Iterative Decoding for 2D ISI Channels Using Message-Passing

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Outline

- Motivation: Advanced Media
- Joint Equalization and Decoding: Linear 2D ISI
- Full Graph Decoding
- Modified Full Graph Decoding: Ordered Subsets
- Density Evolution for Threshold Behavior
- Conclusions

Advanced Media for Storage

- Blu-ray disc
- Patterned media
- Holographic storage
- Two-dimensional optical storage

Two-Dimensional Intersymbol Interference

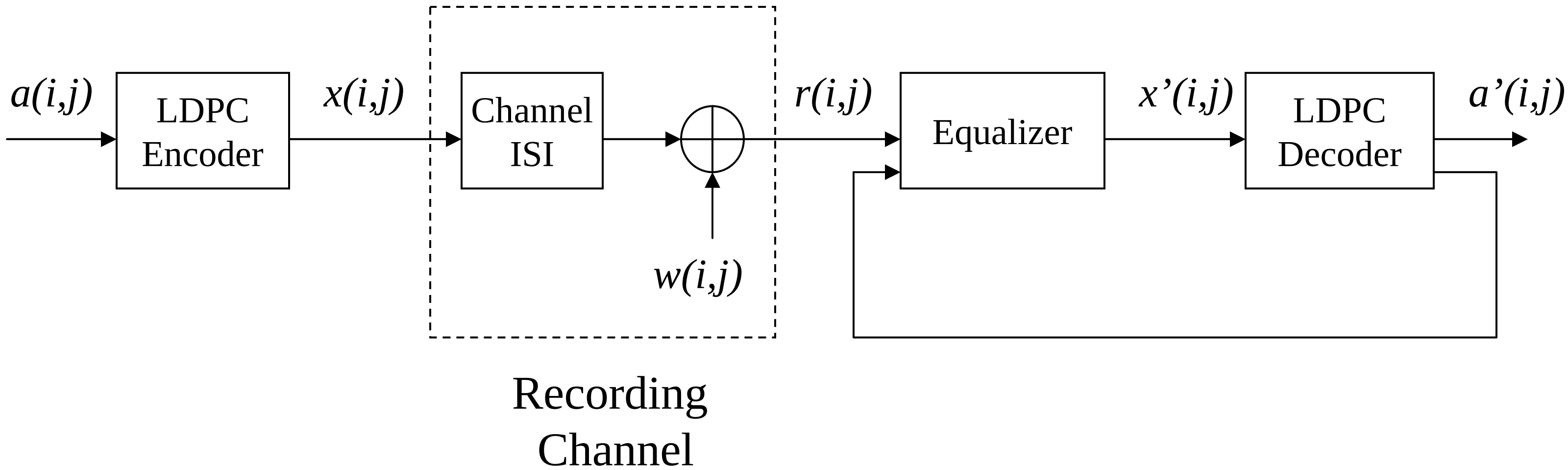
Existing schemes, like the Viterbi algorithm, cannot deal with 2D ISI due to complexity considerations

Joint Equalization and Decoding Schemes for 2D ISI

- Performance can be improved drastically by using error control coding with equalization
- We employ novel message-passing algorithms that take advantage of the 2D dependence

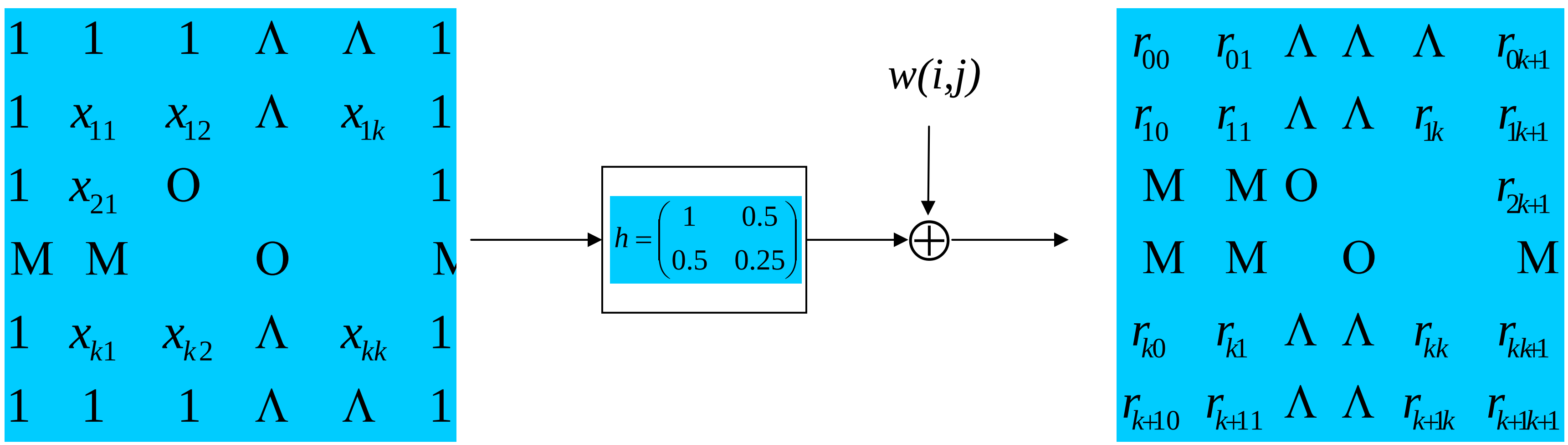
Singla *et al.*, "Iterative decoding and equalization for 2-D recording channels," *IEEE Trans. Magn.*, Sept. 2002.

Channel Model



- Low-density parity-check codes used for error correction
- $x(i,j) \in \{+1,-1\}$
- Channel ISI is 2D
- Noise is assumed to be AWGN

2D Linear Intersymbol Interference

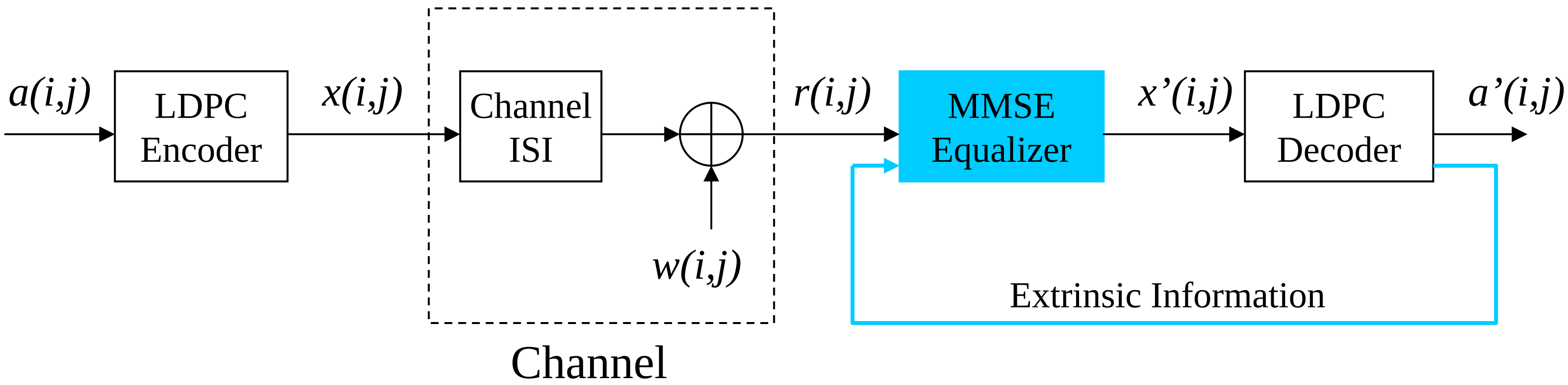


Guard Band

$$r_{i,j} = x_{i,j} + 0.5x_{i-1,j} + 0.5x_{i,j-1} + 0.25x_{i-1,j-1} + w_{i,j}$$

Problem: recover data bits $\boldsymbol{a} \in \{+1,-1\}^{K\times K}$ from $\boldsymbol{r} \in \boldsymbol{R}^{N\times N}$.

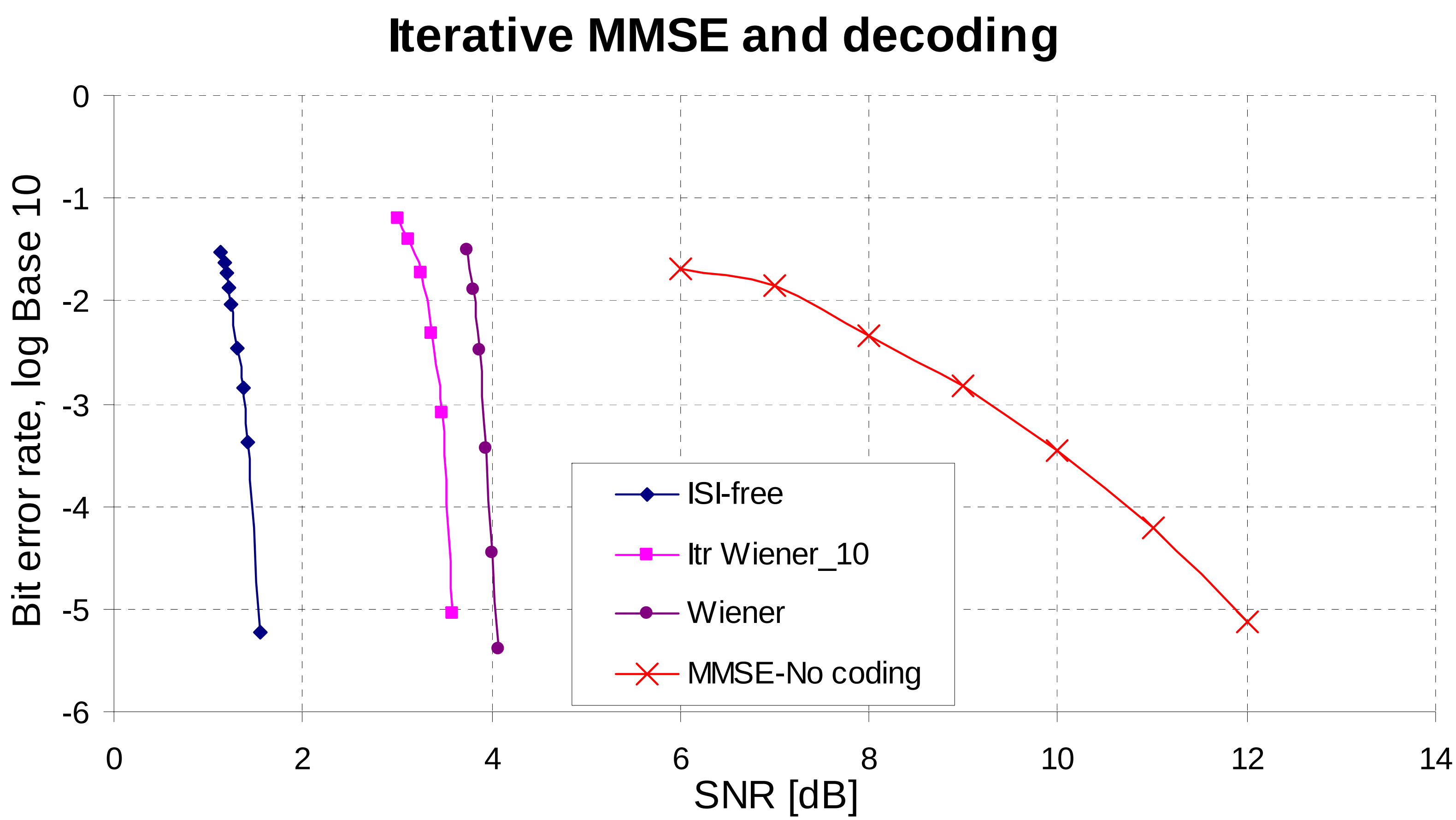
MMSE Equalization



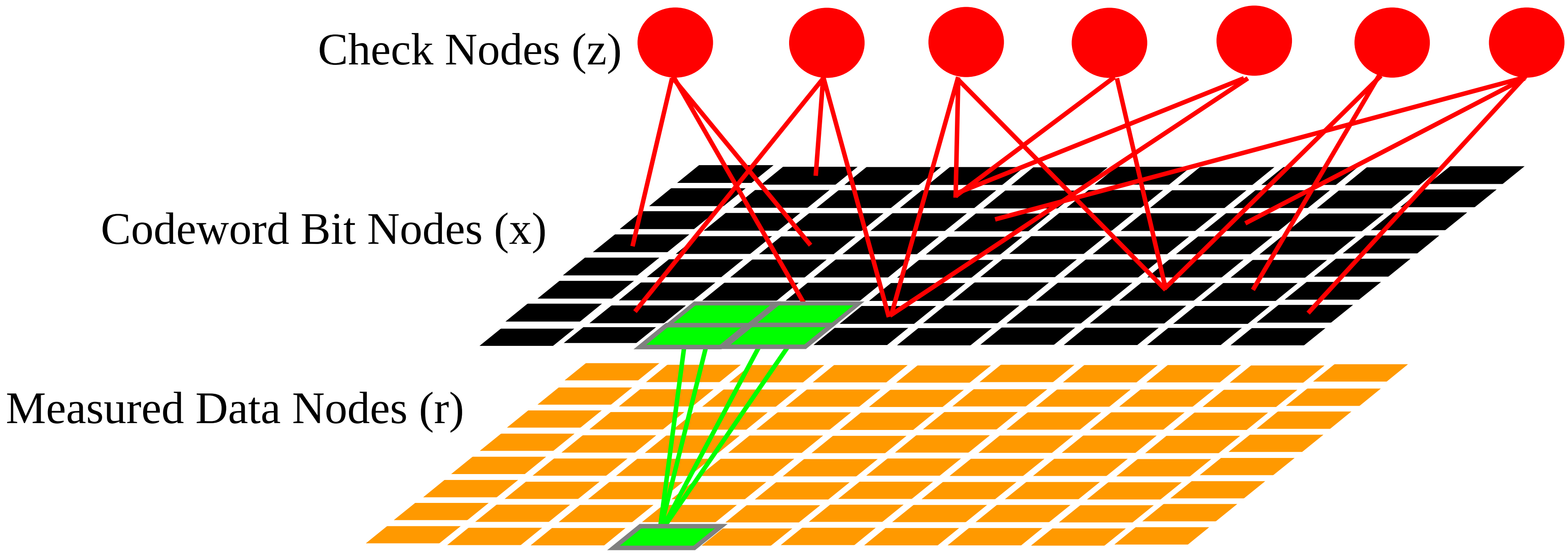
- Equalizer may or may not iterate with the LDPC decoder
- Soft information, estimated mean of the codeword, passed from LDPC decoder to equalizer

Performance

Block length 10000, regular, rate-0.5, LDPC code



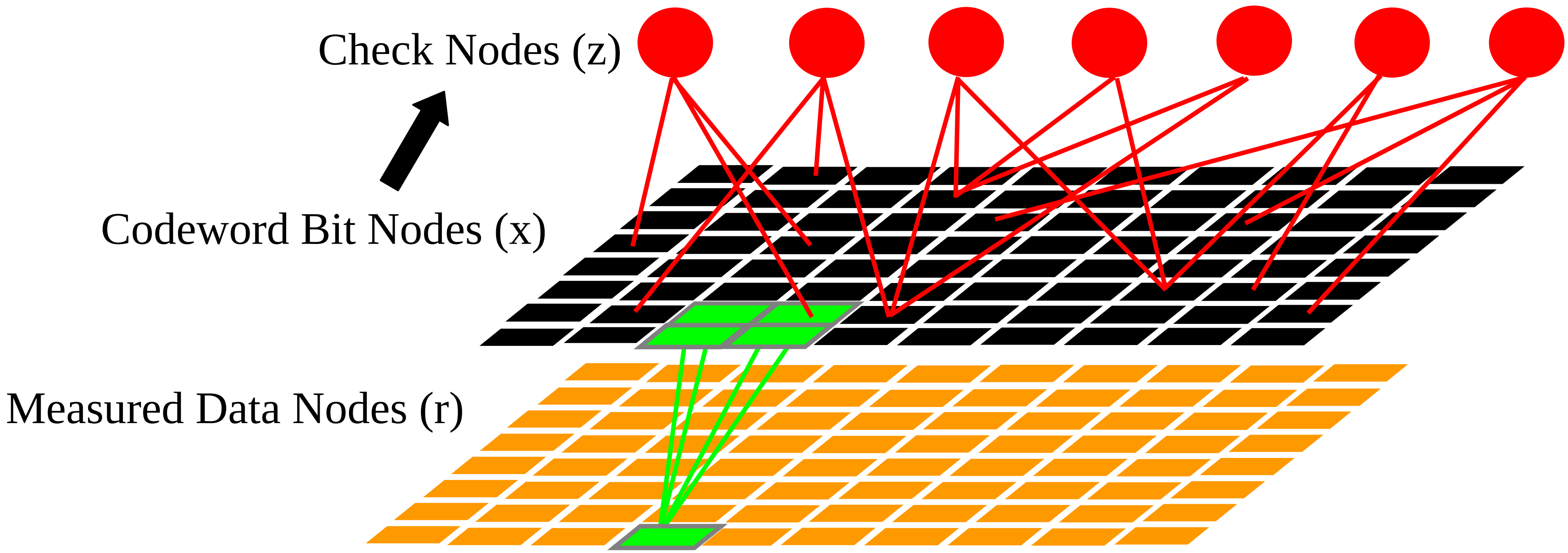
Full Graph Message-Passing



$$h = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 0.25 \end{pmatrix}$$

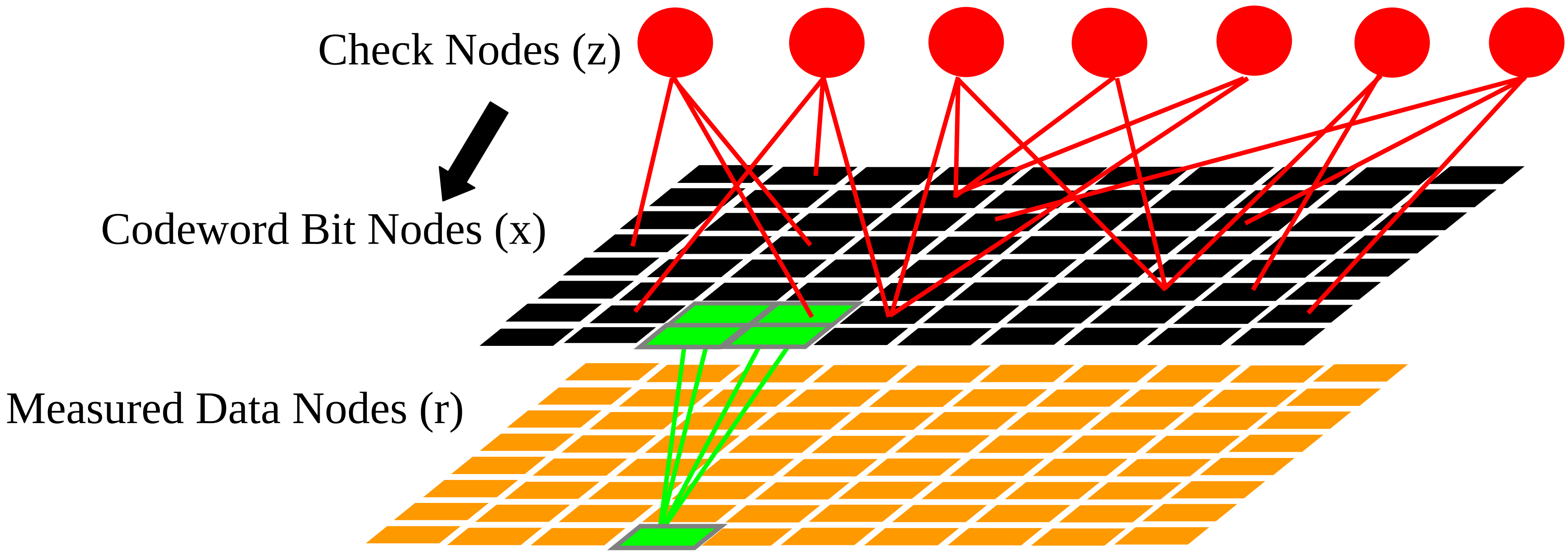
$$r_{i,j} = x_{i,j} + 0.5x_{i-1,j} + 0.5x_{i,j-1} + 0.25x_{i-1,j-1} + w_{i,j}$$

Full Graph Message-Passing



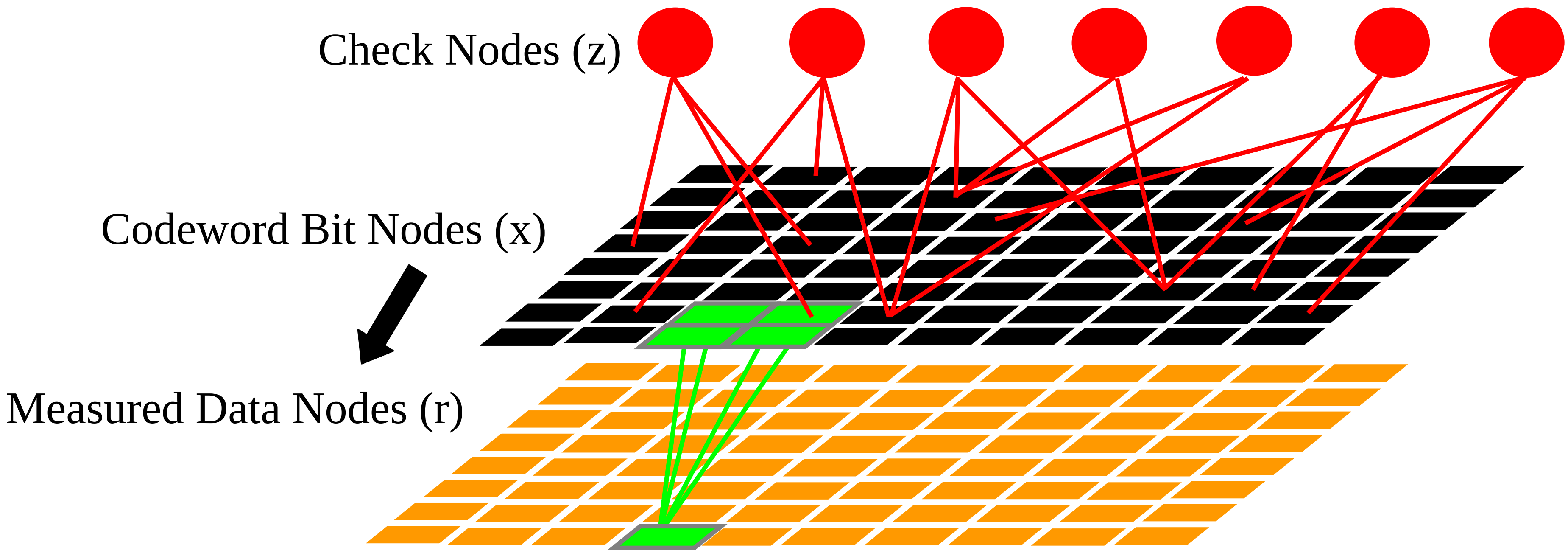
$$L_{x\to z}^{(l)} = \sum_{m\in N(x)} L_{m\to x}^{(l-1)} + \sum_{z'\in N(x)\backslash z} L_{z'\to x}^{(l-1)}$$

Full Graph Message-Passing



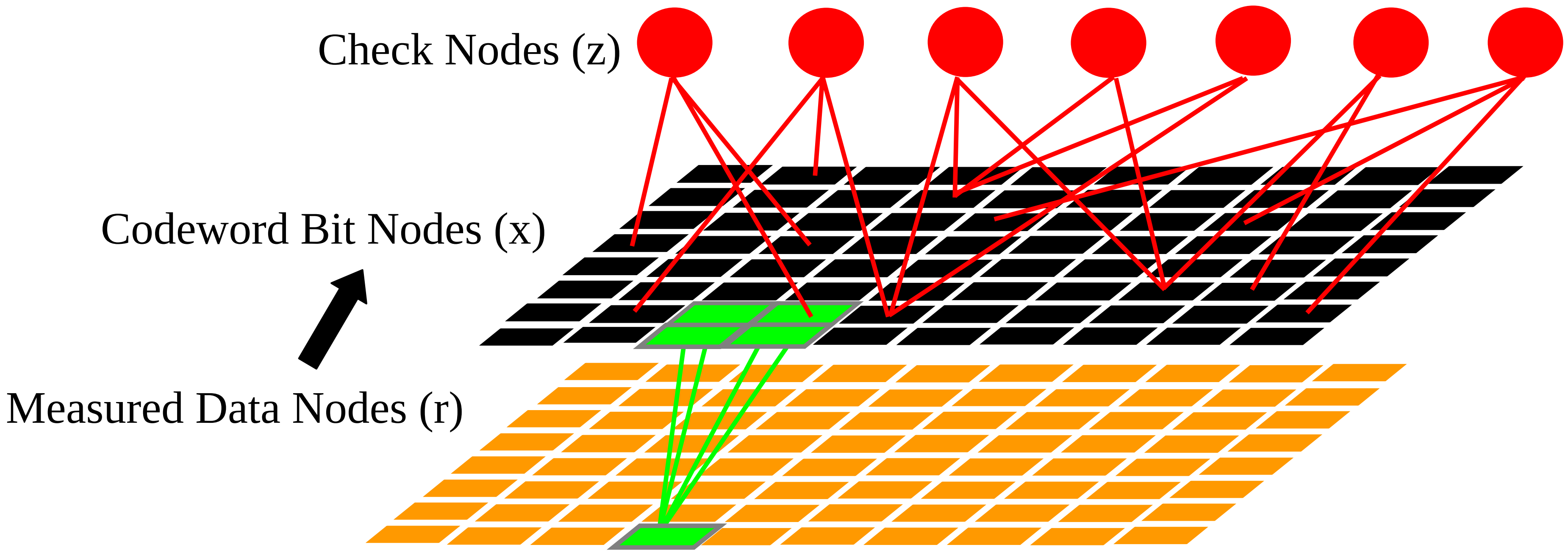
$$\tanh\frac{L_{z\to x}^{(l)}}{2} = (-1)^{z}\prod_{x'\in N(z)\backslash x}\tanh\frac{L_{x'\to z}^{(l-1)}}{2}$$

Full Graph Message-Passing



$$L_{x\to m}^{(l)} = \sum_{m'\in N(x)\backslash m} L_{m'\to x}^{(l-1)} + \sum_{z\in N(x)} L_{z\to x}^{(l)}$$

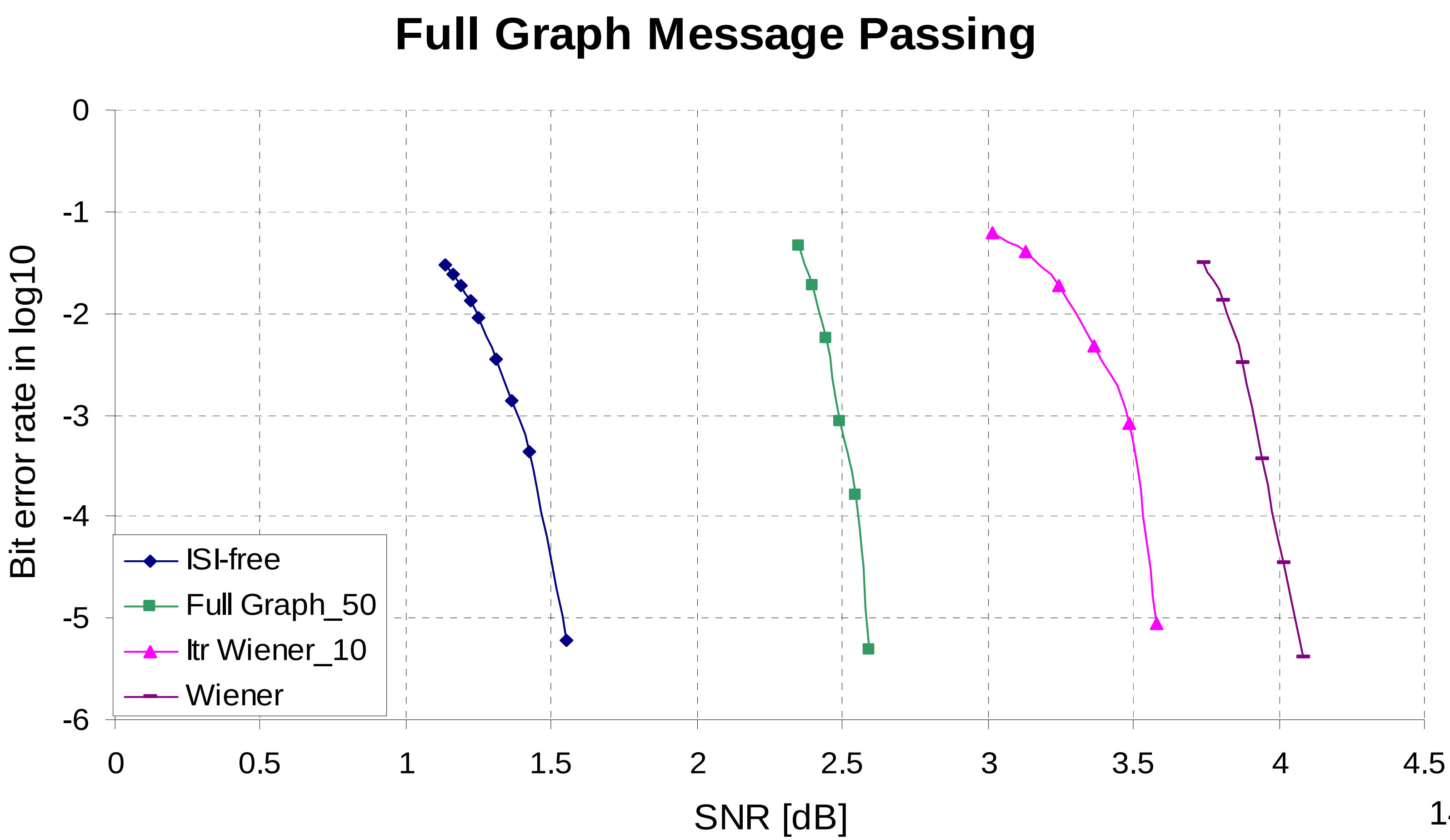
Full Graph Message-Passing



$$L_{m \to x}^{(l)} = f(\{L_{x' \to m}^{(l)} : x' \in N(m) \setminus x\})$$

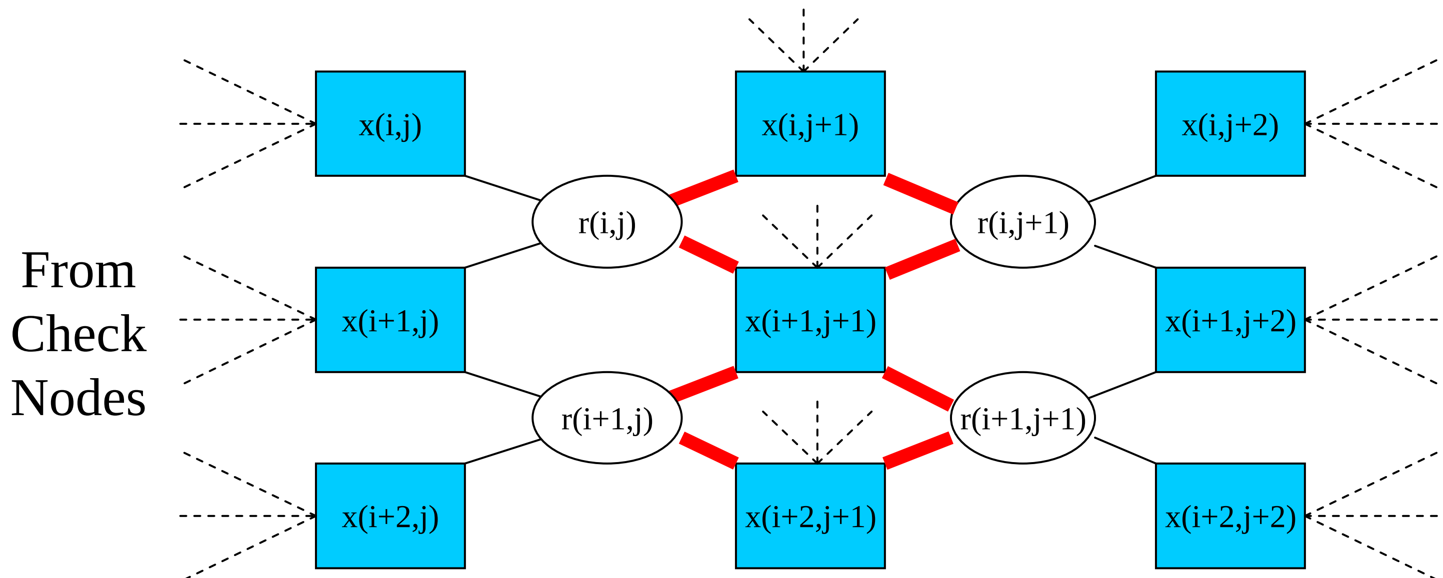
Performance

Block length 10000, regular, rate-0.5, LDPC code



Full Graph Analysis

- Length 4 cycles present which degrade performance of message-passing algorithm



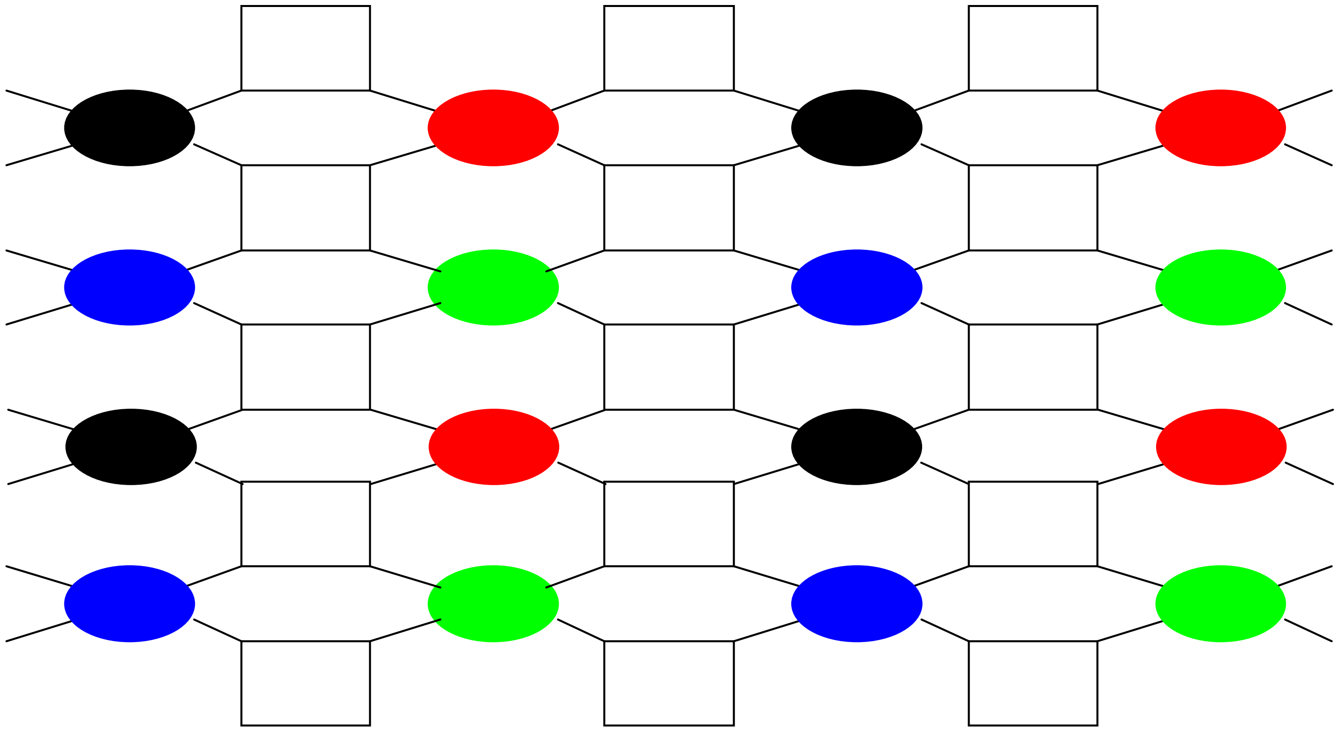
Kschischang *et al.*, "Factor graphs and the sum-product algorithm," *IEEE Trans. Inform. Theory*, Feb. 2001.

Modified Full Graph Message-Passing

- From Imaging – Data set is grouped into subsets to increase rate of convergence
- For Decoding – Measured data is grouped into subsets and a modified schedule is employed: results in increase in girth of full graph

H. M. Hudson, and R. S. Larkin, "Accelerated image reconstruction using ordered subsets of projection data," *IEEE Trans. Med. Img.*, Dec. 1994

Grouped ISI Graph

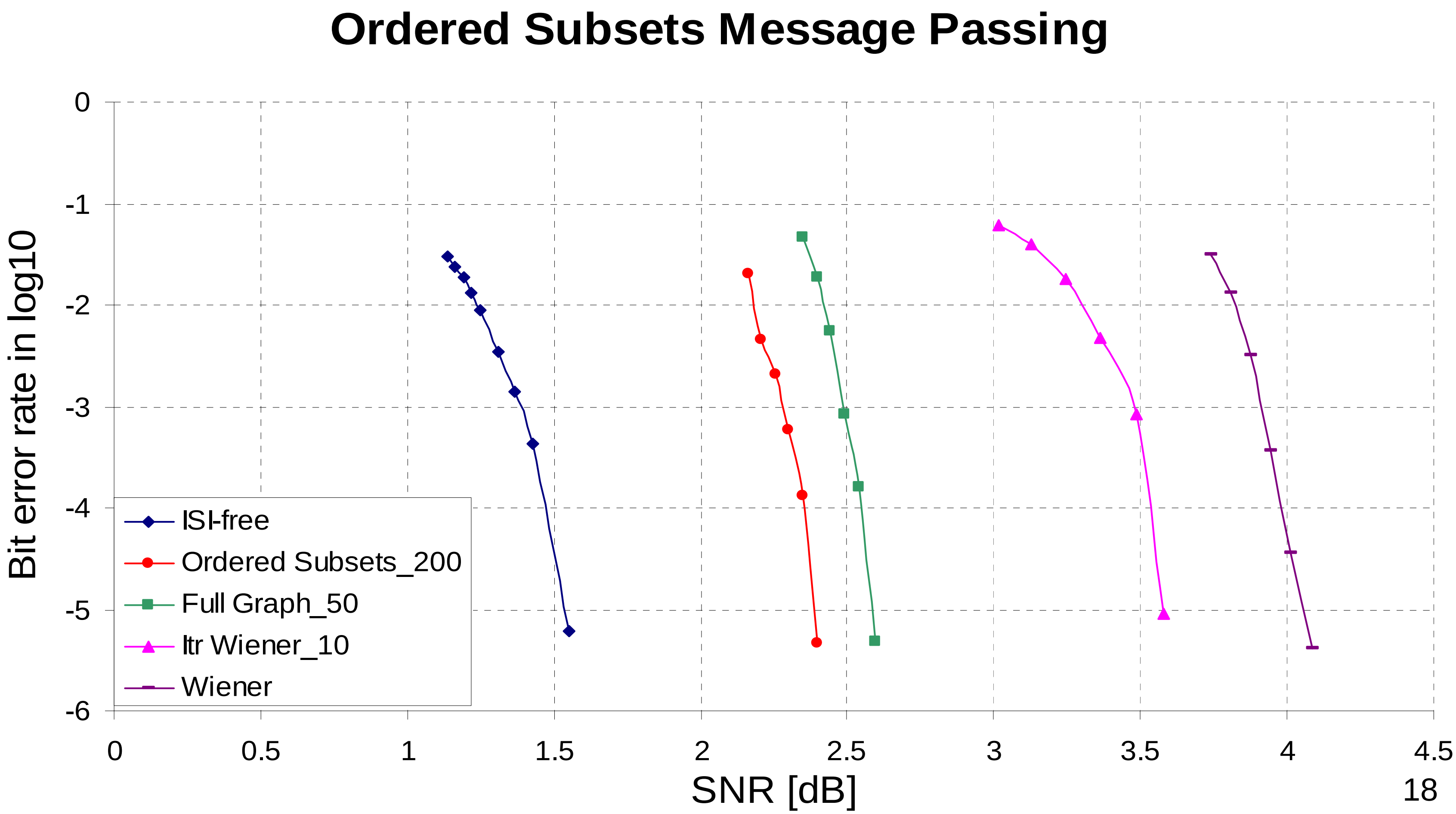


- Labeling of data nodes into 4 subsets
- For each iteration use data nodes of one label only

J. A. O'Sullivan, and N. Singla, "Ordered subsets message-passing," *Int'l Symp. Inform. Theory,* Yokohama, Japan 2003.

Performance

Block length 10000, regular, rate-0.5, LDPC code



Density Evolution

- Assume messages are i.i.d. random variables
- Evolve message densities through the message maps
- If densities converge to desired density, then error-free transmission possible otherwise not
- Gives lower bound on performance of message-passing scheme

T. Richardson and R. Urbanke, "Capacity of low-density parity-check Codes under message-passing decoding," *IEEE Trans. Inform. Theory*, Feb. 2001

Density Evolution for Full Graph Message-Passing

- Codeword bit nodes to check nodes

$$L_{x\to z}^{(l)} = \sum_{m\in N(x)} L_{m\to x}^{(l-1)} + \sum_{z'\in N(x)\backslash z} L_{z'\to x}^{(l-1)} \quad \text{CONVOLUTION}$$

- Check nodes to codeword bit nodes

$$\tanh\frac{L_{z\to x}^{(l)}}{2} = (-1)^{z} \prod_{x'\in N(z)\backslash x} \tanh\frac{L_{x'\to z}^{(l-1)}}{2} \quad \text{LOOKUP TABLE}$$

Density Evolution…

- Codeword bit nodes to measured data nodes

$$L_{x\to m}^{(l)} = \sum_{m'\in N(x)\backslash m} L_{m'\to x}^{(l-1)} + \sum_{z\in N(x)} L_{z\to x}^{(l)}$$

CONVOLUTION

- Measured data nodes to codeword bit nodes

$$L_{m\to x}^{(l)} = f(\{L_{x'\to m}^{(l)} : x'\in N(m)\setminus x\})$$

MONTE CARLO SIMULATION

Density Evolution Results

Thresholds for Full graph algorithms

Code Parameters (d_v,d_c)	Rate	Threshold SNR [dB] Full Graph	Threshold SNR [dB] Modified Full Graph
(3,4)	0.25	1.86	1.69
(3,6)	0.50	1.91	1.73
(3,30)	0.90	4.41	4.21

Conclusions

- Joint iterative equalization and decoding schemes for 2D ISI using message-passing are proposed
- Using the idea of partitioning measured data into subsets improved full graph algorithm is obtained
- Noise tolerance thresholds computed using density evolution are better for modified full graph algorithm