Joint Iterative Equalization and Decoding for 2-D ISI Channels

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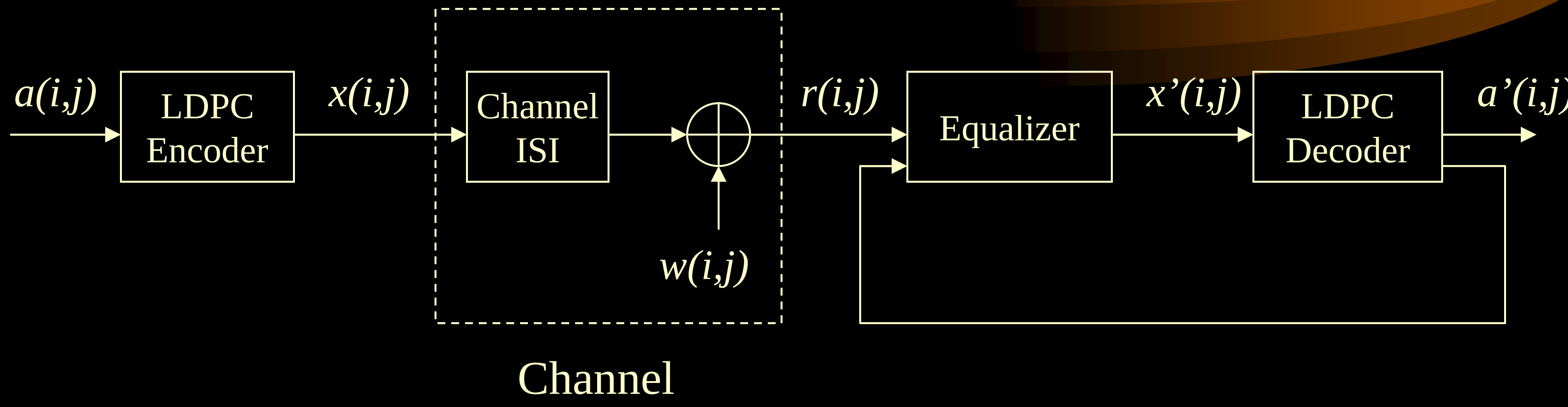
Outline

- Problem description
- Joint equalization and decoding schemes for:
  - General 2-D ISI
    - Using 2-D MMSE equalization and decoding
    - Using novel message passing algorithms that take advantage of the 2-D dependence
  - Separable 2-D ISI
    - Using turbo equalization
- Conclusions

Problem Description

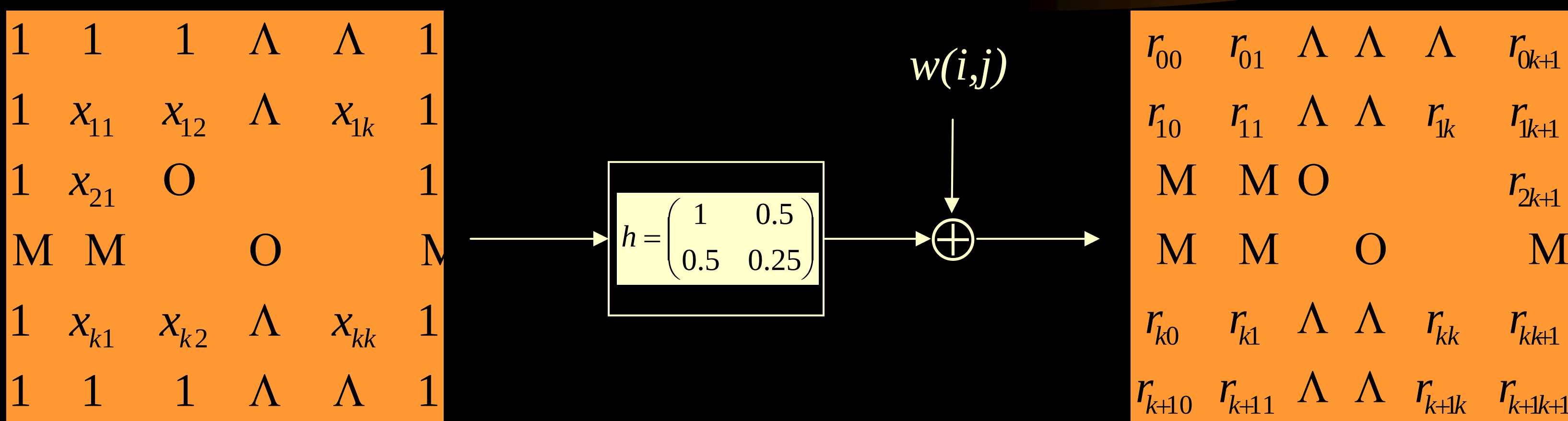
- ❑ Reliable data retrieval from channels having 2-D ISI
  - ➢ As bit aspect ratio reduces inter-track interference becomes significant
  - ➢ Optical memories
  - ➢ Future storage media: 2-D patterned media

Channel Model



- $x(i,j) \in \{+1,-1\}$
- Channel ISI is 2-D and linear
- Noise assumed to be AWGN

2-D Intersymbol Interference

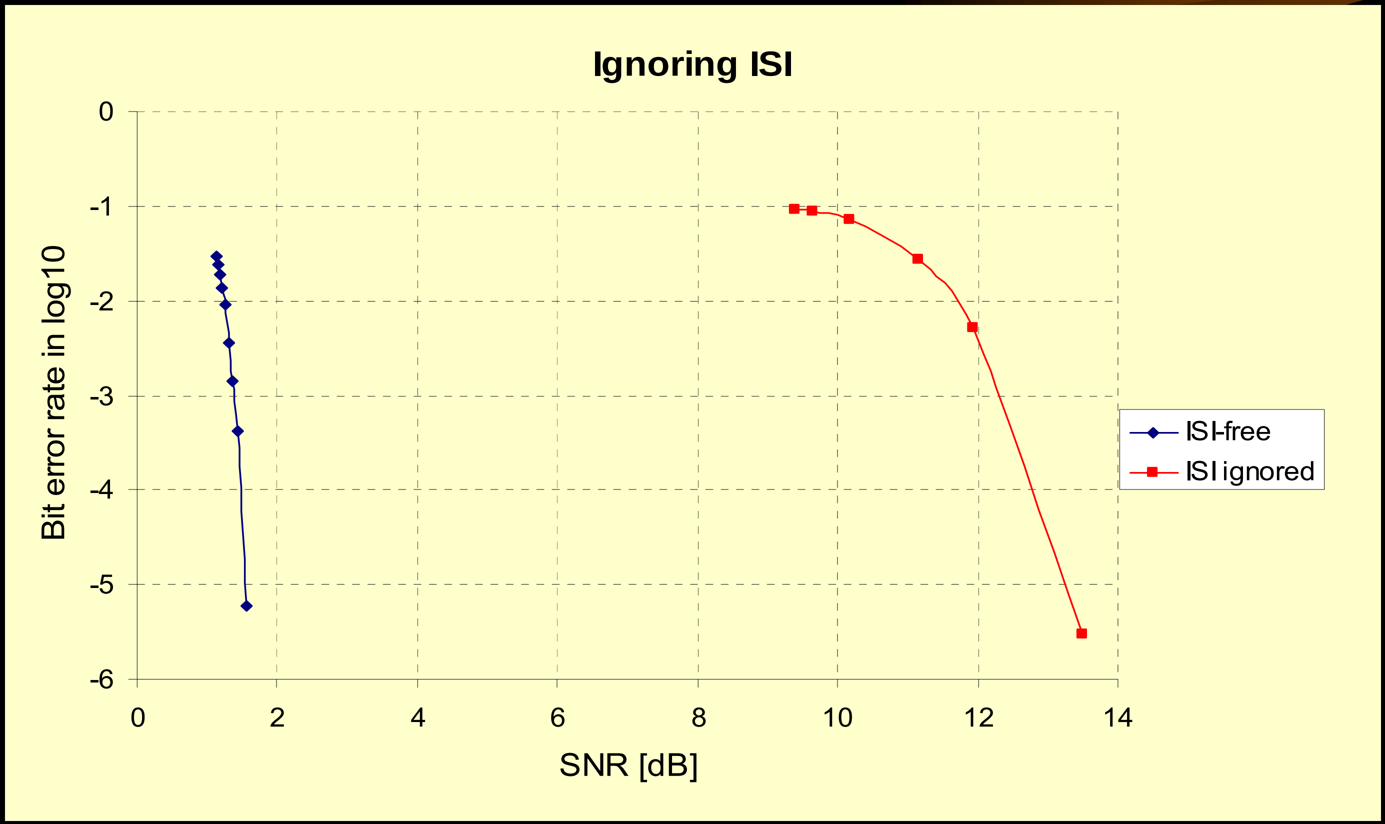


GUARD BAND

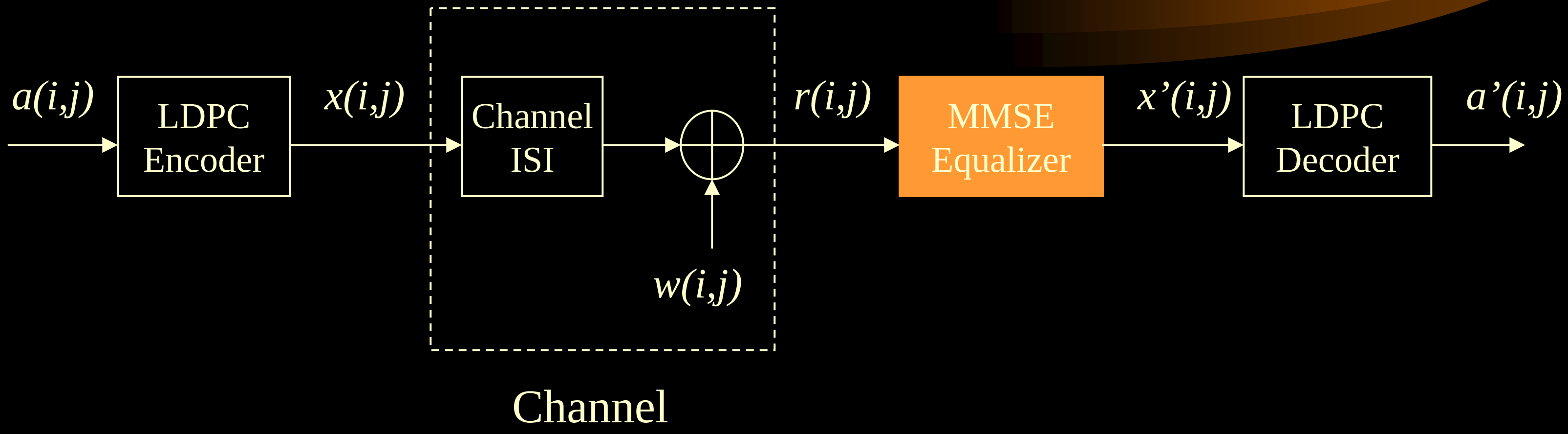
$$r_{i,j} = x_{i,j} + 0.5x_{i-1,j} + 0.5x_{i,j-1} + 0.25x_{i-1,j-1} + w_{i,j}$$

Performance

Block length 10000 regular (3,6) LDPC code

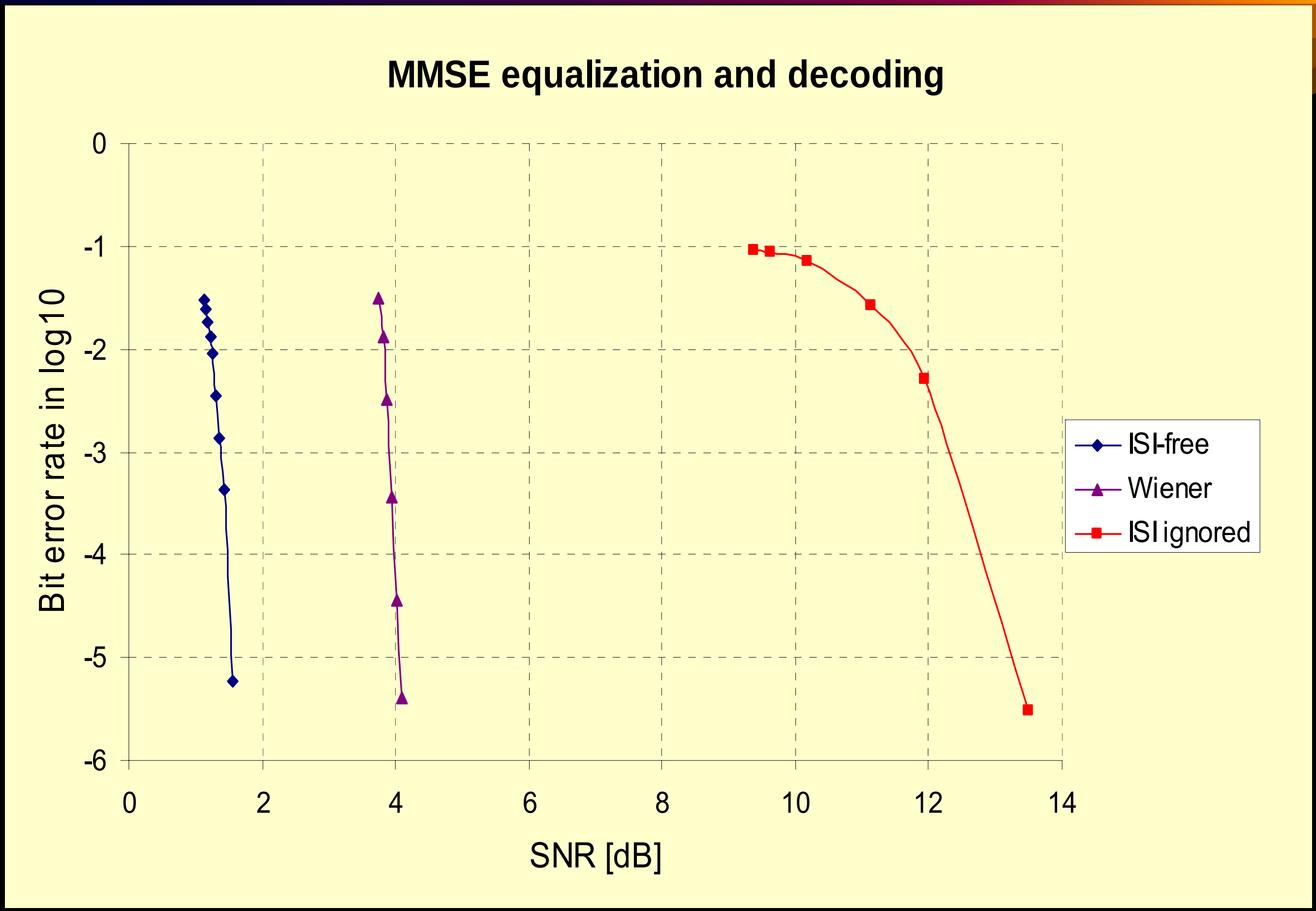


MMSE Equalization

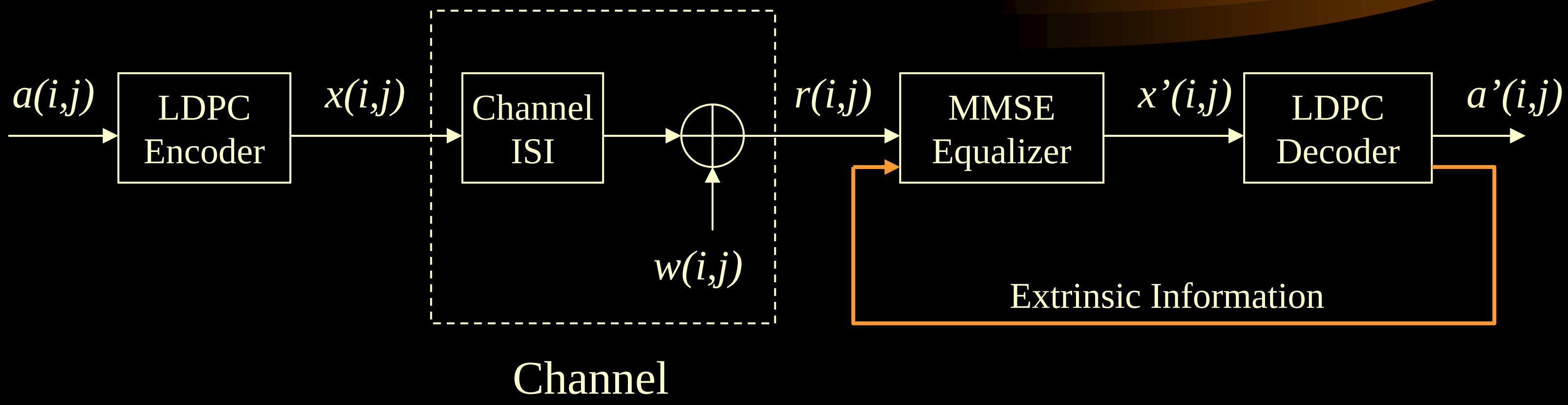


- Equalizer designed assuming inputs to be Gaussian

Performance



Iterative MMSE Equalization

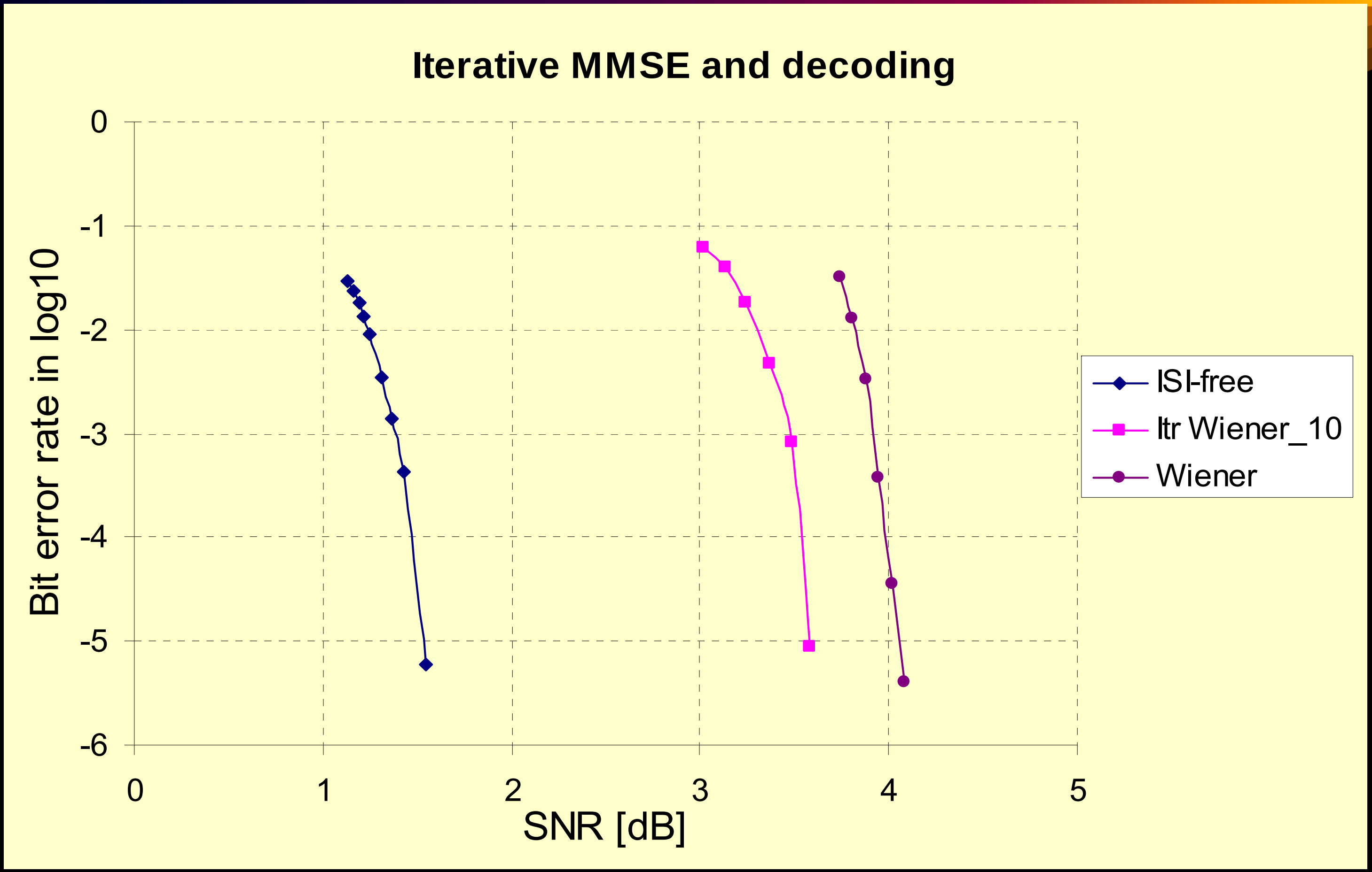


- Soft information, estimated mean of the codeword, passed from LDPC decoder to equalizer

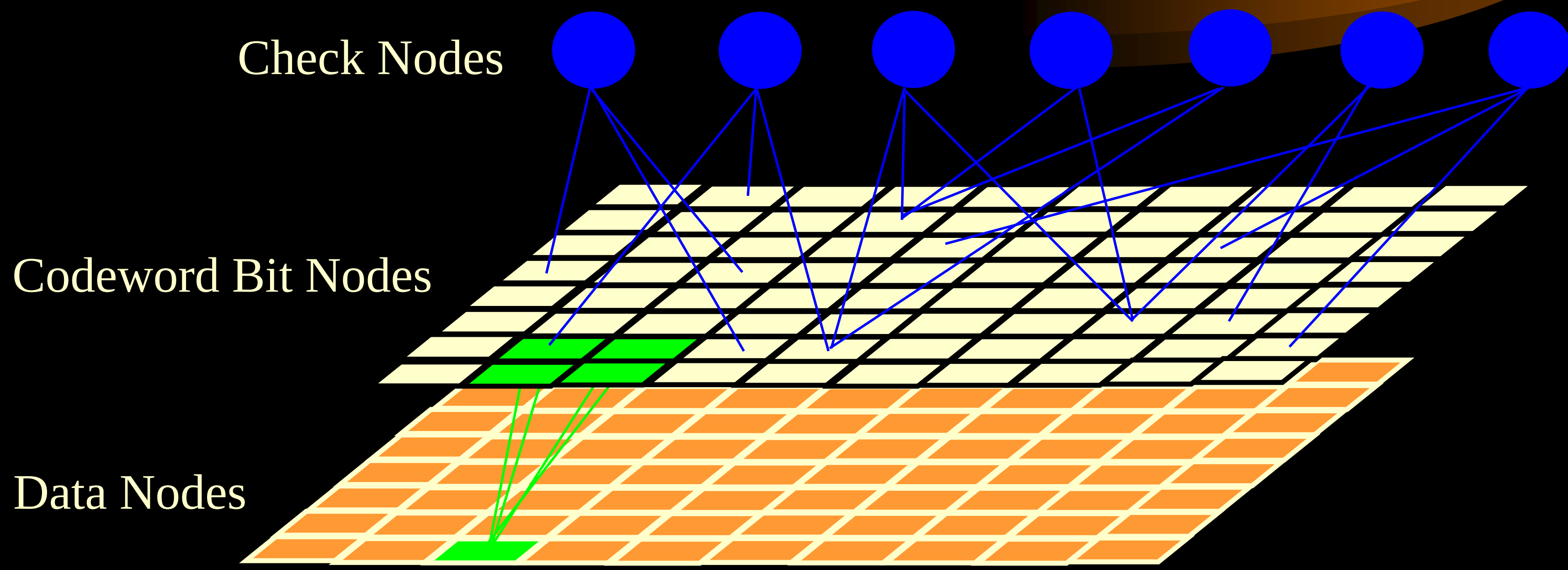
$$E[x] = \Pr(x = 1) - \Pr(x = -1)$$

$$\hat{x} = E[x] + W ** [r - h ** E[x]]$$

Performance

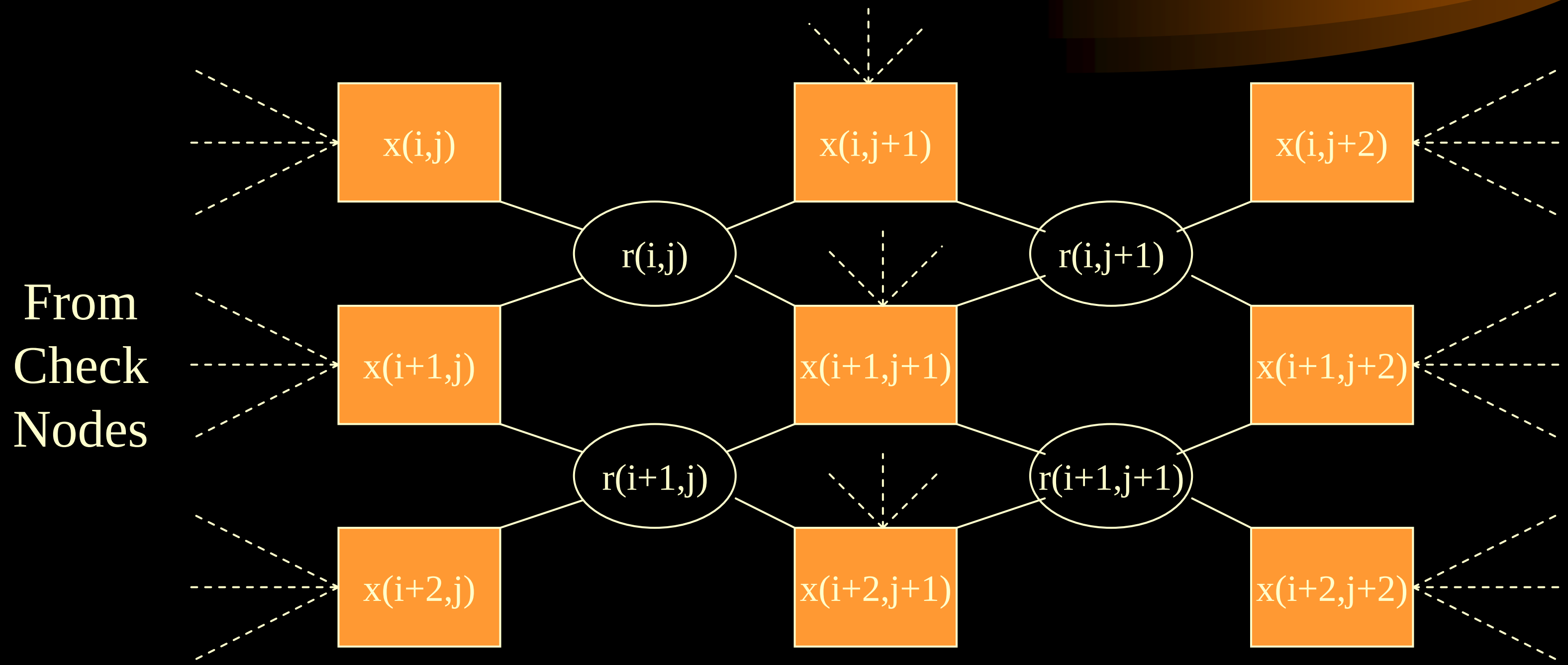


Full Graph Message Passing

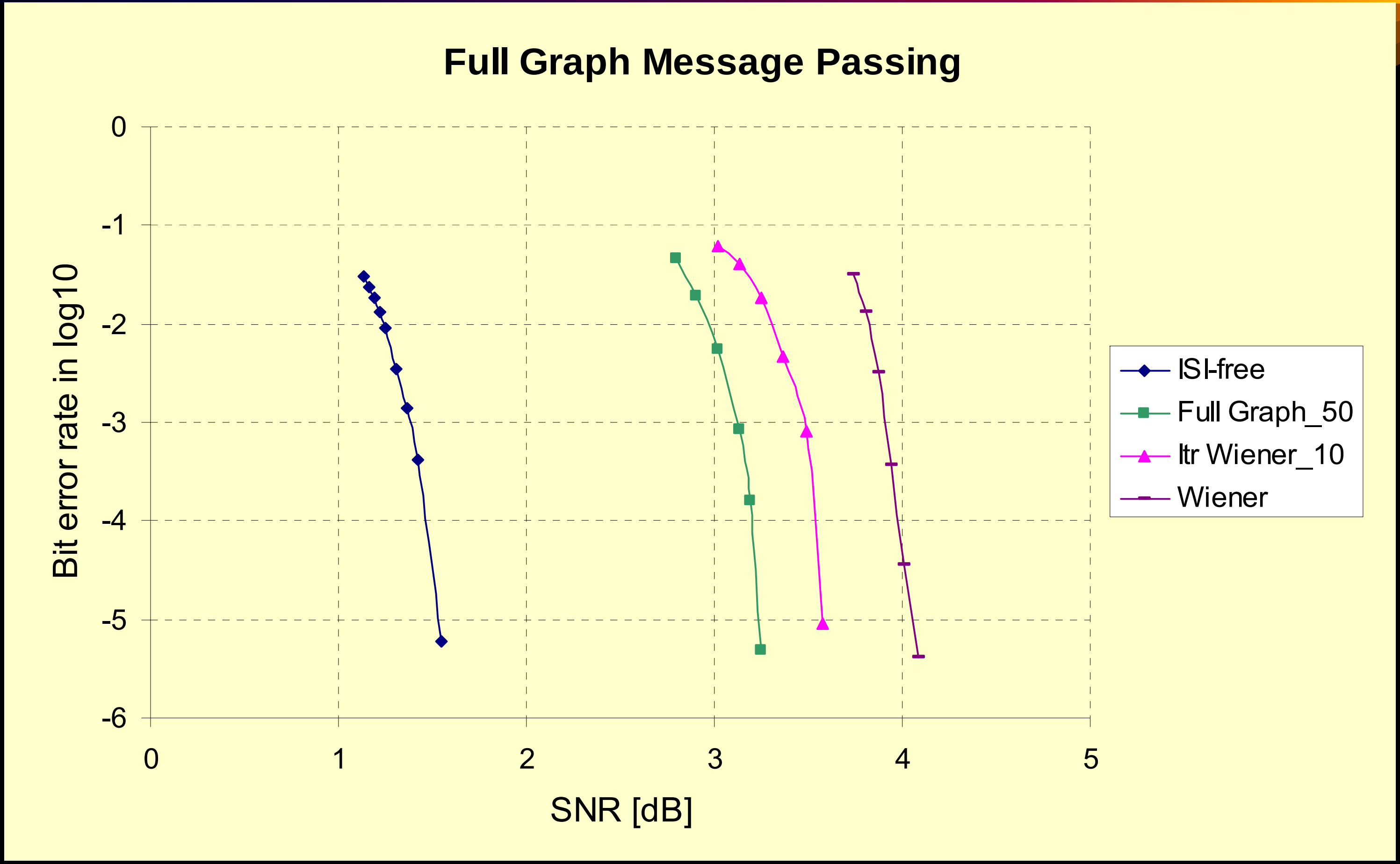


$$r_{i,j} = x_{i,j} + 0.5x_{i-1,j} + 0.5x_{i,j-1} + 0.25x_{i-1,j-1} + w_{i,j}$$

Full Graph

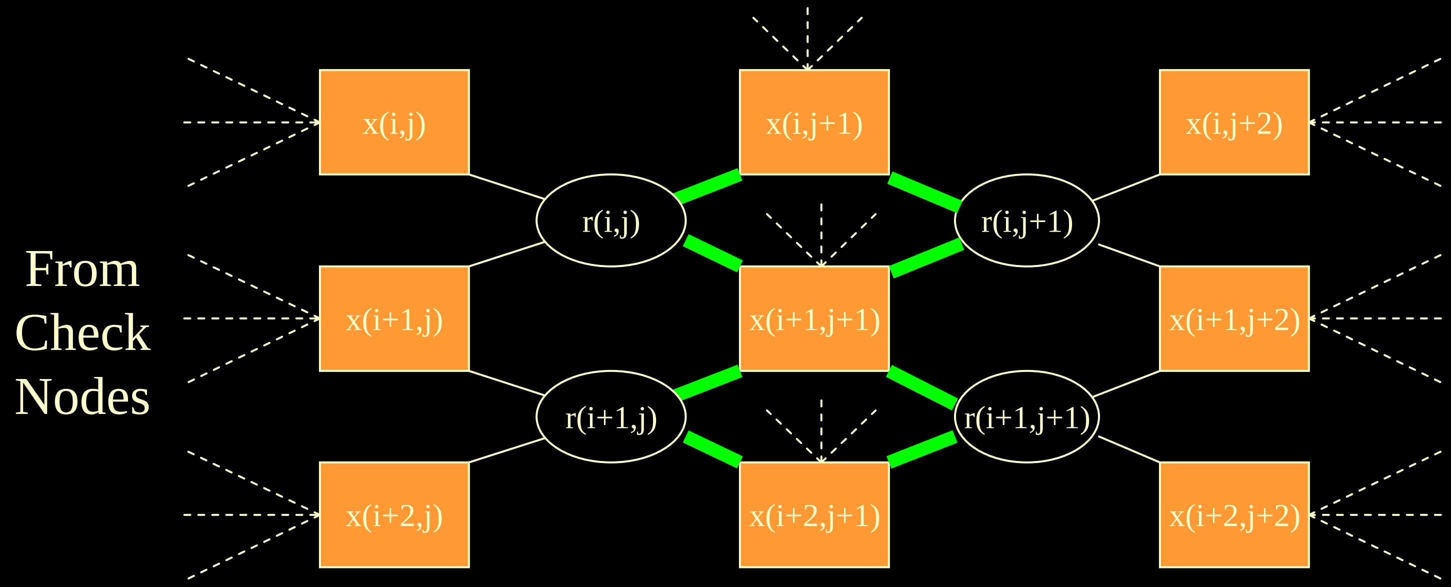

From Check Nodes
x(i,j)
x(i,j+1)
x(i,j+2)
r(i,j)
r(i,j+1)
x(i+1,j)
x(i+1,j+1)
x(i+1,j+2)
r(i+1,j)
r(i+1,j+1)
x(i+2,j)
x(i+2,j+1)
x(i+2,j+2)

Performance



Full Graph Analysis

- ❑Length 4 cycles present which degrade performance of message passing algorithm



Kschischang *et al.*, "Factor graphs and the sum-product algorithm," *IEEE Trans. Inform. Theory*, 47(2), 498-519, Feb. 2001.

Ordered Subsets Message Passing

- From Imaging – Data set is grouped into subsets to increase rate of convergence
- For Decoding – Observed data is grouped into subsets to eliminate short length cycles in the Channel ISI graph

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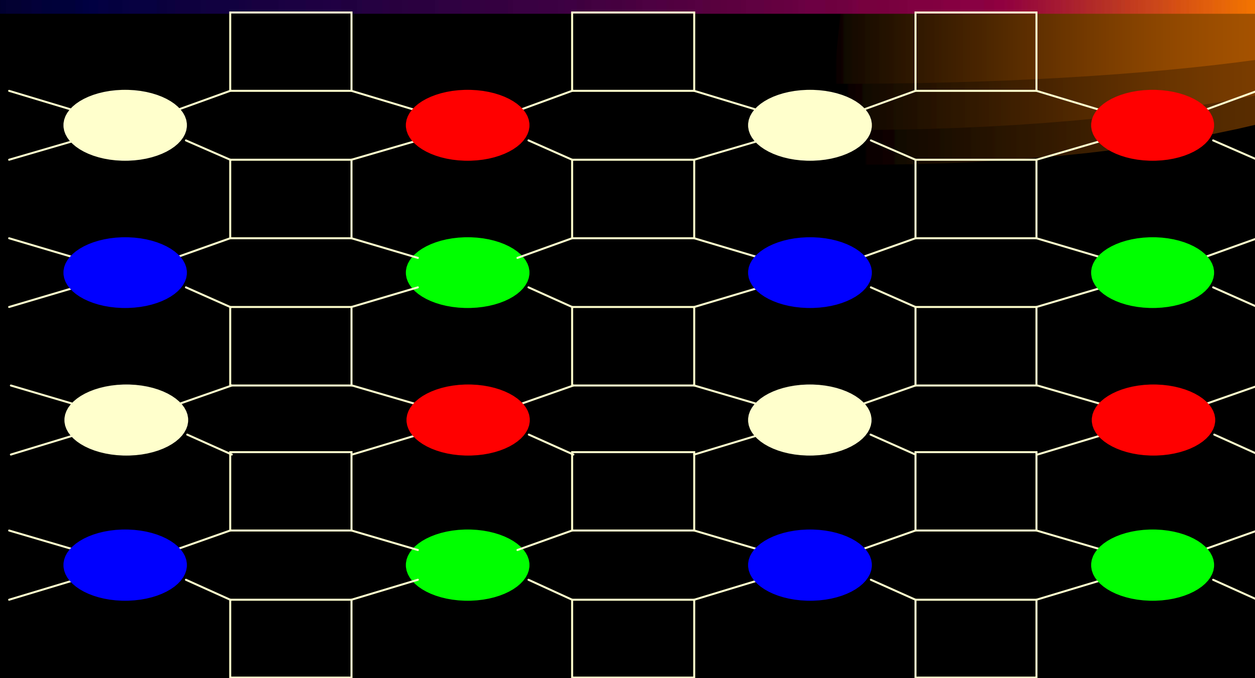
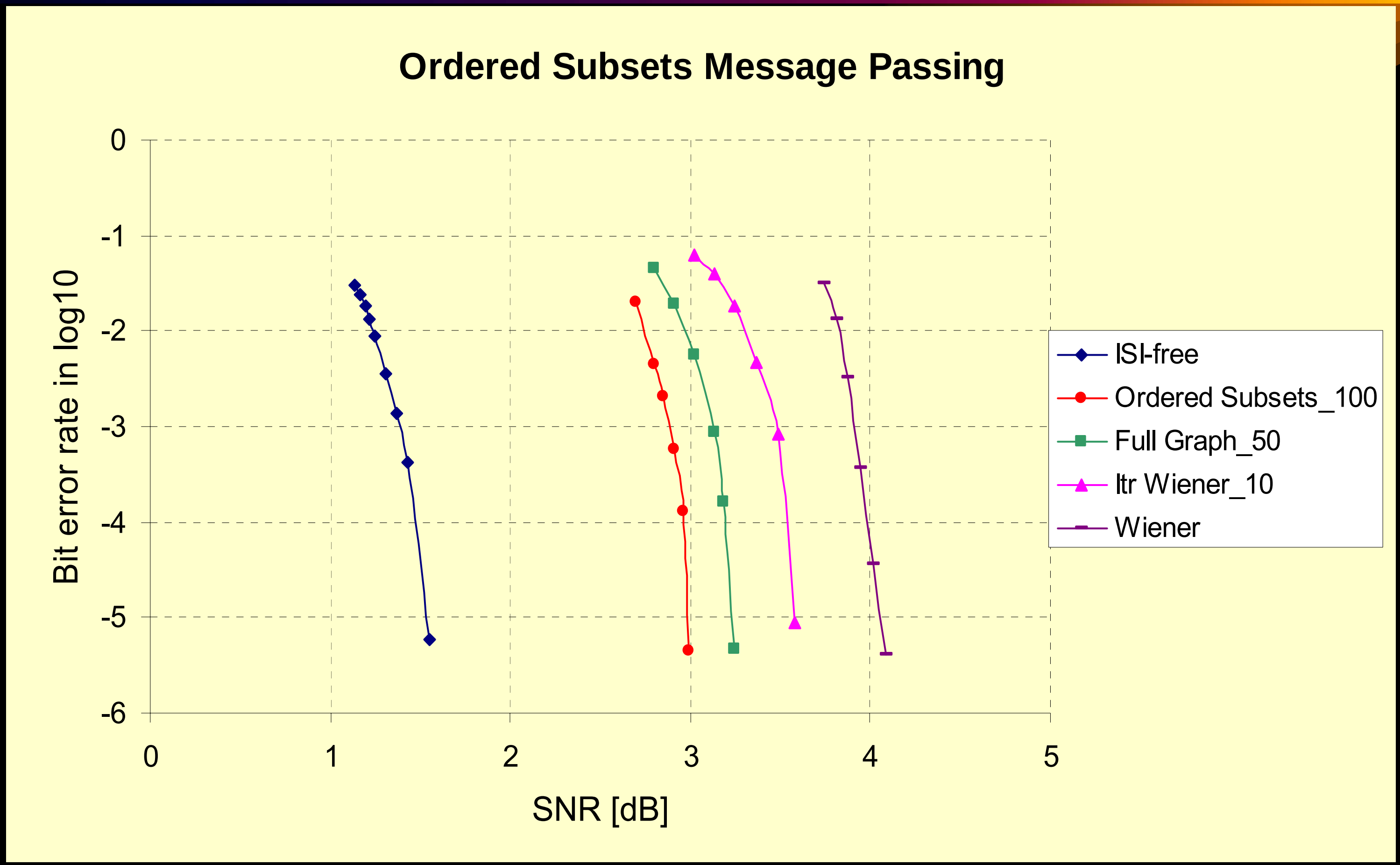
H. M. Hudson, and R. S. Larkin, “Accelerated image reconstruction using ordered subsets of projection data,” *IEEE Trans. Medical Imaging*, 13(4), 601-609, Dec. 1994

Grouped ISI Graph



- Labeling of data nodes into 4 subsets
- For each iteration use data nodes of one label only

Performance



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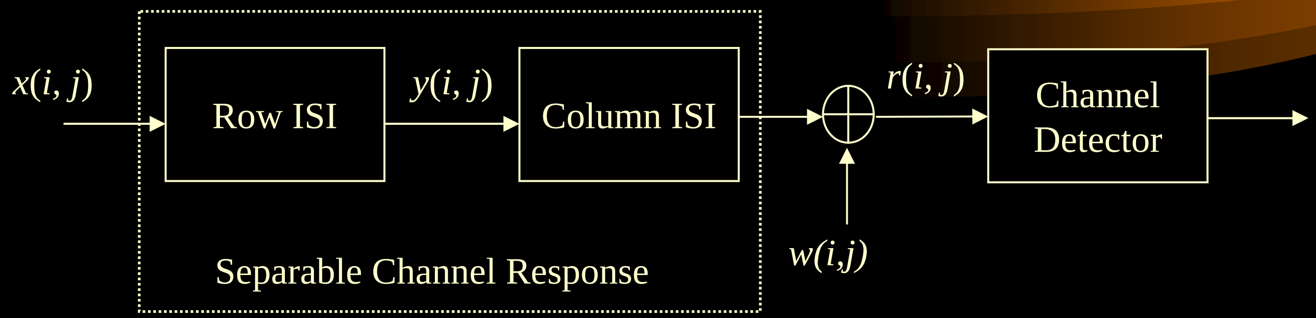
A Separable 2-D ISI

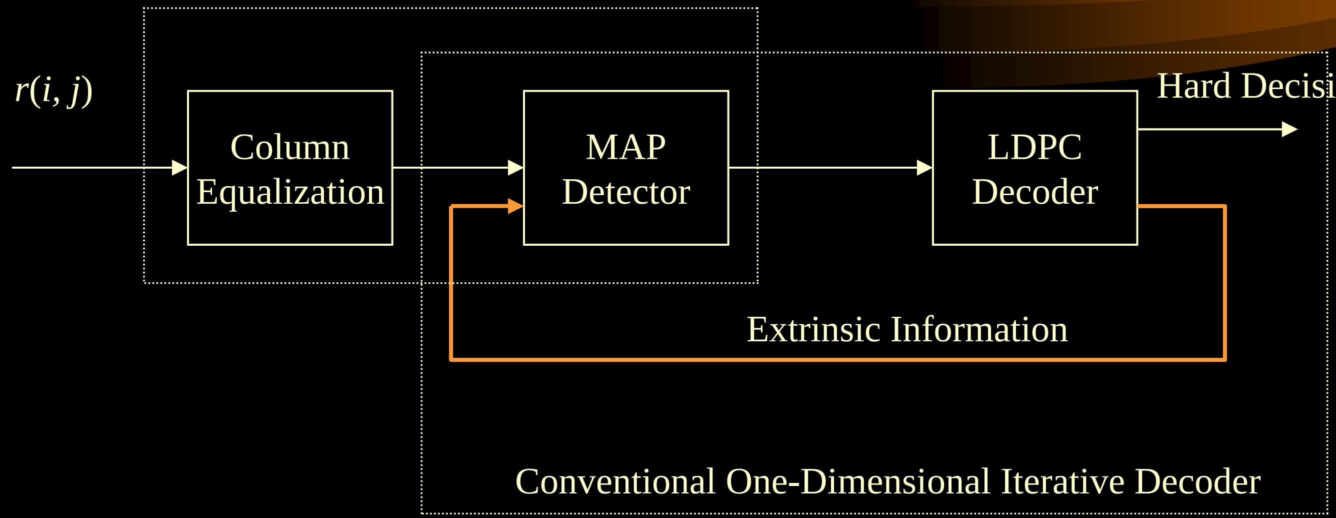


$$h = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 0.25 \end{pmatrix} = \begin{pmatrix} 1 \\ 0.5 \end{pmatrix} \begin{pmatrix} 1 & 0.5 \end{pmatrix}$$

- Advantages of Separable 2-D ISI
  - Reduced Detector Complexity
  - Apply existing one-dimensional equalization methods

Tüchler *et al*., "Turbo equalization: principles and new results," *IEEE Trans. On Comm*., 50(5), May 2002.

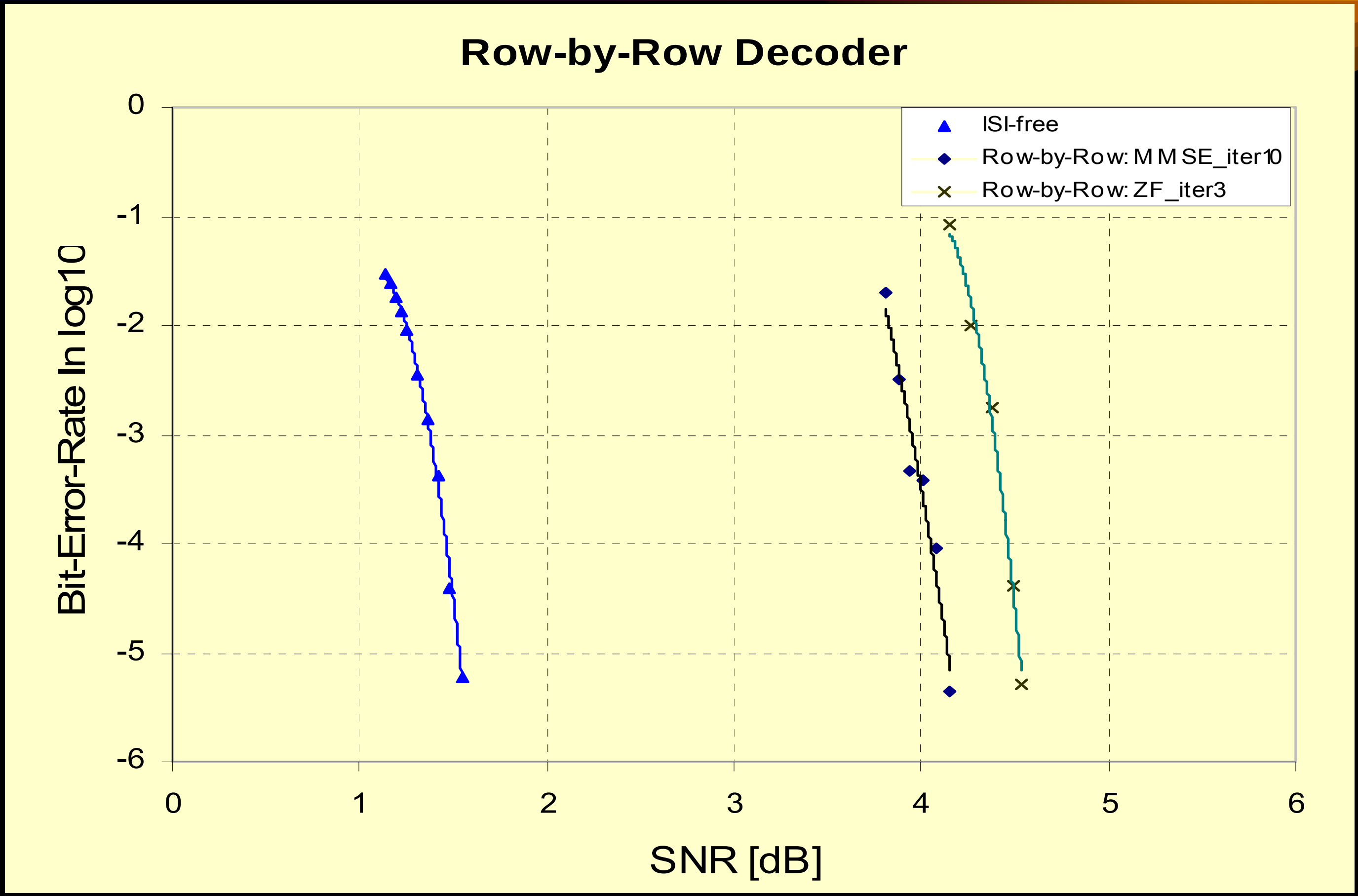
Iterative Decoder Diagram I



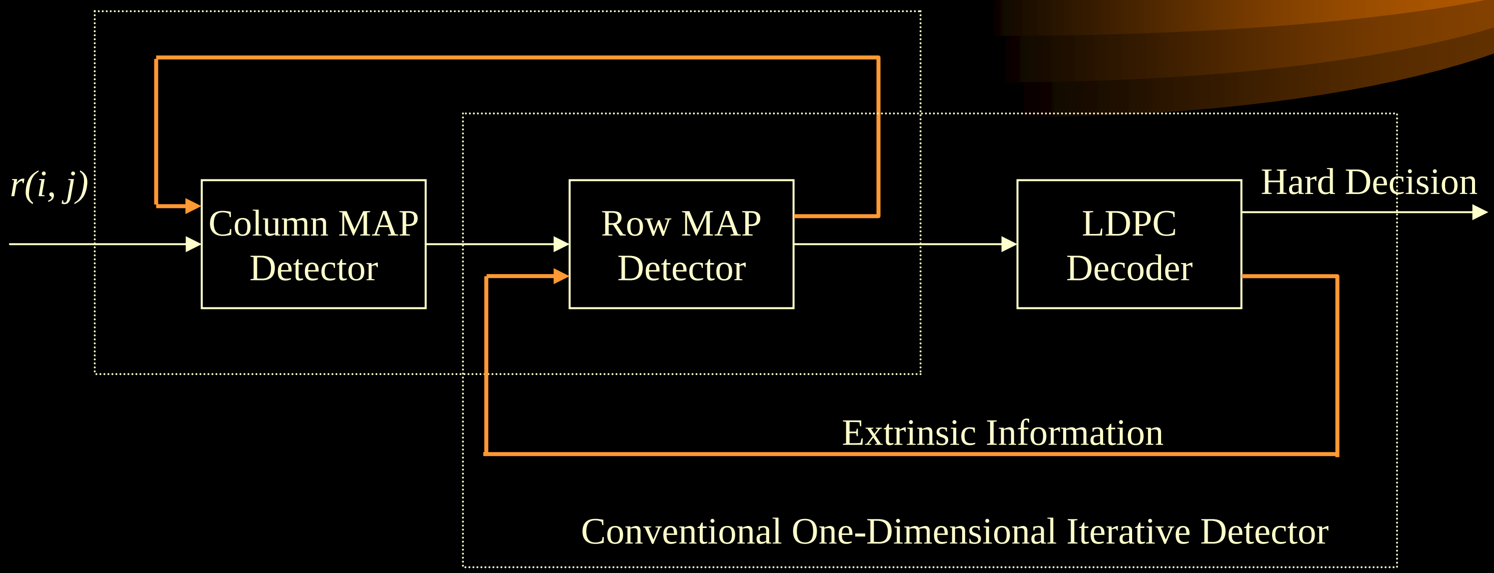
Row-by-Row Decoder

- MMSE and Zero-forcing criteria used for Equalization

Performance



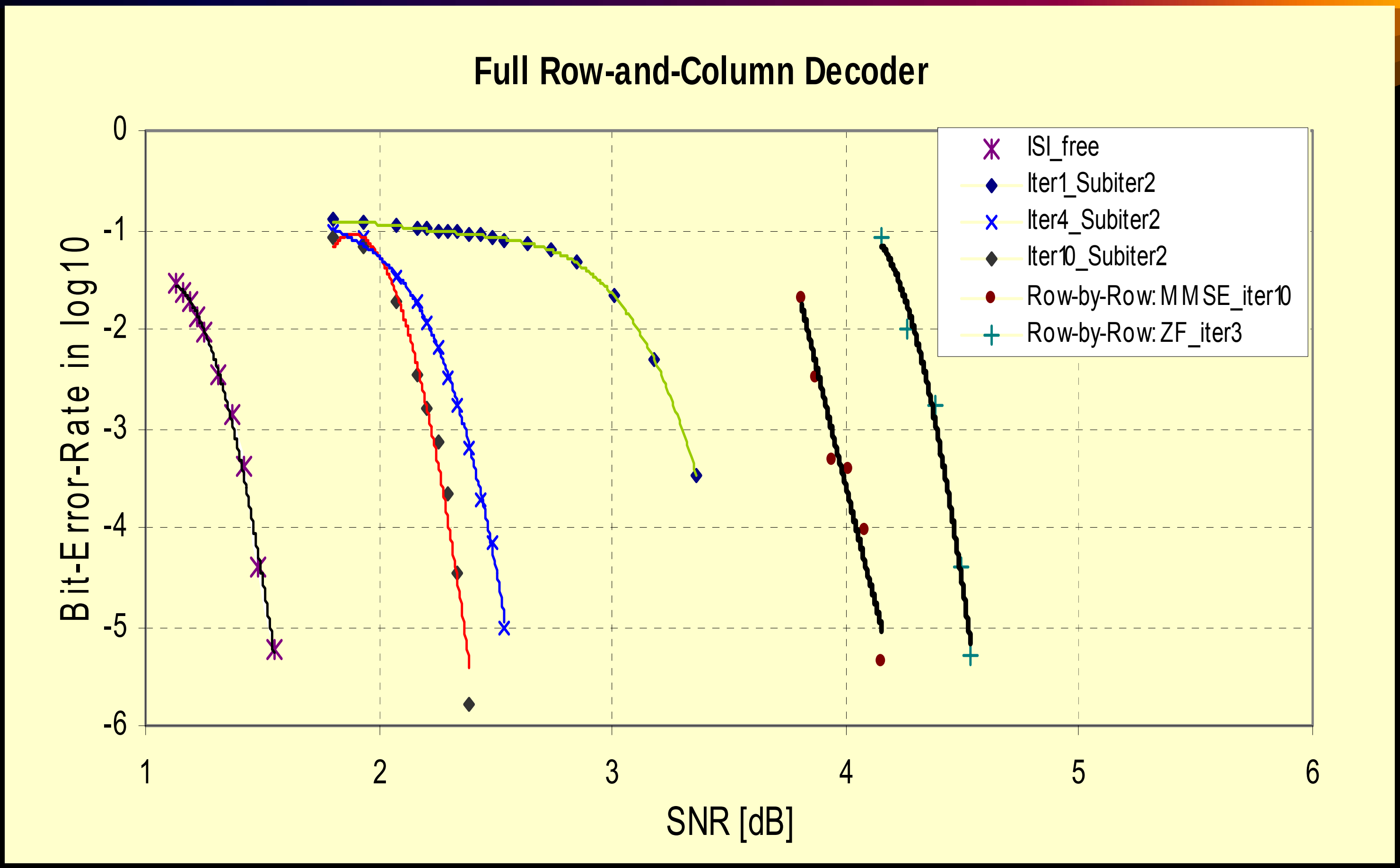
Iterative Decoder Diagram II



Full Row-and-Column Decoder

- Inputs to column detector are not binary

Performance



Conclusions

- MMSE Equalization and Decoding
  - Good Performance with Iterative Equalization
  - Low Complexity FIR Implementation
- Message Passing Algorithms
  - Full Graph algorithm performance deteriorated due to short cycles
  - Ordered Subsets Message Passing gives best performance for general 2-D ISI
- Separable ISI Decoding
  - Best Performance for Separable 2-D ISI
  - Low Complexity
  - Equalize general 2-D ISI to nearby separable channel response

References

- Chen *et al*., "Near-optimal parallel distributed data detection for page-oriented optical memories," *IEEE Journal of selected Topics in Quantum Electronics*, 4(5), 866-879, Sept./Oct. 1998.
- W. Weeks IV, "Full surface data storage," Ph.D. thesis, University of Illinois at Urbana Champaign, 2000.
- Hagenaur *et al*., "Separable MAP "Filters" for decoding of product and concatenated codes," *Proc. of Int. Conf on Comm*., 1740-1745, Aug. 2000
- Kavčić *et al*., "Binary intersymbol interference channels: Gallager codes, density evolution and code performance bounds," submitted to *IEEE Trans. Inform. Theory*.