Ordered Subsets Message-Passing

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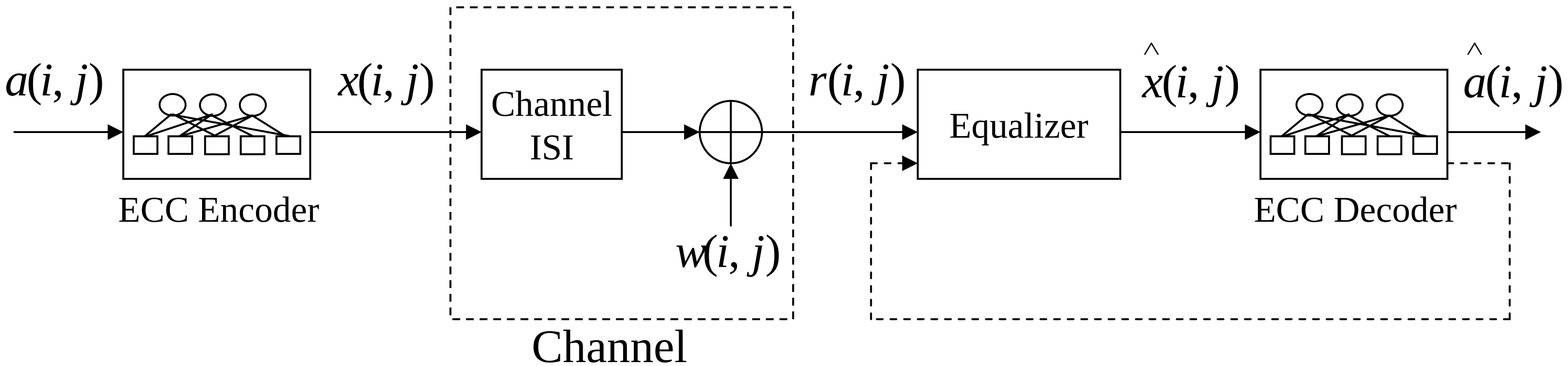
Introduction

- Systems with 2-D ISI
- Joint equalization and decoding for 2-D ISI
  - Full graph message-passing
  - Ordered subsets message-passing
- Density evolution
- Conclusions

Problem Description

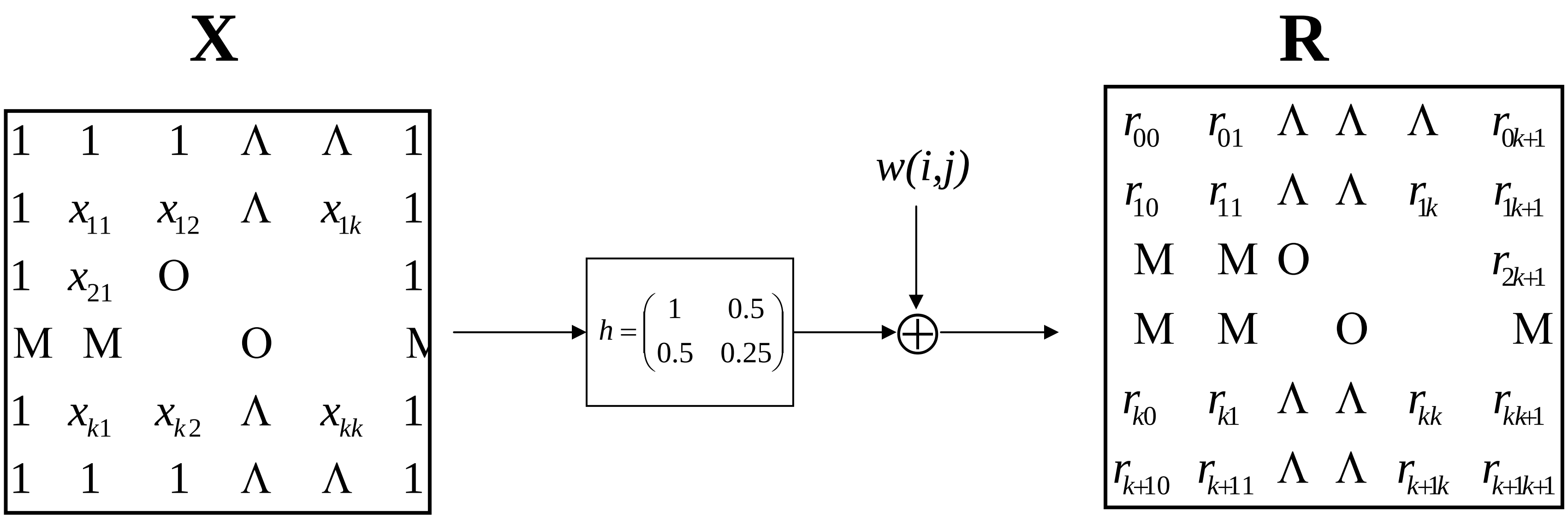
- Joint equalization and decoding for channels having 2-D ISI
  - As bit aspect ratio reduces inter-track interference becomes significant
  - Optical memories
  - Future storage media: Patterned media

System Model



- LDPC codes used for error-correction
- $x(i,j) \in \{+1,-1\}$
- Channel ISI is 2-D and linear
- Noise assumed to be AWGN

2-D Intersymbol Interference

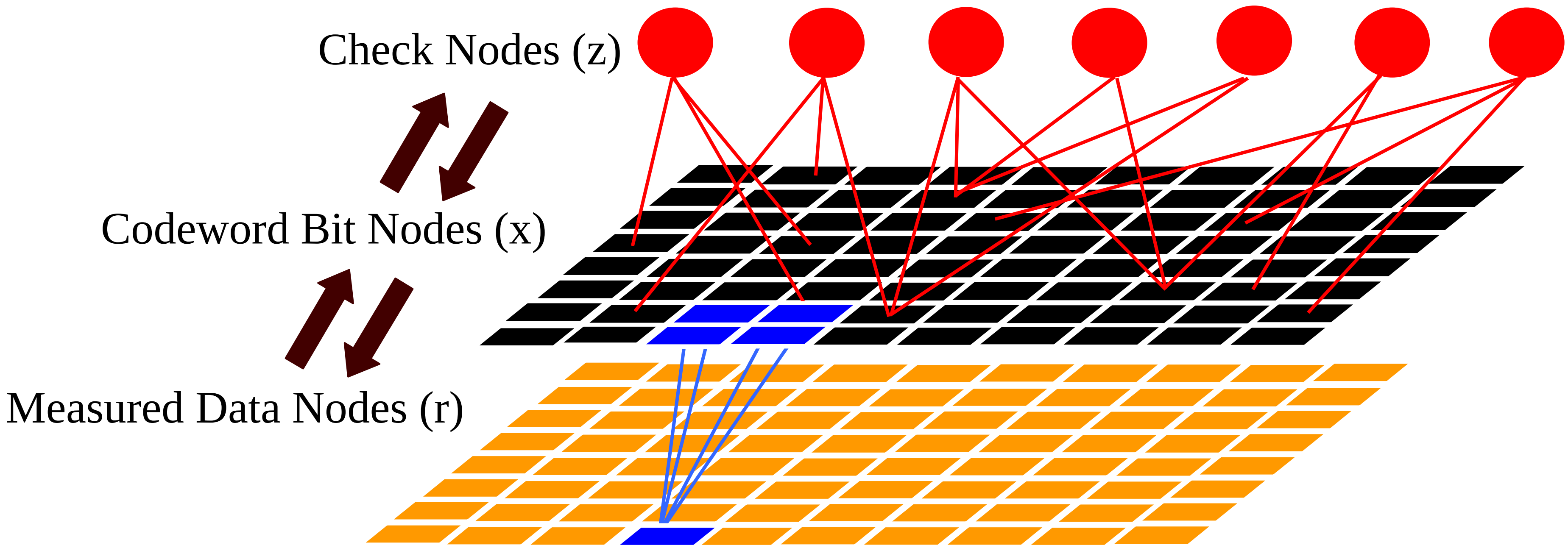


GUARD BAND

$$R = h ** X + W$$

$$r_{i,j} = x_{i,j} + 0.5 x_{i-1,j} + 0.5 x_{i,j-1} + 0.25 x_{i-1,j-1} + w_{i,j}$$

Full Graph Message-Passing



$$h = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 0.25 \end{pmatrix} \qquad r_{i,j} = x_{i,j} + 0.5x_{i-1,j} + 0.5x_{i,j-1} + 0.25x_{i-1,j-1} + w_{i,j}$$

Message-Passing (Sum-Product)

- Codeword bit nodes to check nodes

$$L_{x\to z}^{(l)} = \sum_{m\in N(x)} L_{m\to x}^{(l-1)} + \sum_{z'\in N(x)\backslash z} L_{z'\to x}^{(l-1)}$$

- Check nodes to codeword bit nodes

$$\tanh\frac{L_{z\to x}^{(l)}}{2} = (-1)^{z} \prod_{x'\in N(z)\backslash x} \tanh\frac{L_{x'\to z}^{(l-1)}}{2}$$

Message-Passing (Sum-Product)

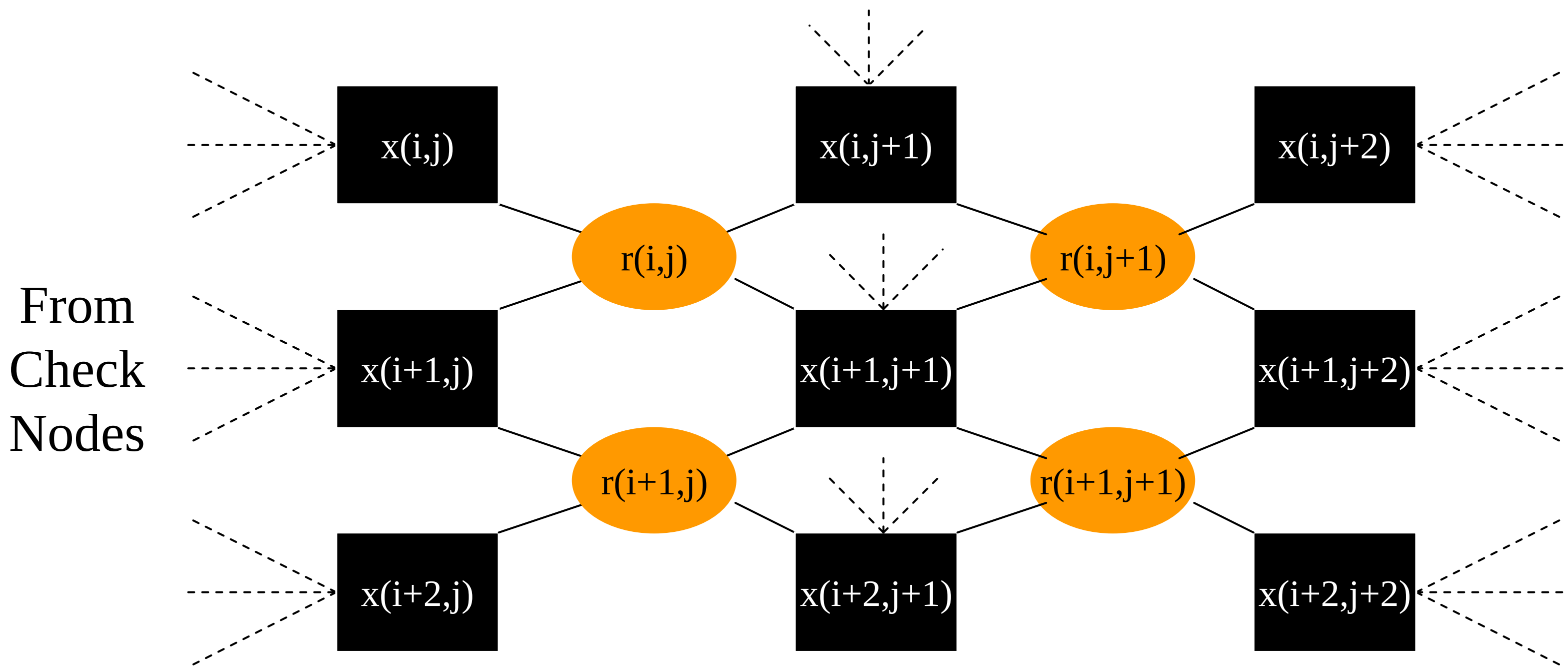
- Codeword bit nodes to measured data nodes

$$L_{x\rightarrow m}^{(l)} = \sum_{m'\in N(x)\backslash m} L_{m'\rightarrow x}^{(l-1)} + \sum_{z\in N(x)} L_{z\rightarrow x}^{(l)}$$

- Measured data nodes to codeword bit nodes

$$L_{m\rightarrow x}^{(l)} = f(\{L_{x'\rightarrow m}^{(l)} : x' \in N(m)\backslash x\})$$

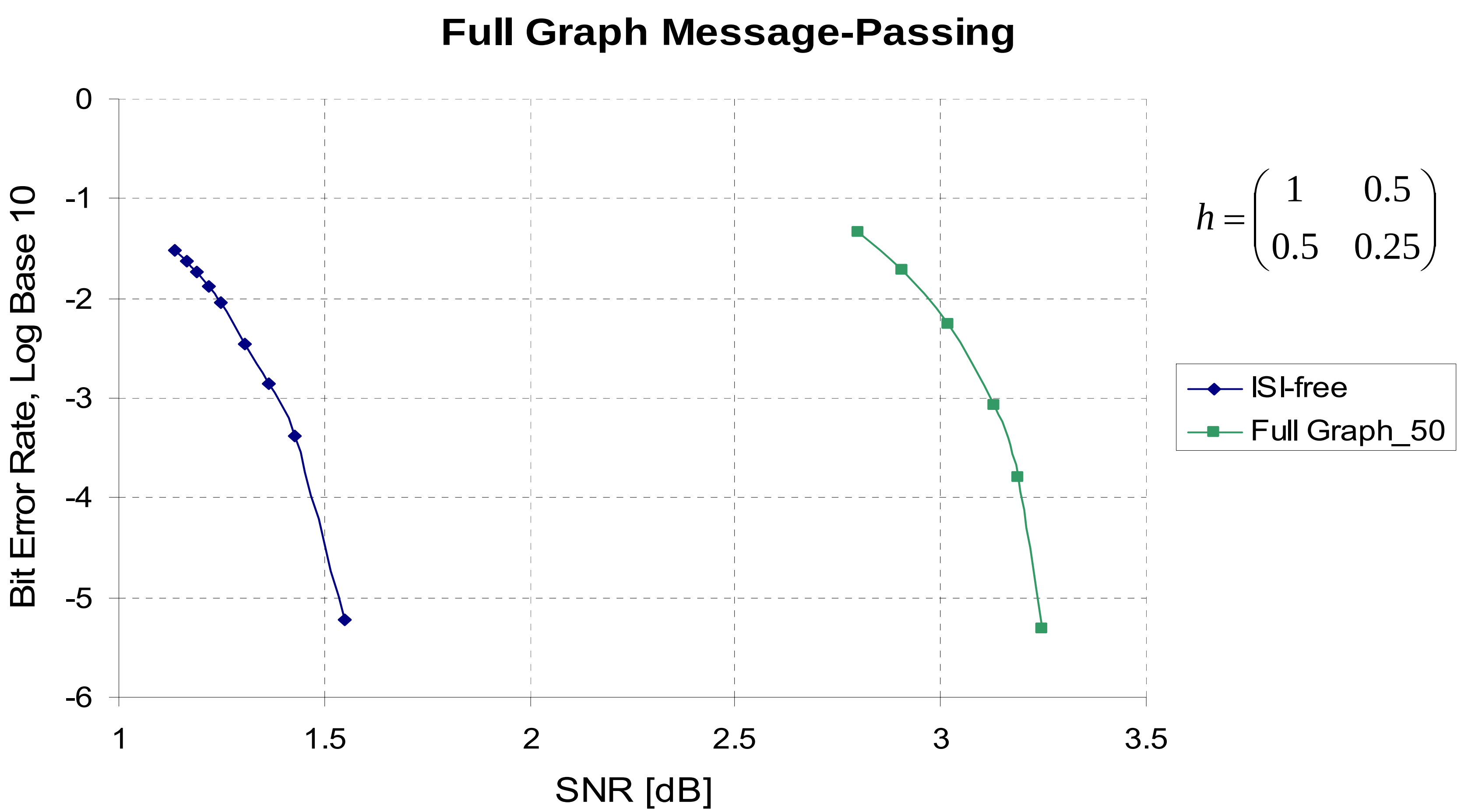
Full Graph



Singla *et al.*, "Iterative decoding and equalization for 2-D recording channels," *IEEE Trans. Magn.*, Sept. 2002.

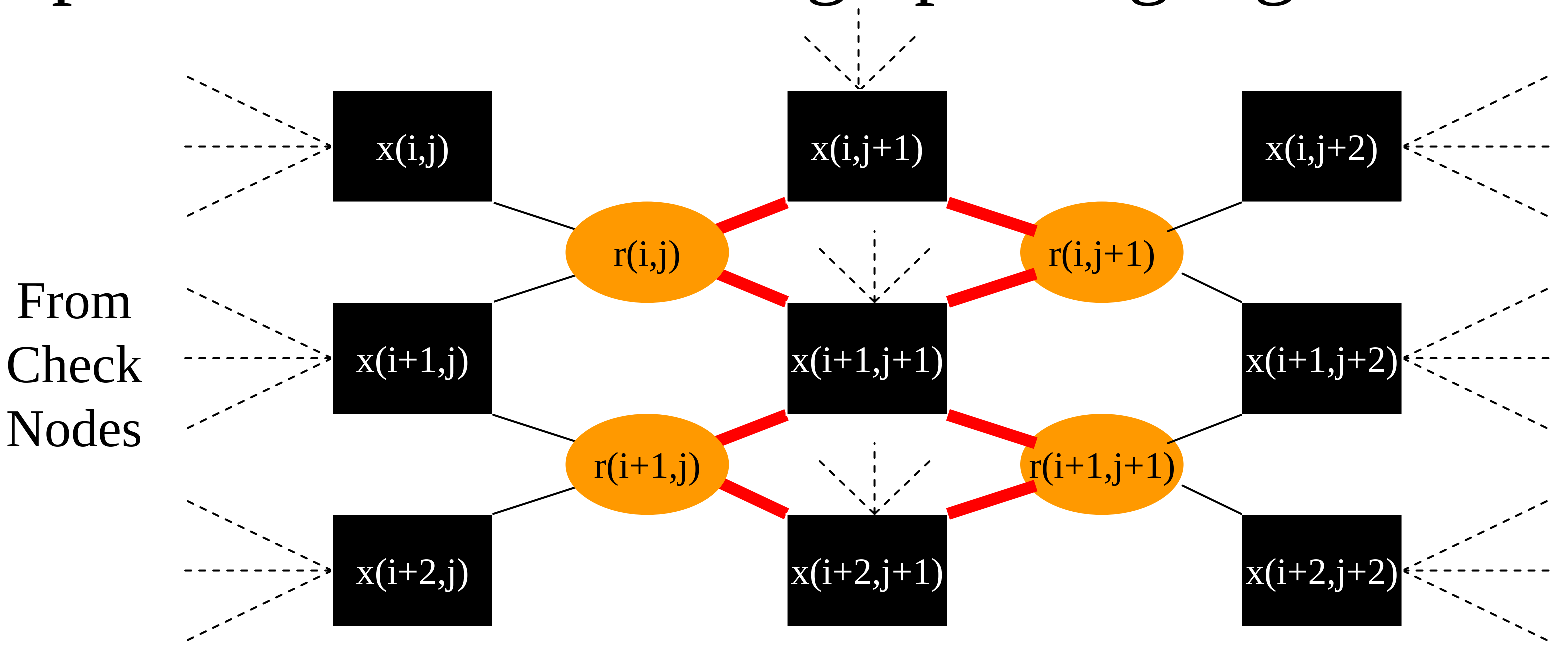
Performance

Block length 10000 regular (3,6) LDPC code



Full Graph Analysis

- Length 4 cycles present which degrade performance of message-passing algorithm



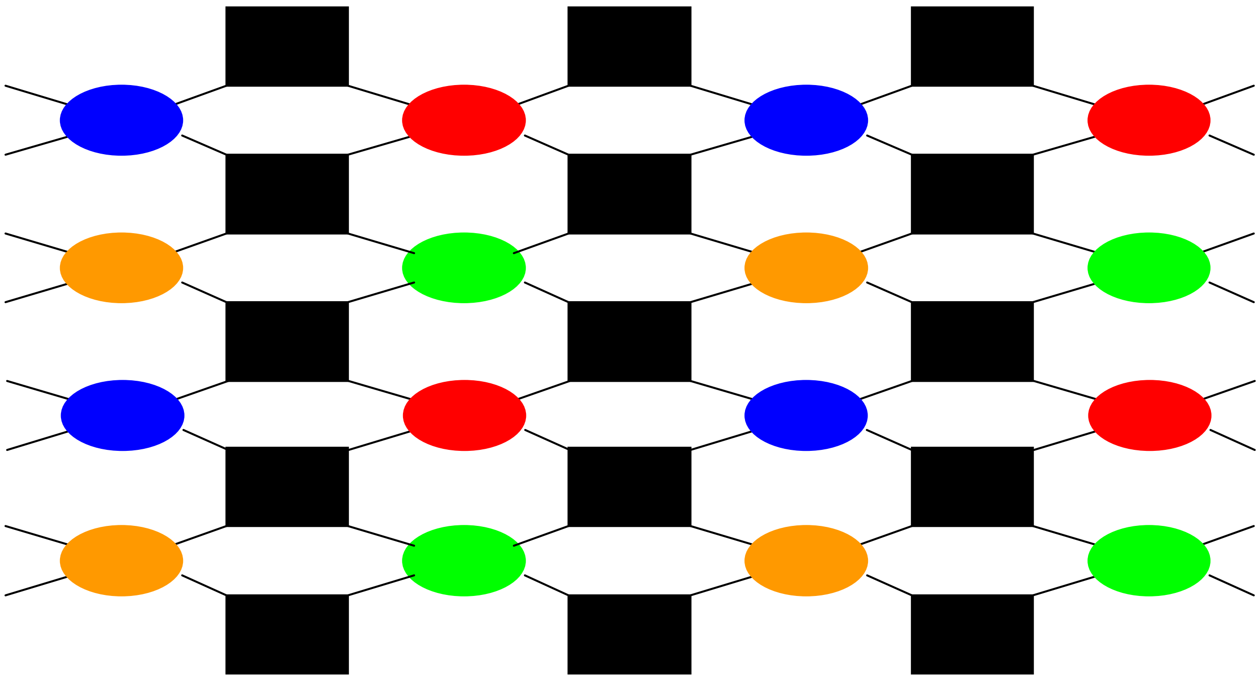
Kschischang *et al.*, "Factor graphs and the sum-product algorithm," *IEEE Trans. Inform. Theory*, Feb. 2001.

Ordered Subsets Message-Passing

- From Imaging – Measured data is grouped into subsets to increase rate of convergence
- For Decoding – Measured data is grouped into subsets to eliminate short length cycles in the channel ISI graph

H. M. Hudson and R. S. Larkin, "Accelerated image reconstruction using ordered subsets of projection data," *IEEE Trans. Medical Imaging*, Dec. 1994.

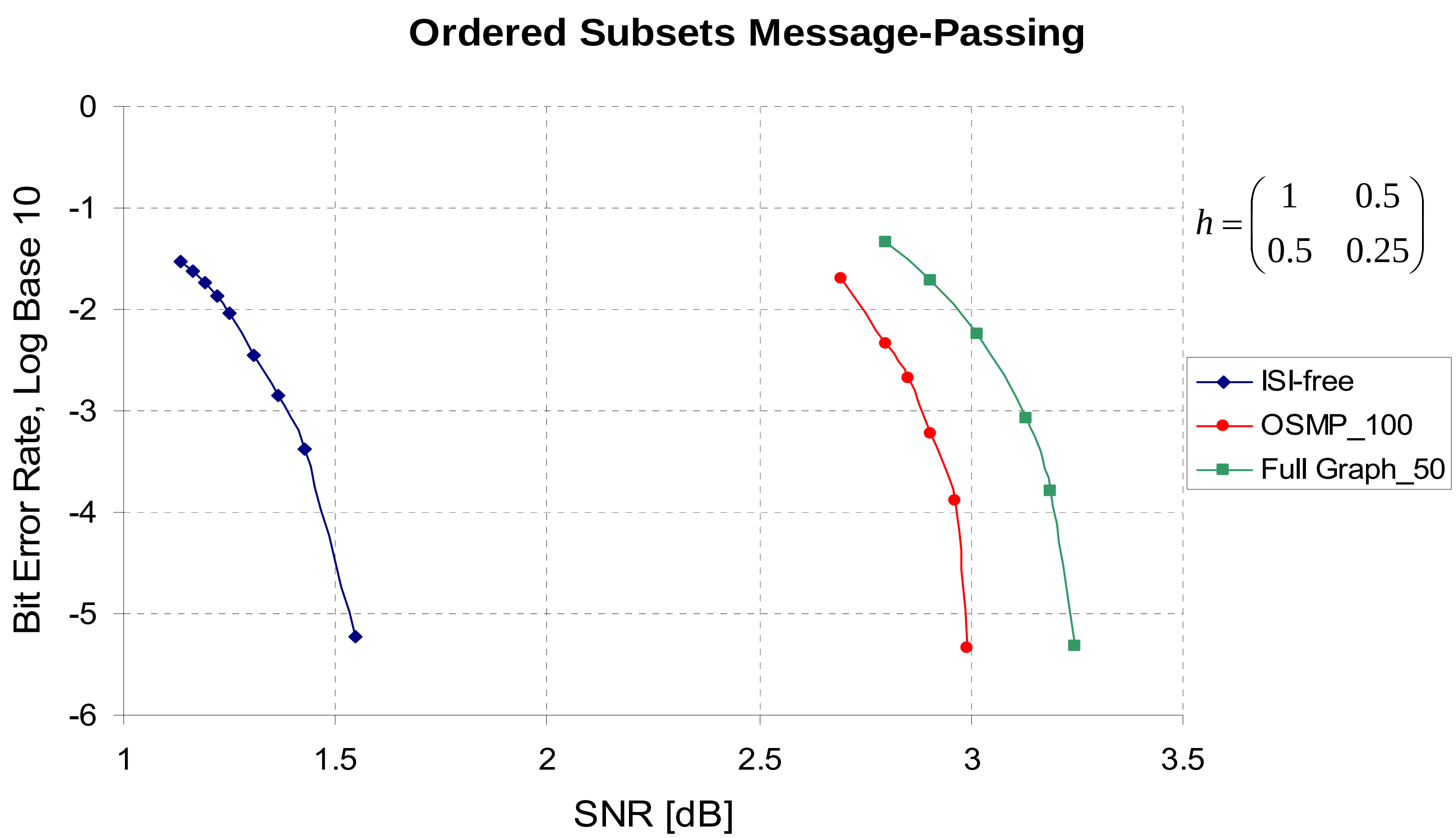
Labeled ISI Graph



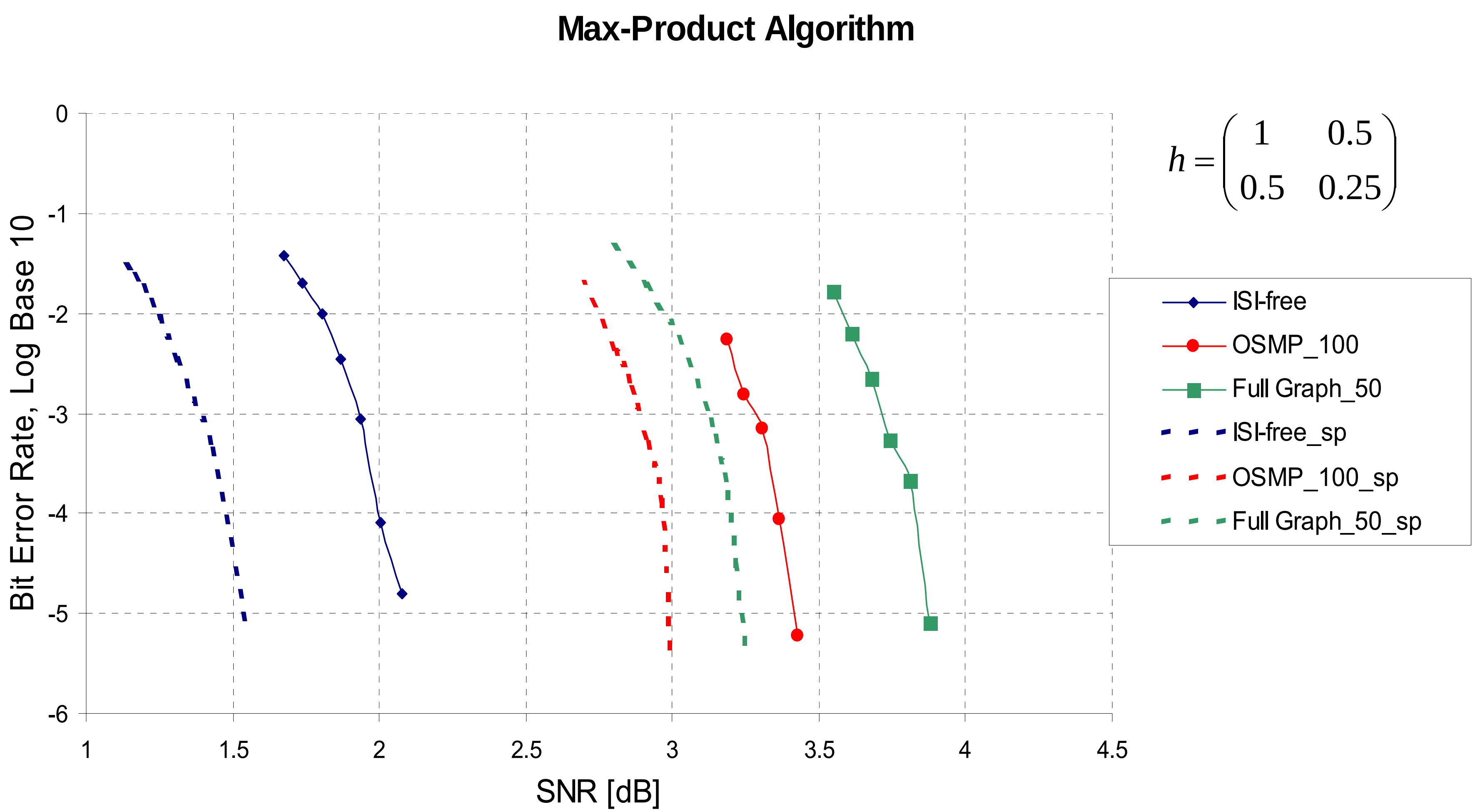
$$h = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 0.25 \end{pmatrix}$$

- Labeling of measured data nodes into 4 subsets
- For each iteration use measured data nodes of one label only

Performance



Max-Product Decoding



Density Evolution

- Codeword bit nodes to check nodes

$$L_{x\to z}^{(l)} = \sum_{m\in N(x)} L_{m\to x}^{(l-1)} + \sum_{z'\in N(x)\backslash z} L_{z'\to x}^{(l-1)}$$ CONVOLUTION

- Check nodes to codeword bit nodes

$$\tanh\frac{L_{z\to x}^{(l)}}{2} = (-1)^{z} \prod_{x'\in N(z)\backslash x} \tanh\frac{L_{x'\to z}^{(l-1)}}{2}$$ LOOKUP TABLE

S.-Y Chung *et al.*, "On the design of low-density parity-check codes within 0.0045 dB of the Shannon limit," *IEEE Comm. Letters*, 2002.

Density Evolution

- Codeword bit nodes to measured data nodes

$$L_{x \to m}^{(l)} = \sum_{m' \in N(x) \setminus m} L_{m' \to x}^{(l-1)} + \sum_{z \in N(x)} L_{z \to x}^{(l)}$$

CONVOLUTION

- Measured data nodes to codeword bit nodes

$$L_{m \to x}^{(l)} = f(\{L_{x' \to m}^{(l)} : x' \in N(m) \setminus x\})$$

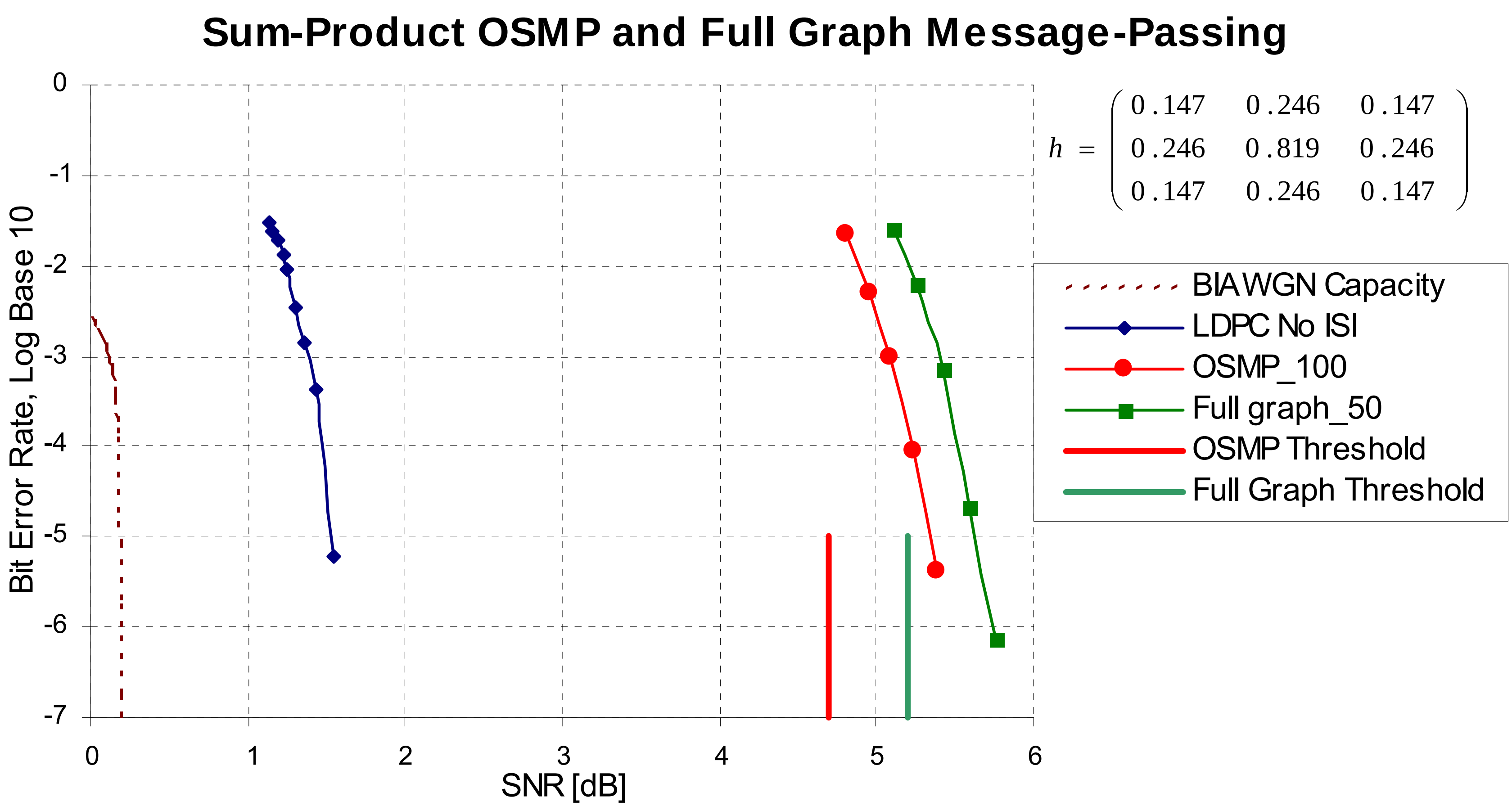
MONTE CARLO SIMULATION

Density Evolution Results

Code Parameters (d_v,d_c)	Threshold OSMP (σ_o)	Threshold Full Graph (σ_f)	SNR Difference $20\log(\sigma_o / \sigma_f)$
(3,4)	1.41	1.34	0.442
(3,6)	0.97	0.91	0.459
(3,30)	0.56	0.52	0.644

$$h = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 0.25 \end{pmatrix}$$

Performance



$$h = \begin{pmatrix} 0.147 & 0.246 & 0.147 \\ 0.246 & 0.819 & 0.246 \\ 0.147 & 0.246 & 0.147 \end{pmatrix}$$

Generalized Belief Propagation

- Link between message-passing algorithms and statistical physics
- Using knowledge of statistical physics to modify message-passing algorithms to achieve better performance in loopy graphs

Is there a link between the OSMP algorithm and GBP?

Yedidia *et al.*, "Generalized Belief Propagation", *Advances in Neural Information Processing Systems (NIPS)*, Dec. 2000.

Conclusions

- Ordered subsets message-passing using the idea from imaging
- Used for joint equalization and decoding for 2-D: gives better performance than full graph algorithm
- Similar results for max-product algorithm
- Density evolution results corroborate simulation results

References

- Kavčić *et al.*, “Binary intersymbol interference channels: Gallager codes, density evolution and code performance bounds,” *IEEE Trans. Inform. Theory*, July 2003.
- T. Richardson and R. Urbanke, “The capacity of low-density parity-check codes under message-passing decoding,” *IEEE Trans. Inform. Theory*, Feb. 2001.

Max-Product Decoding

- Real summation is replaced by “max” operator
- Determines which codeword has largest APP
- In the negative log-likelihood domain max-product becomes “min-sum” which is the well-known Viterbi algorithm

N. Wiberg, “Codes and decoding on general graphs,” Ph. D. dissertation, Linköping Univ., Linköping, Sweden, 1996.