically reduces IDB performance, (6%–20% MPAE and 9%–38% MPN) regardless of the use of prior information. The deterministic signal acquisition model assumed by IDB causes noise from the low photon-count regions to propagate throughout the image, producing artifacts approaching the severity of FBP streaking. In contrast, AM is able to form much smoother, streak-free images without the aid of regularization. The streaking artifacts that result when prior information is ignored are unaffected by the presence of noise.

Our numerical experiments demonstrate that a realistic treatment of detector counting statistics is essential to any reconstruction algorithm that hopes to overcome the metal artifact-streaking problem. Noise degrades the performance of deterministic iterative algorithms as well as FBP when dense metal objects are present. Although beam hardening and scatter also reduce the accuracy of both IDB and FBP images, the presence of scatter does mitigate noise propagation errors.

Figures 5 and 6 illustrate the performance of AM and FBP with more realistic data models. The AM algorithm incorporating beam hardening and scatter corrections yields high quality images (MPNs of 2%–3%) even when the photon fluence is unrealistically low [Fig. 5(c)]. With clinically realistic levels of emitted photons/detector, MPN falls below 1%. Incorporating knowledge of the metal rods and their attenuating properties reduces MPAE’s to less than 1% and eliminates nearly all evidence of streaking [Figs. 5(e) and 6(b)]. However, when this information is not used [Fig. 5(b)], significant streaking artifacts arise, resulting in mean percent absolute errors of 6% and 2% in the Lucite cylinder and water annulus, respectively. The local errors in the streaks range from −10% to +23% [Fig. 6(c)]. Finally, applying the monoenergetic AM algorithm to polyenergetic data yields average absolute errors of 9% to 11%, which is comparable to the MPAE resulting from application of FBP to noiseless polyenergetic data. Examination of image profiles [Figs. 6(d) and 6(e)] reveals nearly identical “cupping” artifacts.
IV. DISCUSSION

Our study demonstrates several important findings. With regard to FBP images, commonly seen metal streaking artifacts are not due solely or even mostly to “missing” sinogram data. Indeed, Fig. 3(a) demonstrates that in the presence of nearly opaque structures, FBP images are quite good when operating on otherwise idealized noiseless, scatter-free, monoenergetic sinograms that match the simplified signal formation model assumed by FBP. Rather, streaking artifacts arise from system nonlinearities (scatter and beam hardening). In addition, noise, in the form of large statistical fluctuations in photon-starved sinogram regions shadowed by...
dense metal objects, is an important source of streaking artifacts. We use the phrase “detector-model mismatch” to denote discrepancies between the nonlinear response of CT detectors to attenuation line integrals along the source-detector ray path and the idealized linear detector response assumed by the reconstruction algorithm. As noted in the Introduction, each of these detector-model mismatch mechanisms has been previously identified by multiple authors as causes of FBP image artifacts in general. Besides this paper, only DeMan’s recent simulation study\(^{30}\) claims that detector-model mismatch is the dominant cause of FBP metal streaking artifacts specifically.

Our experience demonstrates that incorporating the known position and attenuation map of metal objects to constrain the iterative image formation process does improve image quality for both the deterministic iterative deblurring and stochastic AM algorithms. However, in the presence of beam hardening, noise, and scatter, IDB images exhibit streaking artifacts comparable to FBP images. As the FBP and IDB algorithms suffer from identical detector model mismatches, this finding demonstrates that iterative as well as FBP algorithms require accurate modeling of the CT signal acquisition process to mitigate metal artifacts. In contrast, the AM algorithm, which accounts for scatter, the polyenergetic photon spectrum, and Poisson counting statistics, yields images with markedly reduced metal artifacts when

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<th>% abs error</th>
<th>% rel noise</th>
<th>% rel bias</th>
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Fig. 7. Comparison of sinograms calculated by discrete forward projection (FP) of the image, Fig. 4(c), (AM reconstruction without rod location knowledge) and the original discrete image used to form the simulated sinogram measurements for Fig. 4. The comparison is based upon 500 AM iterations. (a) Sinogram profiles ($\sigma=0$) from forward projections of Fig. 4(c) (broken line) and original image (solid line); (b) image of difference between noiseless Fig. 4 data sinogram and forward projection of Fig. 4(c).
presented with synthetic sinograms. Using a different maximum-likelihood (ML) reconstruction algorithm with beam hardening corrections, DeMan\textsuperscript{27} has also demonstrated that reducing detector-model mismatches mitigates metal streaking artifacts.

In contrast to previous studies, we have found that without using a priori knowledge of metal rod locations and attenuation maps as constraints, significant streaking artifacts persist in images formed by iterative reconstruction algorithms, even when the sinogram data is exactly matched to the algorithm’s assumptions as in the case of AM [Figs. 4(e) and 4(g) for idealized monoenergetic sinograms and Fig. 5(b) for more realistic sinograms]. These residual streaks introduce mean percent absolute errors of 5%–6% in reconstructed Lucite core attenuation coefficients. Simulations based on the convex ML algorithm\textsuperscript{40} exhibit nearly identical residual streaks. DeMan’s\textsuperscript{51,27} simulations, based upon yet another ML statistical algorithm,\textsuperscript{31} exhibit qualitatively similar artifacts. In contrast, IDB images formed without prior knowledge exhibit much less pronounced residual streaking artifacts. Because our sinogram model is derived from a voxelized phantom and a discrete forward projection operator that exactly matches the projection operators used by the algorithms themselves, edge-gradient effects cannot explain our findings. We conclude that eliminating detector-model mismatch does not, in itself, sufficiently mitigate metal artifacts on images formed by iteratively maximizing the Poisson transmission log-likelihood, $\ln[P(d|e)]$. In addition, prior knowledge of metal object attenuation maps must be used to constrain ML iterations. For images consisting only of normal anatomic constituents, AM is able to reconstruct nearly artifact-free images without the aid of prior knowledge.

Figure 7 demonstrates that the discrete forward projection of the streaky image [Fig. 4(e)] formed by unconstrained AM iterations is nearly identical to that of the “truth” image, used to synthesize the model sinogram, except for small deviations adjacent to the high density metal. Derivatives of the objective function for the image in Fig. 4(e) are near zero, suggesting that AM is approaching a fixed point, representing either a local or global minimum. As the number of AM iterations increases, the differences between the two sinograms continue to decrease, as do the streaks, although streaks are still evident after 400,000 iterations. A possible explanation for the poor convergence of AM, or convergence to an image other than the truth image, is that the iterative solution may contain some component of a null space image. The null space is the set of $\mu(x)$ such that $\int h(y|x)\times[\mu(x) + \mu_\alpha(x)]dx = \int h(y|x)\mu(x)dx$. From prior work on the Radon transform,\textsuperscript{52–54} we know that the null space of $h(y|x)$ contains many nonzero functions for discrete $y$ which depend on pixel size, detector discretization, and the approximation selected for evaluation of the forward operator. Avoidance of null space solutions may be a mechanism by which prior information improves statistical algorithm convergence. Unlike the nonlinear AM iterations, FBP and IDB are essentially linear transformation algorithms, with solution spaces orthogonal to the null space. We are continuing to explore the structure of the null space and its relation to the AM iterations to deepen our understanding of this phenomenon and to identify more effective and flexible forms of prior knowledge constraints.

Our study has a number of important limitations. Because our sinogram data models are based upon discrete forward projections of discrete phantoms, our analysis does not include edge-gradient effects, finite focal spot effects, or motion blur, some of which\textsuperscript{30} have been shown to be important contributors to metal artifacts. Secondly, our AM simulations are based upon idealized phantoms consisting of water equivalent materials [$N = 1$ in Eq. (4)]. Our efforts\textsuperscript{55} to use dual energy CT imaging to measure photon cross sections in the energy range of interest to brachytherapy (20–1000 keV) indicate that at least two constituents are needed to accurately represent the photon cross sections of biological tissues. Finally, this study omits treatment of regularization.

Regularization, either in the form of sieves\textsuperscript{56} or penalty-driven regularization schemes\textsuperscript{57} (modified objective functions that penalize undesirable solution behavior, e.g., large voxel-to-voxel fluctuations), could significantly reduce metal streaking and noise-related artifacts usually at the expense of other characteristics such as spatial resolution. Finally, AM, along with all other published ML transmission CT algorithms, ignores the fact that modern CT detectors are energy integrating rather than photon-counting detectors. A more complex compound Poisson distribution is needed to describe the stochastic behavior of energy-integrating detectors,\textsuperscript{44} although the simple Poisson distribution may be an acceptable approximation under many circumstances.

An important goal of our future research is to test AM iterative reconstruction on sinograms experimentally acquired from clinical scanners with metal objects present in the phantom scanned. Preliminary tests demonstrate that images reconstructed from experimentally scanned rod phantoms exhibit streaks similar to Fig. 4(e). Integration of AM reconstruction into the OCCT framework will permit iterative localization of high density rod locations, so that the value of constrained AM iterative image reconstruction can be assessed. Validation of AM-based metal artifact reduction requires extending the algorithm to multiple-constituent media and may require addition of regularization and modeling of edge-gradient effects. In parallel with our algorithm developments, we are pursuing modeling of measured sinograms, to ensure that all important sources of detector mismatch are identified.

V. CONCLUSIONS

Contrary to the dominant view of the literature, the familiar metal streaking artifacts evident in filtered backprojection images are almost completely due to discrepancies between the simplified detector model assumed by the algorithm and the actual CT signal formation process. Photon scatter, hardening of the polychromatic spectrum, and especially noise in photon-starved areas of the sinogram, shadowed by dense metal objects, all contribute to this detector model mismatch phenomenon. We have evaluated the performance of a novel maximum-likelihood algorithm, the alternating minimization
approach, 20 which combines an iterative search for the pose artifact-free images. Utilization of such prior information is strain AM’s iterative solutions, we are able to obtain nearly operator. By using prior information regarding the location, part of the null space of the discrete forward projection operator. When model mismatch is completely eliminated, significant by minimizing detector-model mismatch. However, even CT detector response and accounts for beam hardening and metal objects with AM iterations to estimate the tissue attenuation map.

ACKNOWLEDGMENTS

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