2D Coding for Future Perpendicular and Probe Recording

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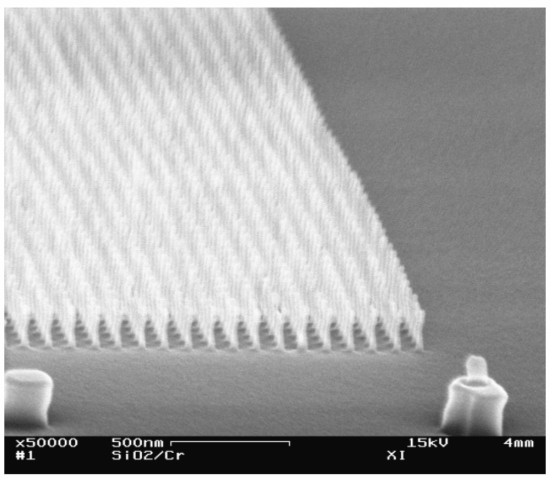

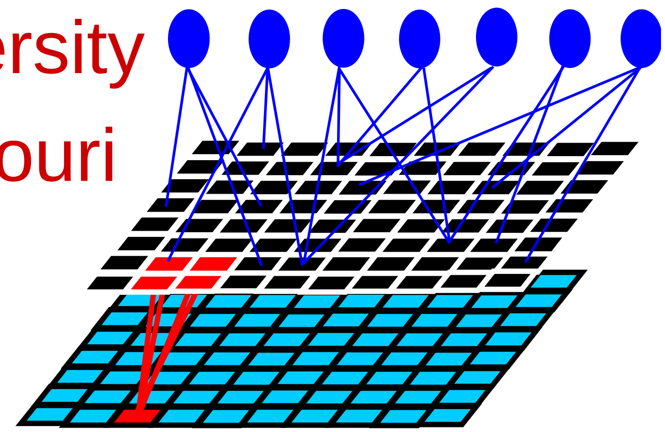

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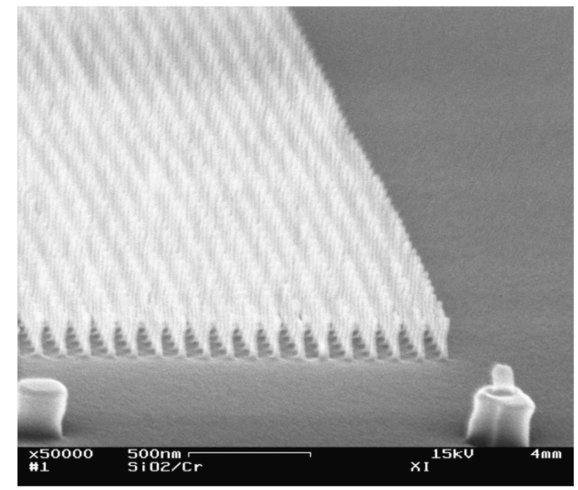
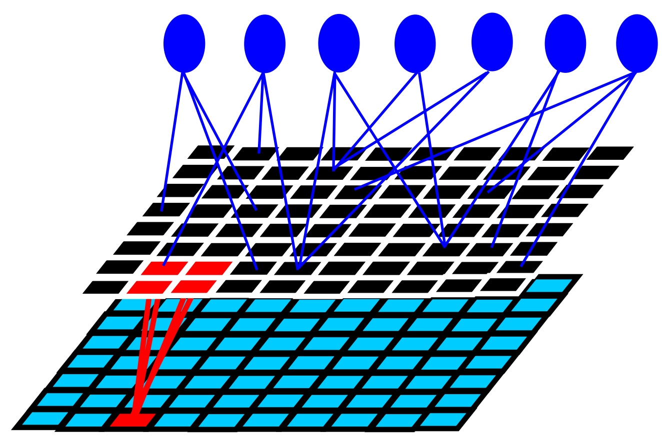
Outline: 2D Coding for 2D ISI

- Motivation:
  - 2D Intersymbol Interference (ISI+ITI)
  - 2D Patterned/Self-Organized Media
- Iterative Equalization and Decoding
  - LDPC Codes
  - Performance Results
- Extensions
- Performance Predictions
  - thresholds from density evolution
- Conclusions





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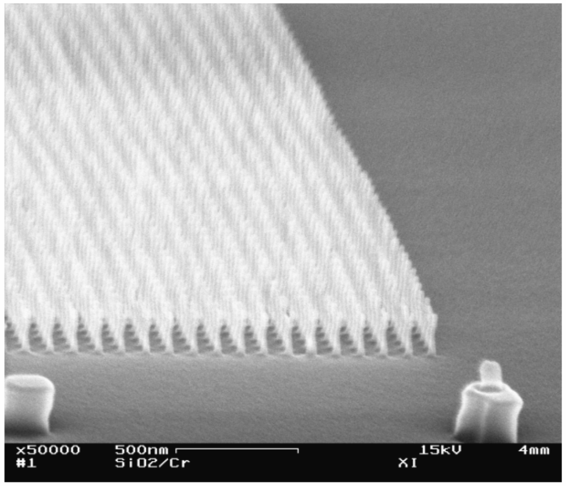
Outstanding Issue: Two-Dimensional ISI



- Patterned media
  - interference from neighbors in 2D
- Conventional magnetic storage
  - reduce bit-aspect-ratio increases ITI
- Probe recording
  - probe's point spread function
- Optical recording
  - laser spot spread; page oriented optical memories
  - nonlinear ISI

Existing schemes (Viterbi, BCJR) do not directly extend to 2D ISI

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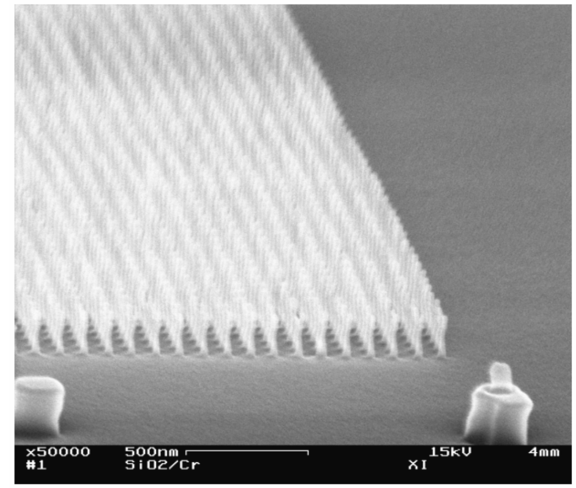

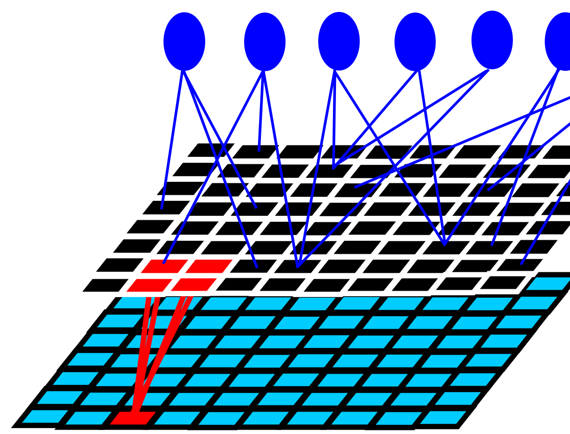

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Joint Equalization and Decoding Schemes for 2D ISI

- General 2D ISI
  - using 2D MMSE equalization and decoding
  - using novel message-passing algorithms that take advantage of the 2D dependence
- Separable 2D ISI
  - using turbo equalization

Singla *et al.*, "Iterative decoding and equalization for 2-D recording channels," *IEEE Trans. Magn.*, Sept. 2002.

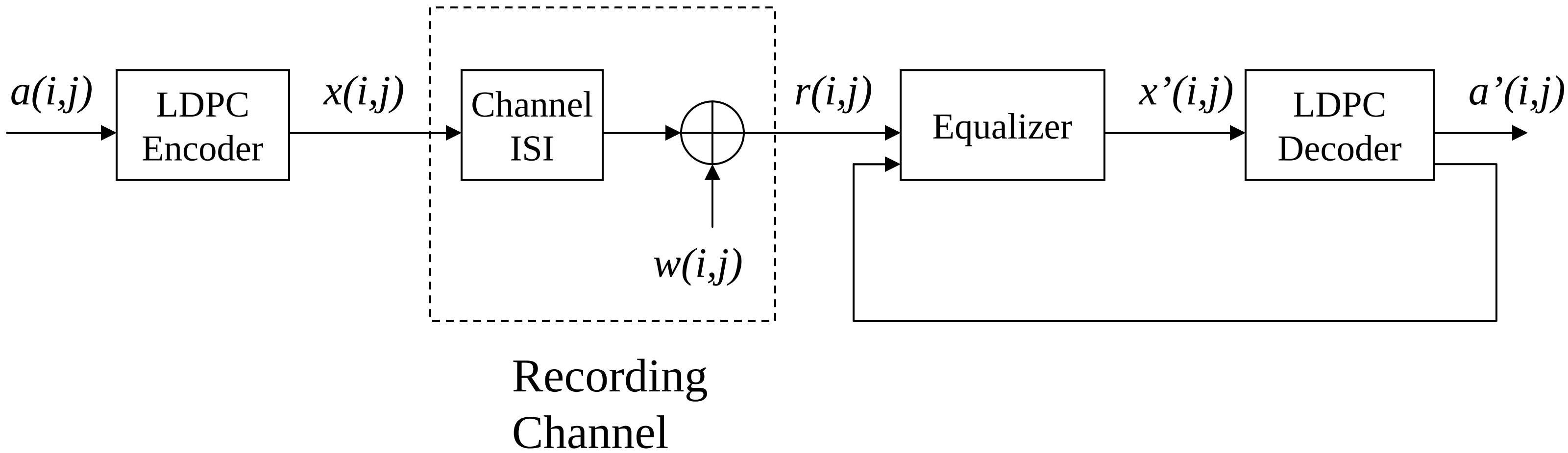
Joint Equalization and Decoding Schemes for 2D ISI

Performance can be improved by combining error control coding with equalization

- Base on existing equalization schemes
- Jointly model channel ISI and parity check matrix for error control code—three level graph
- Employ novel message-passing algorithms that take advantage of the 2D dependence
- Separable ISI: Reduced Complexity
  Turbo Equalization

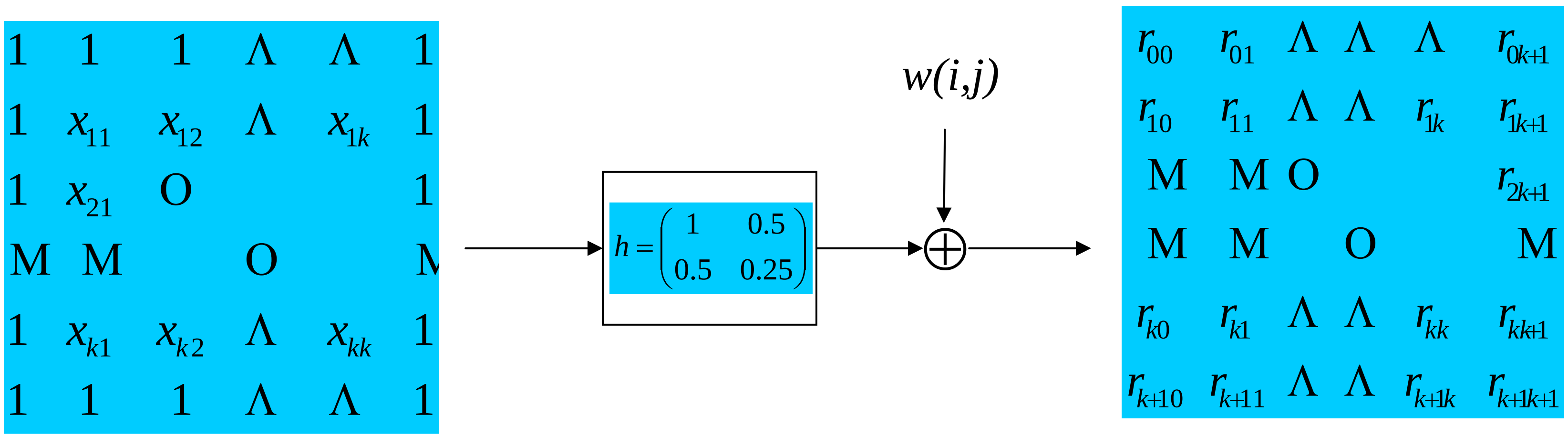
Singla *et al.*, "Iterative decoding and equalization for 2-D recording channels," *IEEE Trans. Magn.*, Sept. 2002.

Channel Model



- $x(i,j) \in \{+1,-1\}$
- Channel ISI is 2D and linear
  - Extensions to nonlinear
- Noise assumed to be AWGN

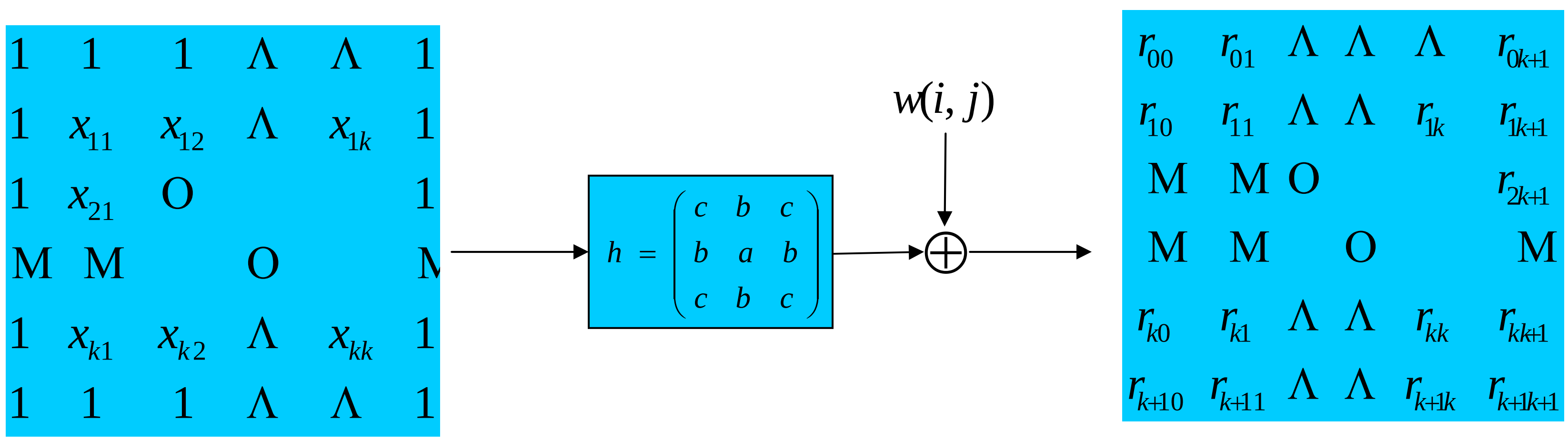
2D Linear Intersymbol Interference



Includes
guard band

$$r_{i,j} = x_{i,j} + 0.5x_{i-1,j} + 0.5x_{i,j-1} + 0.25x_{i-1,j-1} + w_{i,j}$$

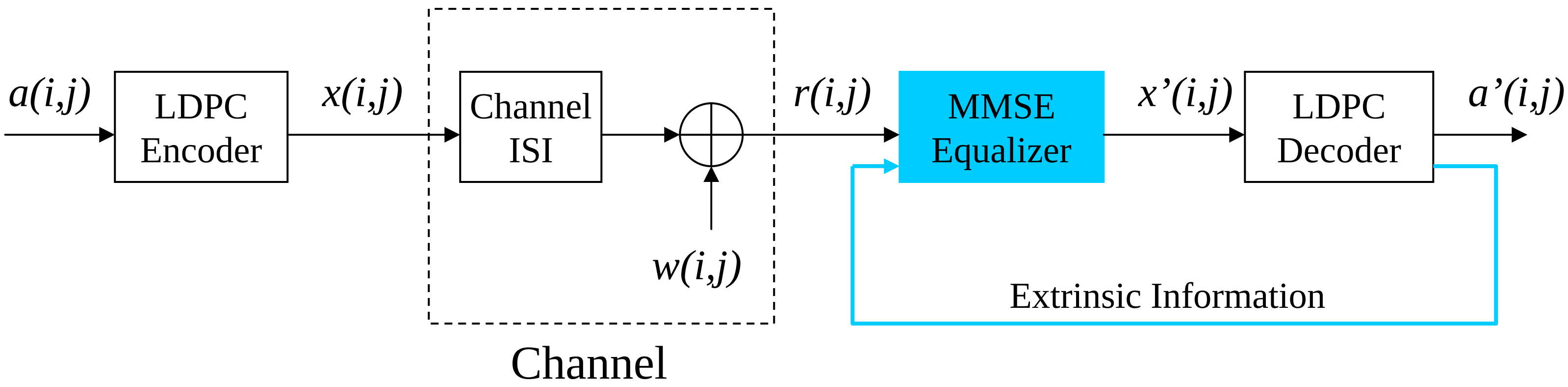
2D Linear Intersymbol Interference



GUARD BAND

$$r(i,j)=\sum_{m,n=0}^{2} x(i-m,j-n)h(m,n)+w(i,j)$$

MMSE Equalization



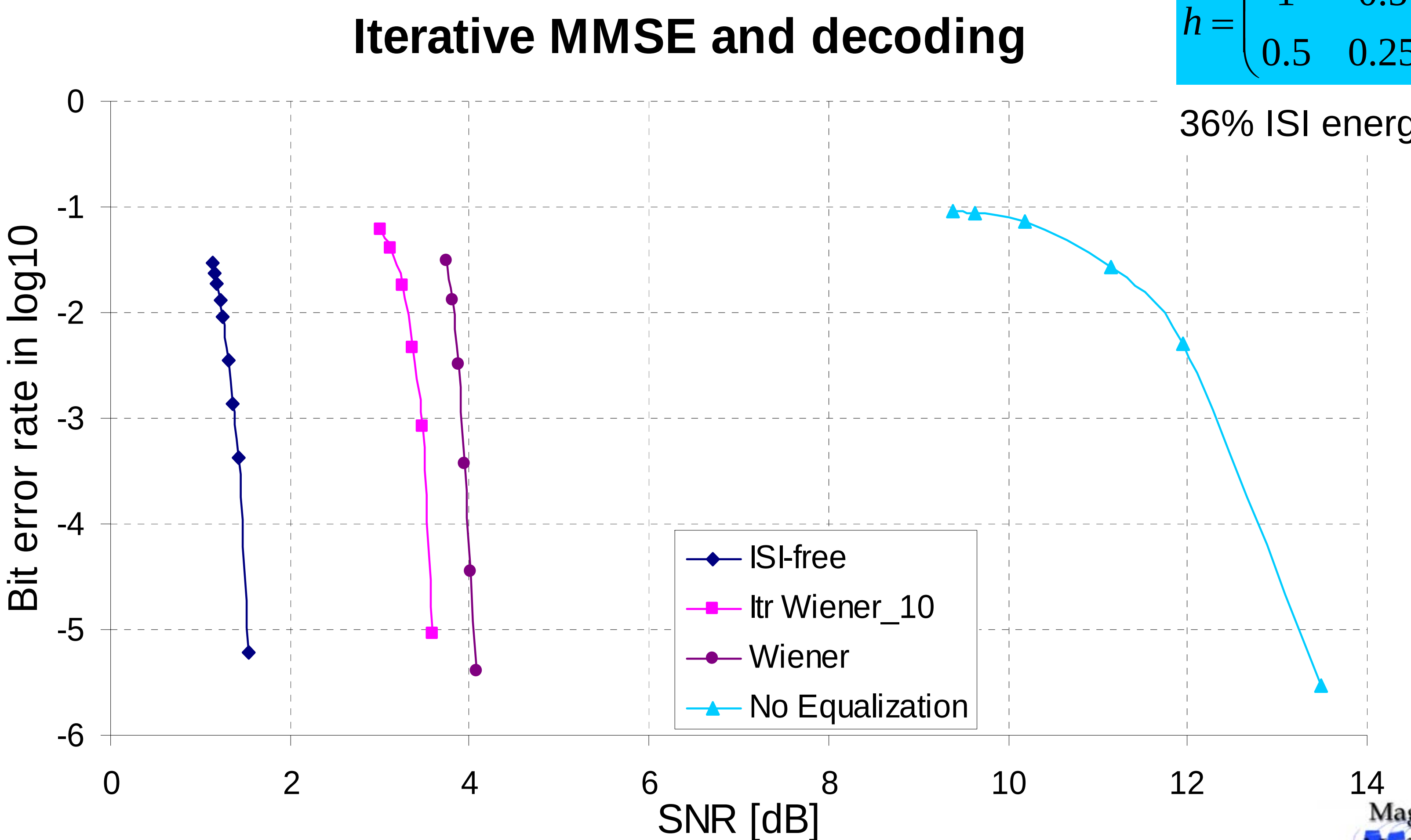
- Equalizer may or may not iterate with the LDPC decoder
- Soft information, estimated mean of the codeword, passed from LDPC decoder to equalizer

Performance

Block length 10000 regular (3,6) LDPC code

$$h = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 0.25 \end{pmatrix}$$

36% ISI energy



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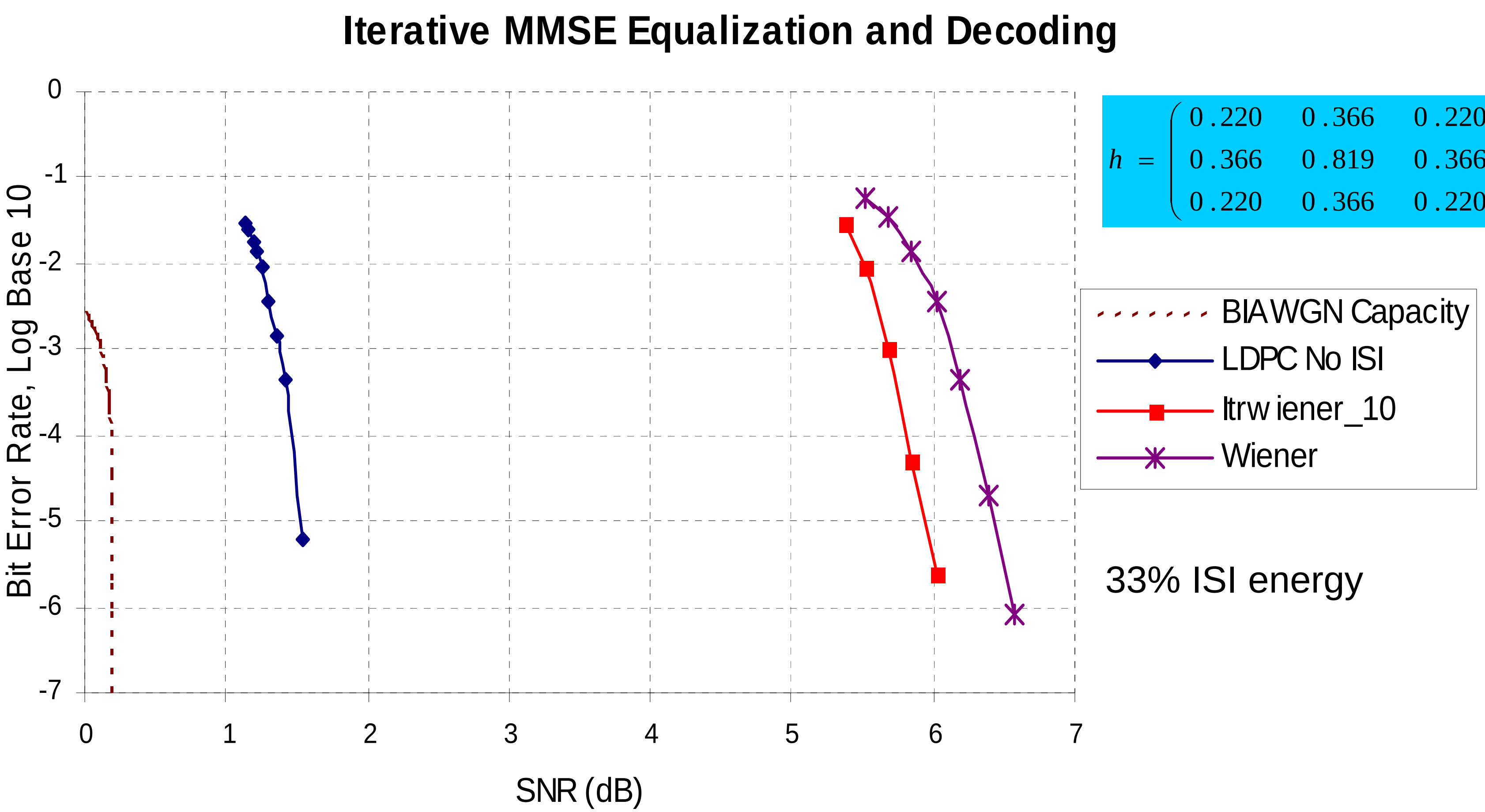
Performance

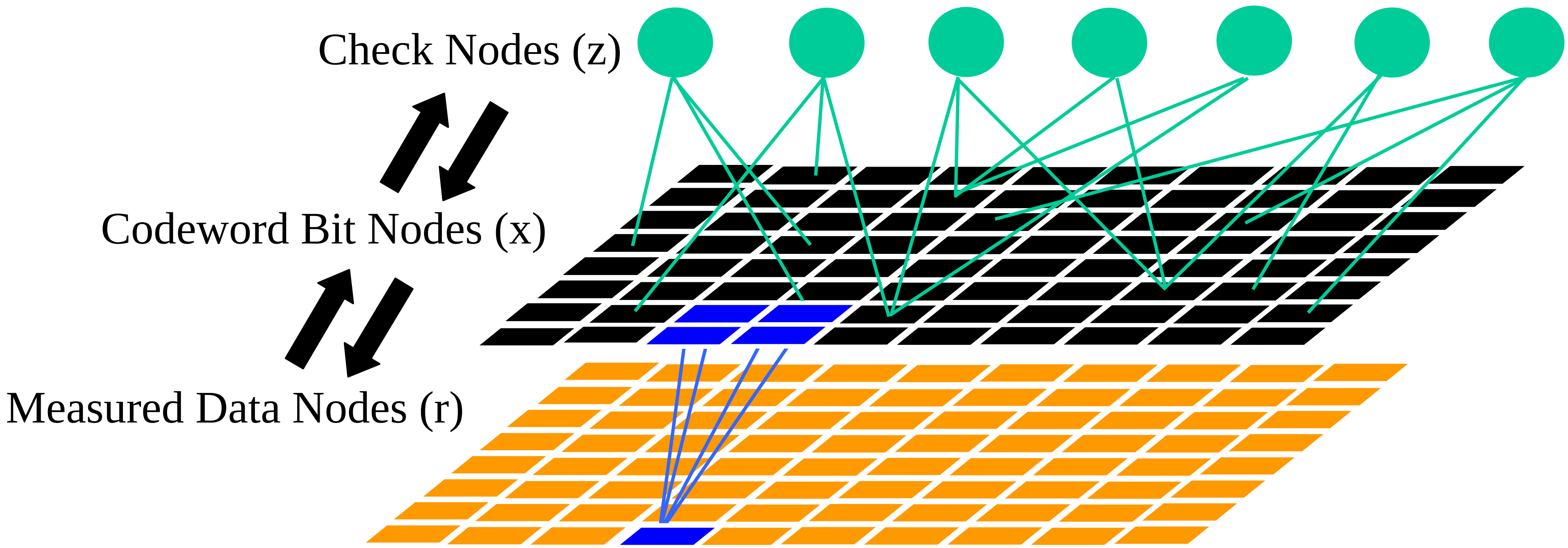


$$h = \begin{pmatrix} 0.220 & 0.366 & 0.220 \\ 0.366 & 0.819 & 0.366 \\ 0.220 & 0.366 & 0.220 \end{pmatrix}$$

33% ISI energy

 J. A. O’Sullivan, et al. PMRC 2004

Full Graph Message-Passing



$$h = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 0.25 \end{pmatrix}$$

$$r_{i,j} = x_{i,j} + 0.5x_{i-1,j} + 0.5x_{i,j-1} + 0.25x_{i-1,j-1} + w_{i,j}$$

Performance

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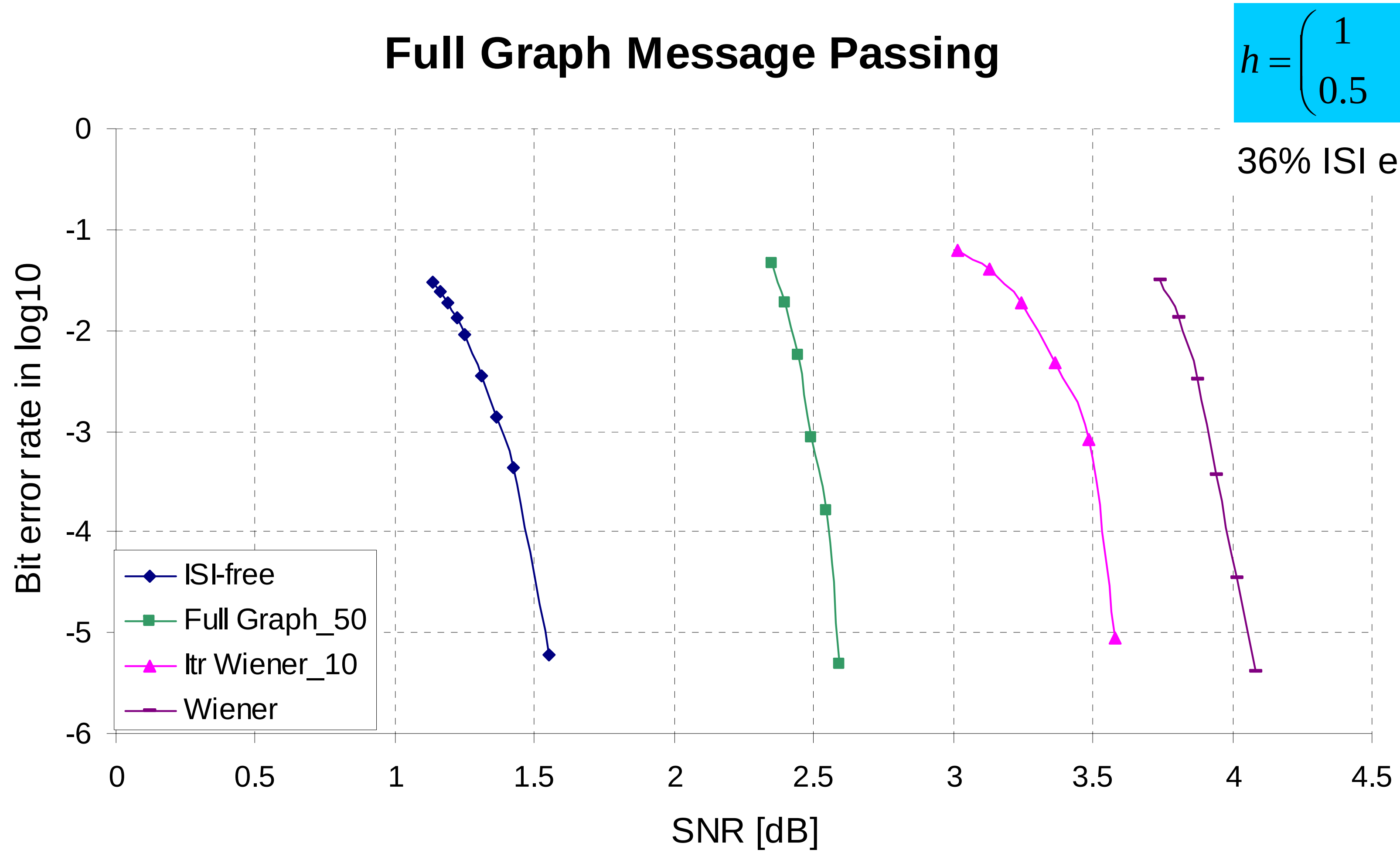

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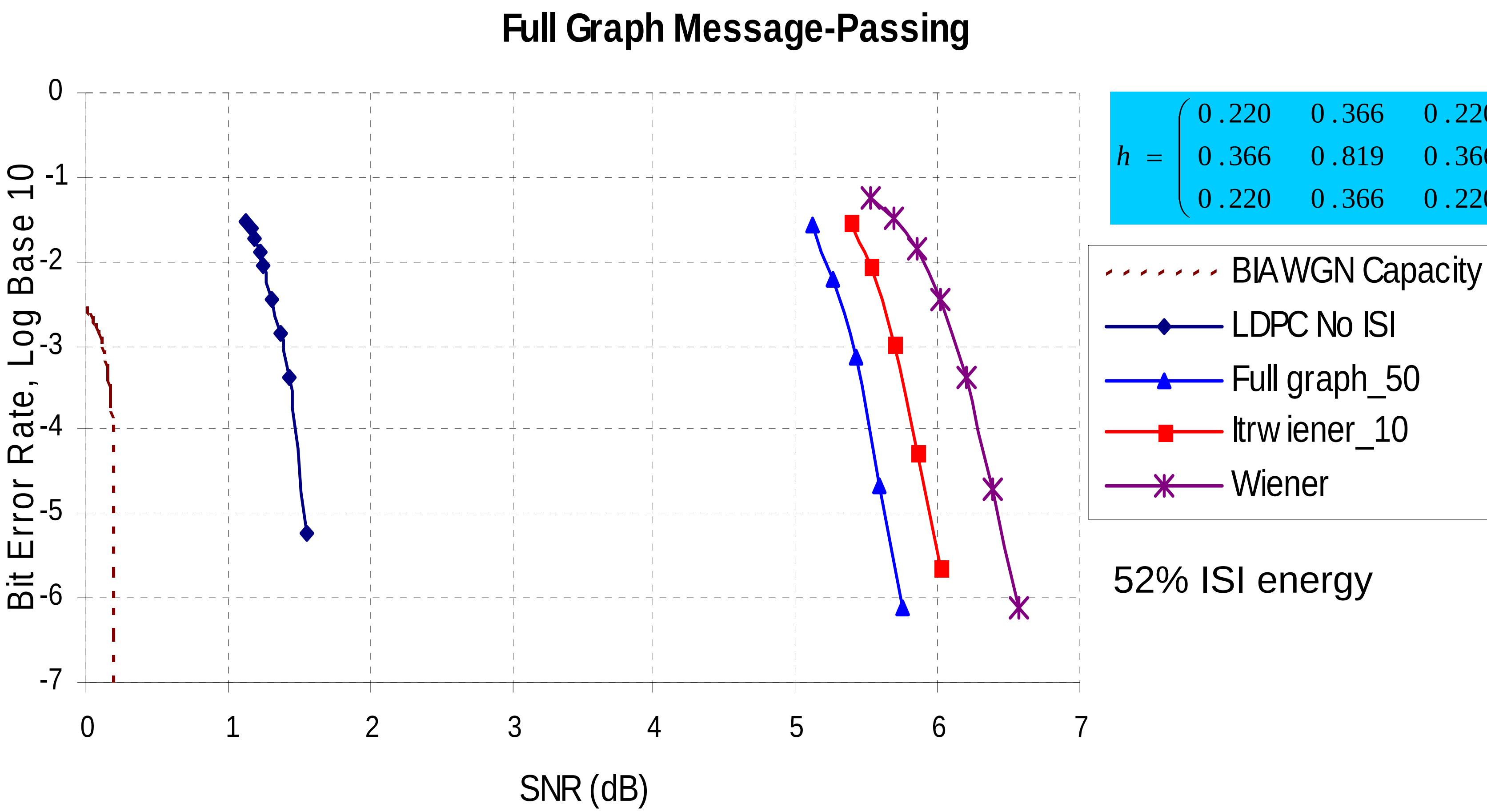
Performance



$$h = \begin{pmatrix} 0.220 & 0.366 & 0.220 \\ 0.366 & 0.819 & 0.366 \\ 0.220 & 0.366 & 0.220 \end{pmatrix}$$

52% ISI energy

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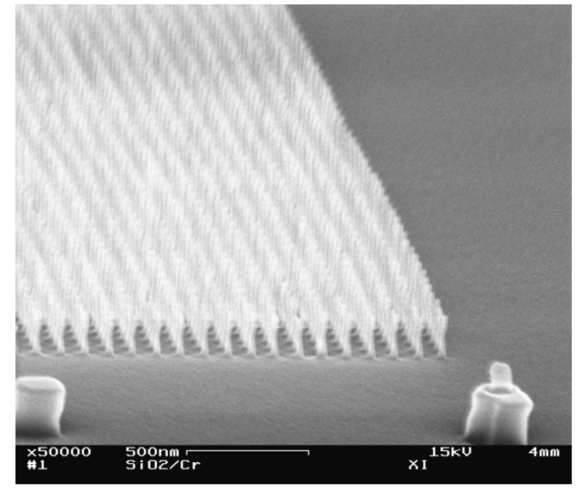

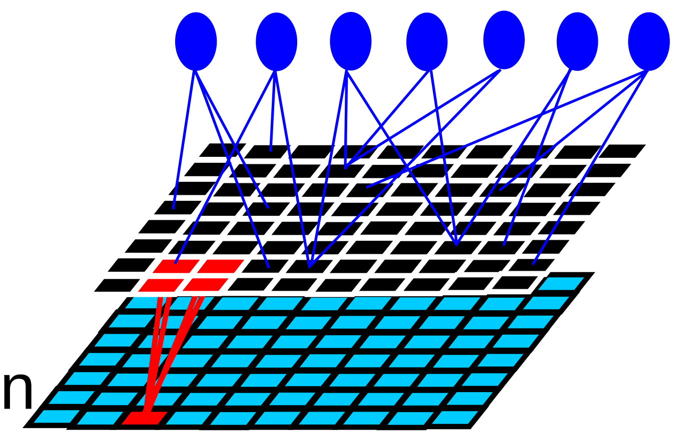

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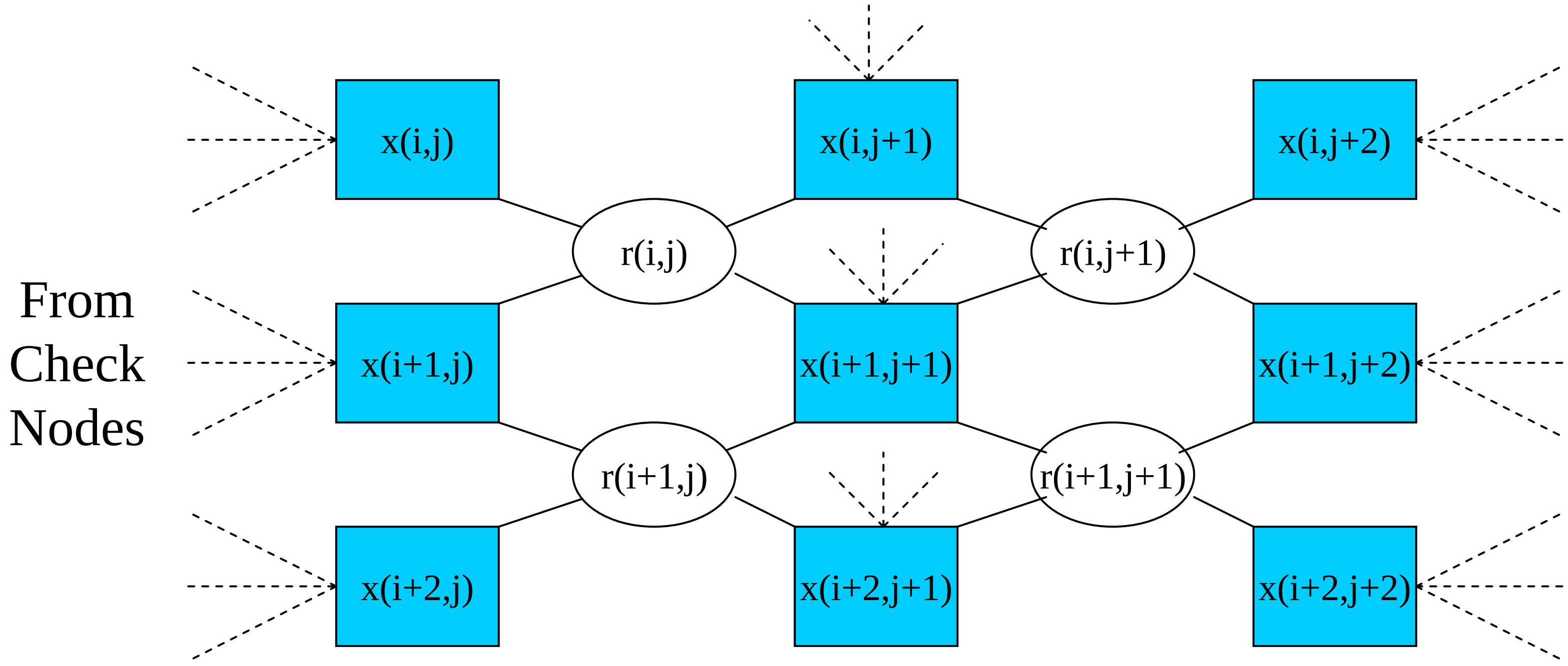
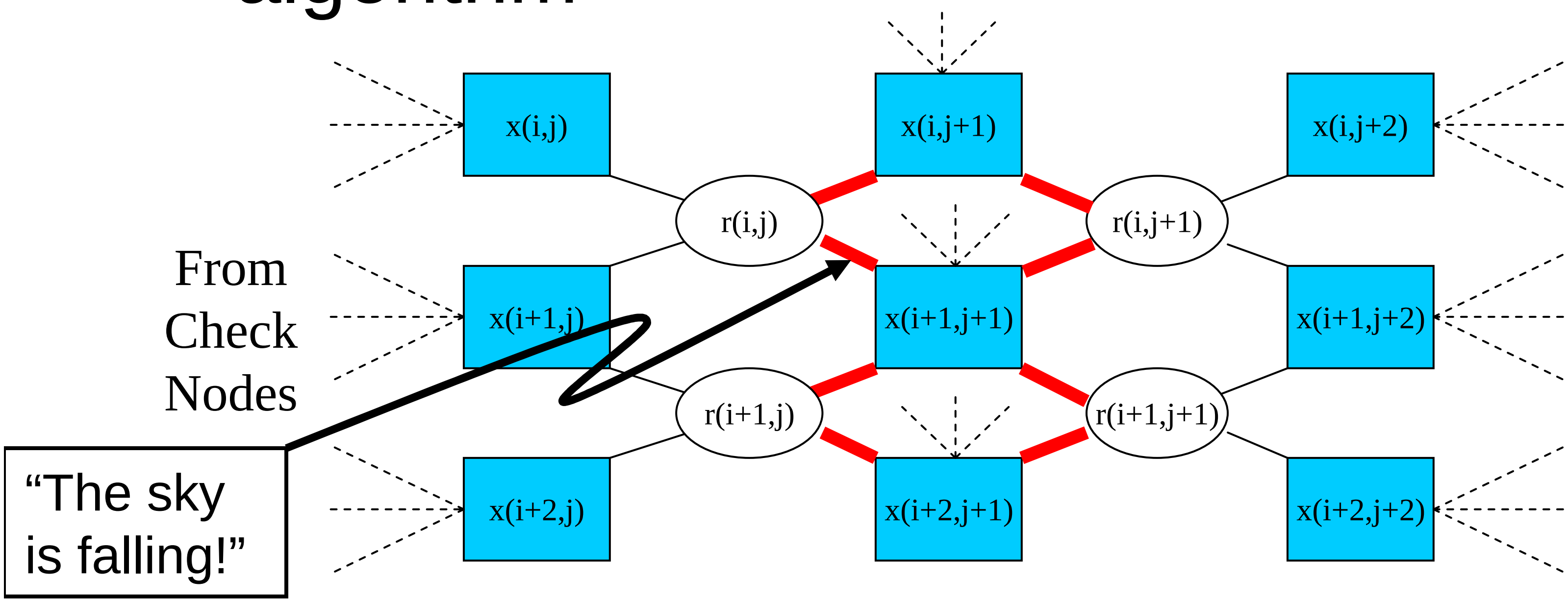
Full Graph



Same as before – just a different representation

Full Graph Analysis

- Length 4 cycles present which degrade performance of message-passing algorithm



Kschischang *et al*., “Factor graphs and the sum-product algorithm,” *IEEE Trans. Inform. Theory*, Feb. 2001.

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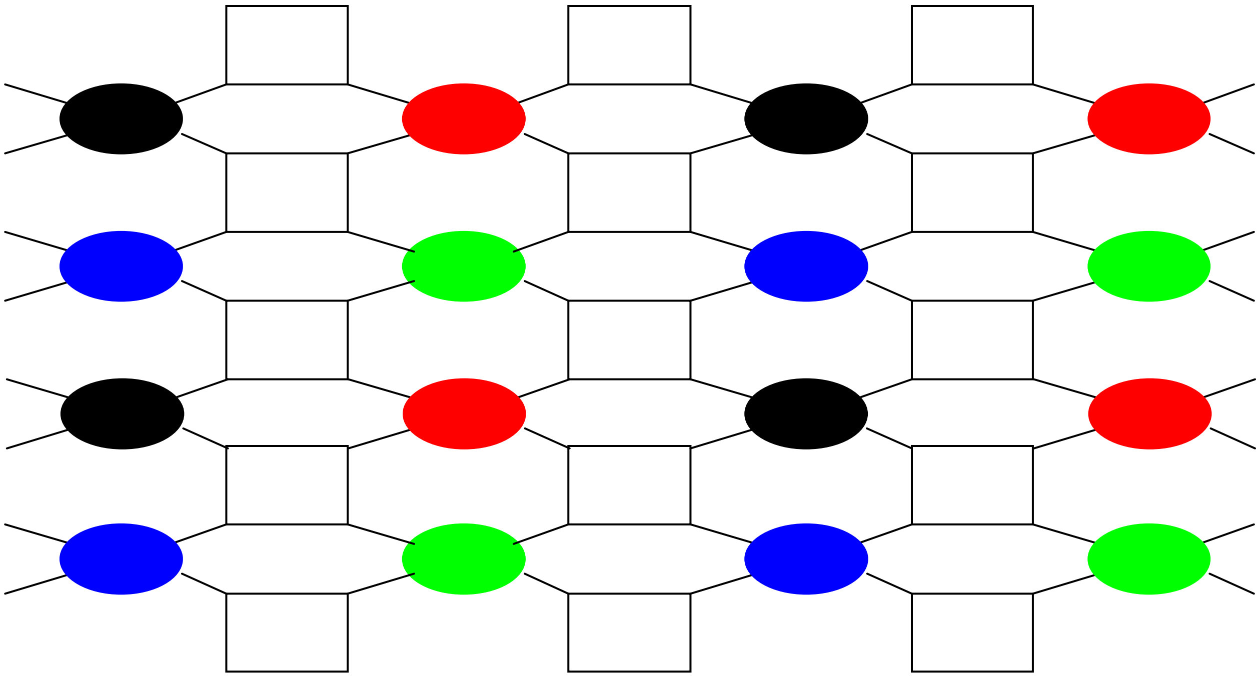
Ordered Subsets Message-Passing

- From Imaging – Data set is grouped into subsets to increase rate of convergence
- For Decoding – Observed data is grouped into subsets to eliminate short length cycles in the Channel ISI graph

H. M. Hudson, and R. S. Larkin, "Accelerated image reconstruction using ordered subsets of projection data," *IEEE Trans. Medical Imaging*, Dec. 1994

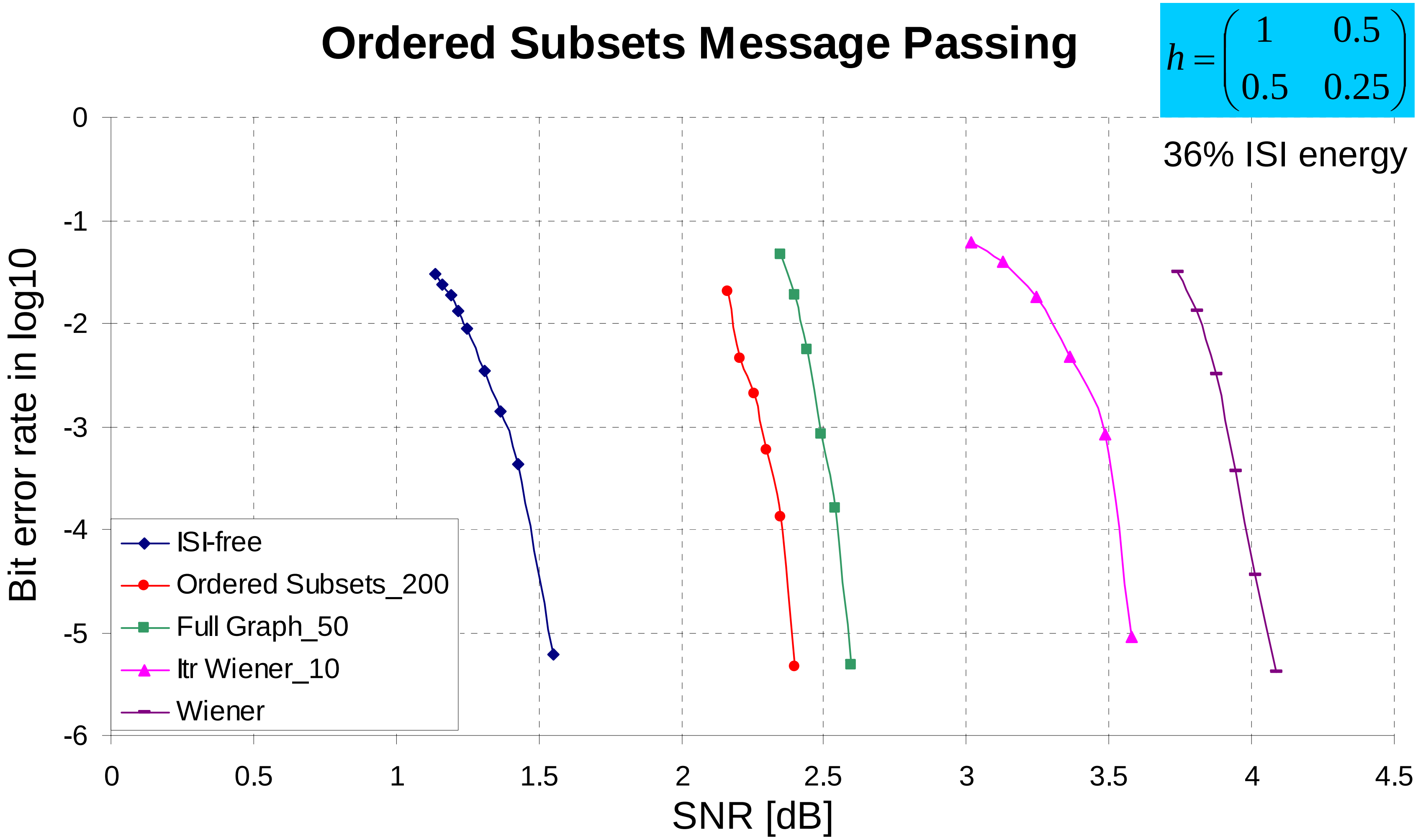
Grouped ISI Graph



- Labeling of data nodes into 4 subsets
- For each iteration use data nodes of one label only

J. A. O'Sullivan, and N. Singla, "Ordered subsets message-passing," *Int'l Symp. Inform. Theory,* Yokohama, Japan 2003.

Performance



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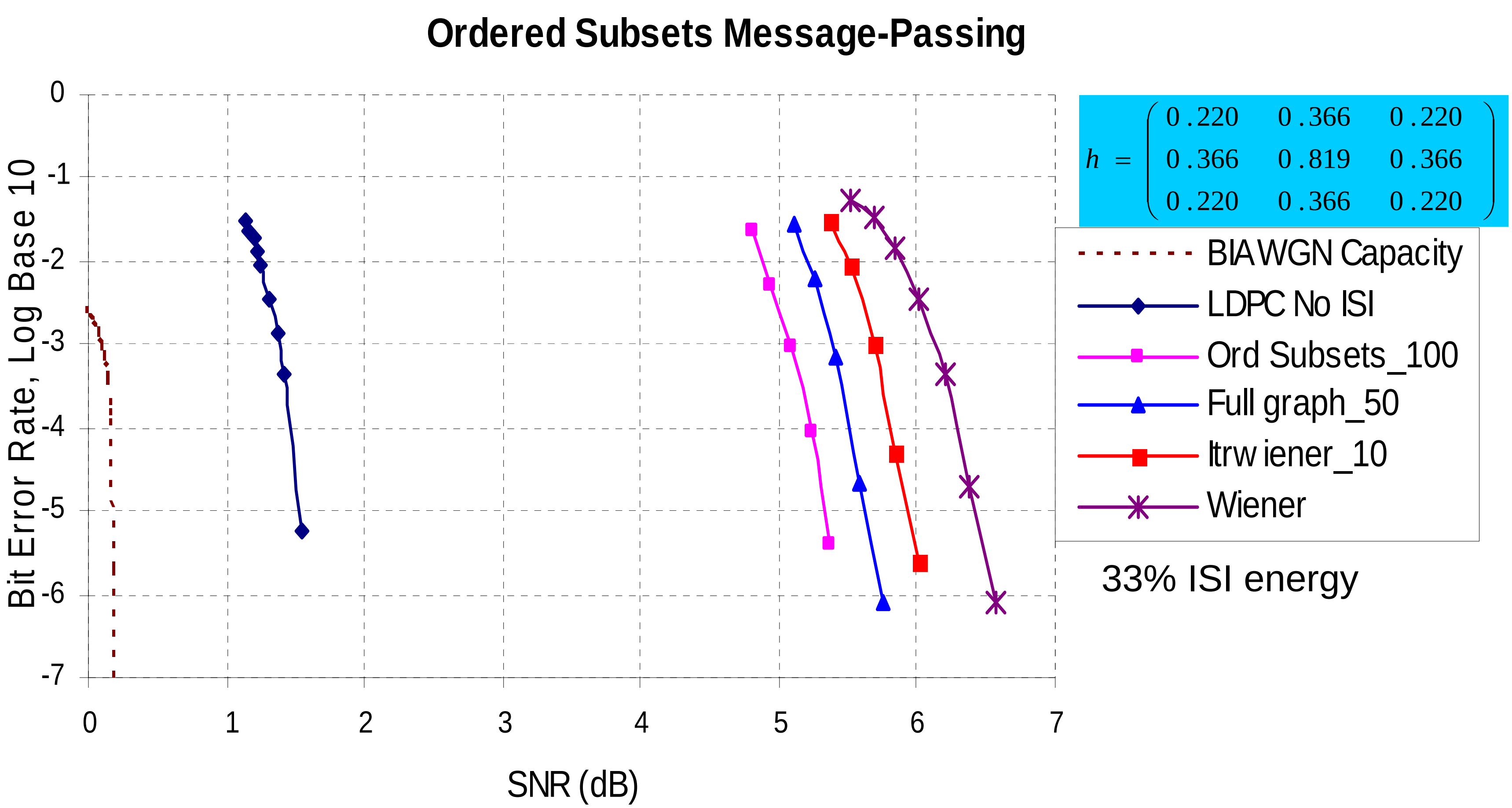
Performance



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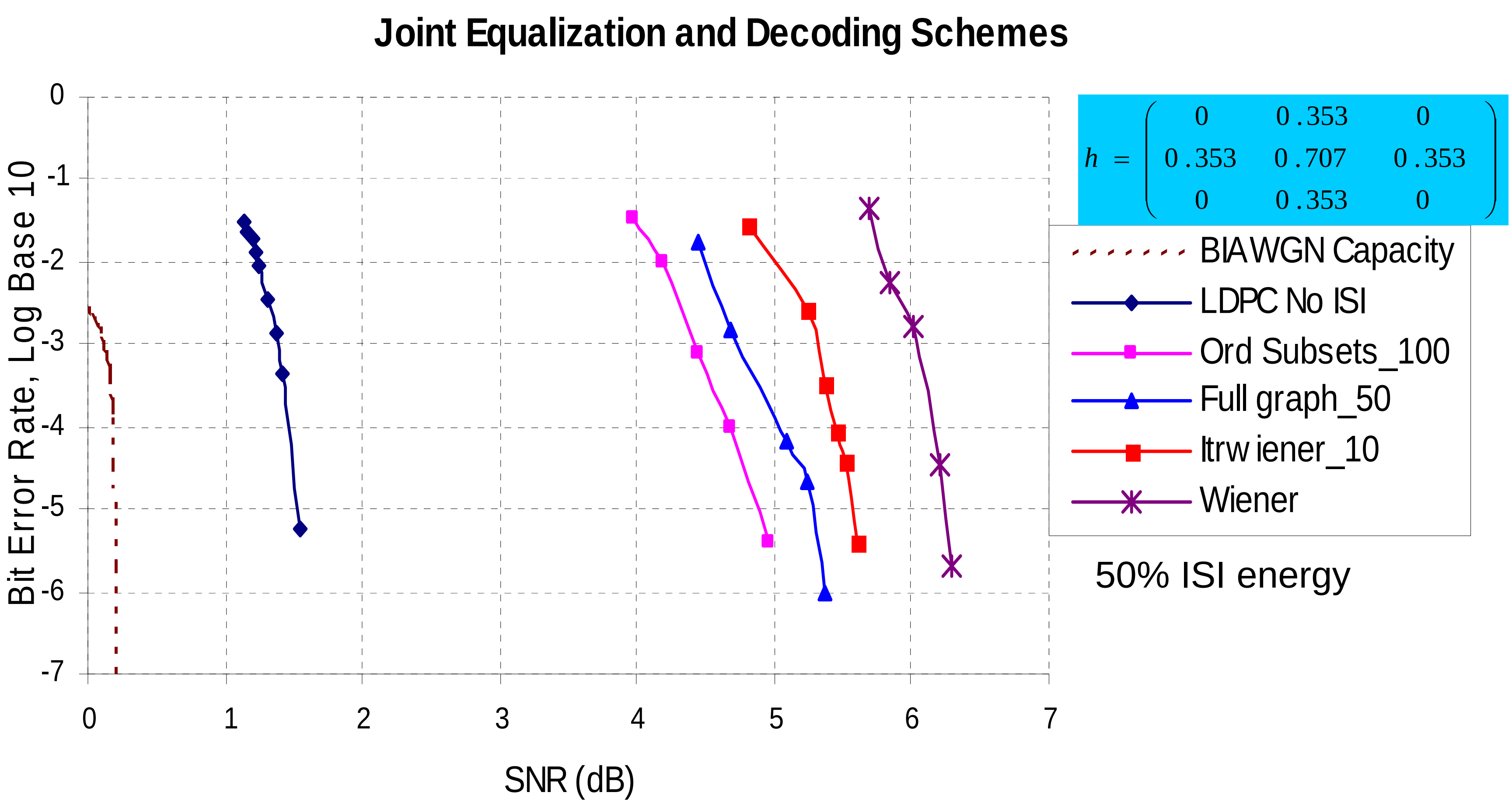

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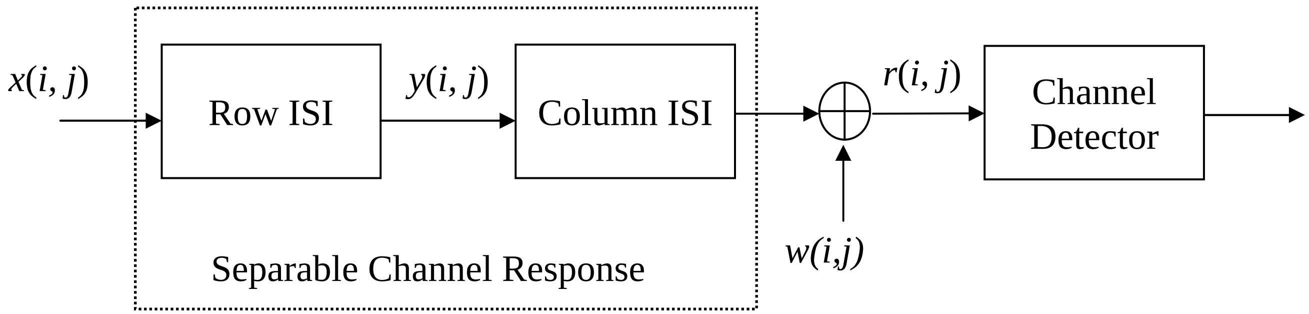
A Separable 2D ISI

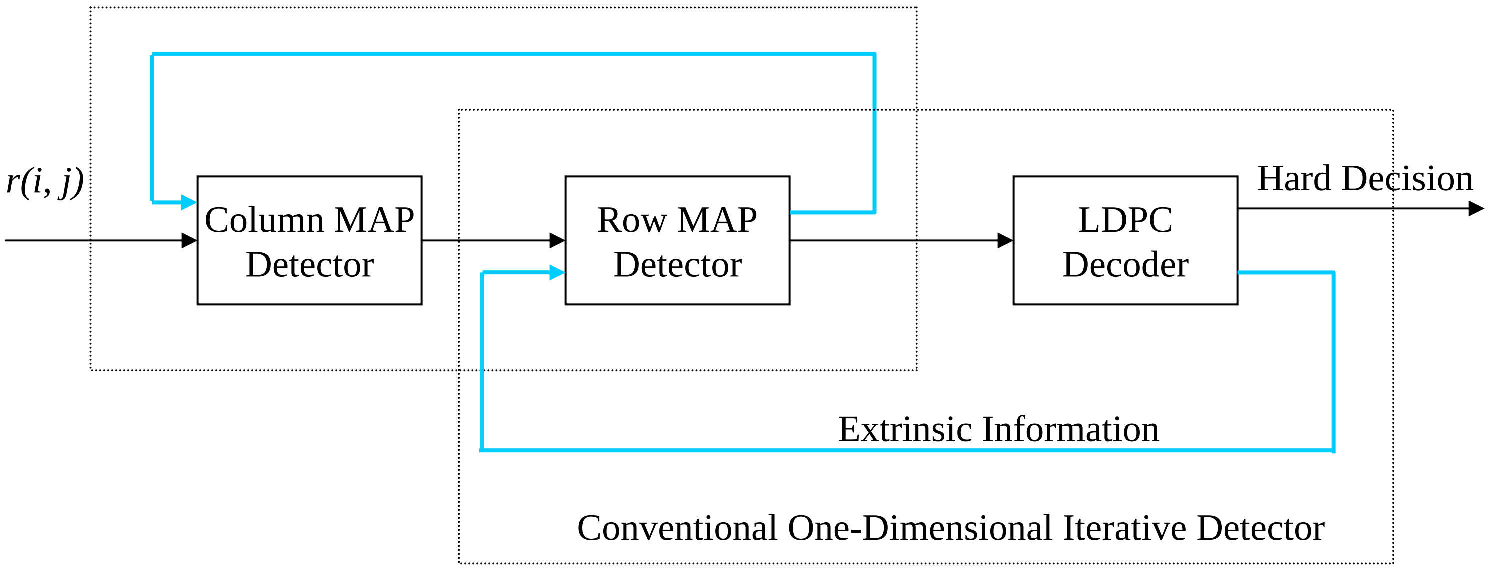


$$h = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 0.25 \end{pmatrix} = \begin{pmatrix} 1 \\ 0.5 \end{pmatrix} \begin{pmatrix} 1 & 0.5 \end{pmatrix}$$

- Advantages of separable 2D ISI
  - apply existing one-dimensional equalization methods
  - reduced detector complexity

Tüchler *et al*., "Turbo equalization: principles and new results," *IEEE Trans. On Comm*., May 2002.

Row-Column Decoder Diagram



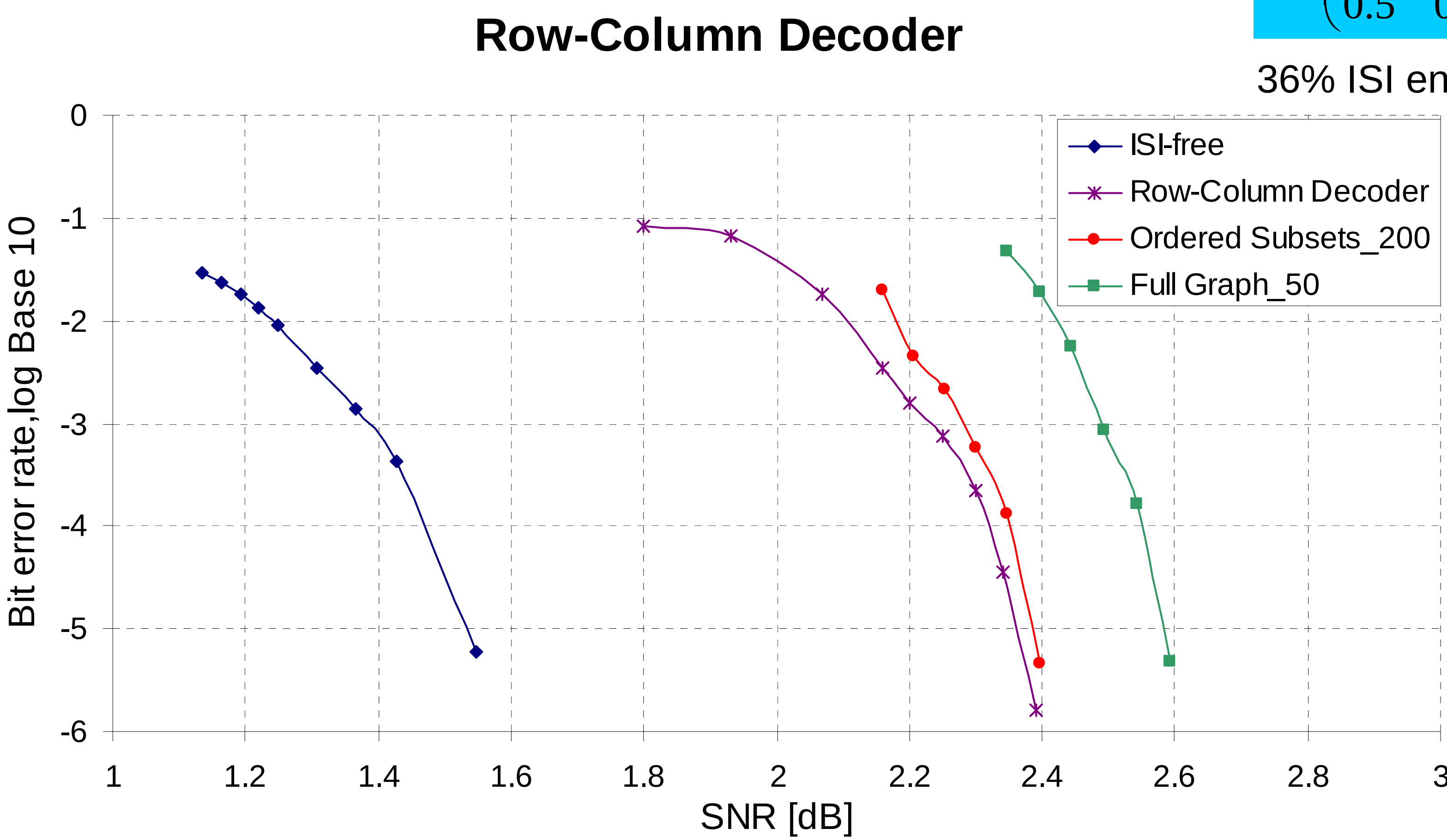
- Inputs to column detector are not binary

Wu *et al.*, "Iterative detection and decoding for separable two-dimensional intersymbol interference" *IEEE Trans. Magn.*, July 2003.

Performance

$$h = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 0.25 \end{pmatrix}$$

36% ISI energy



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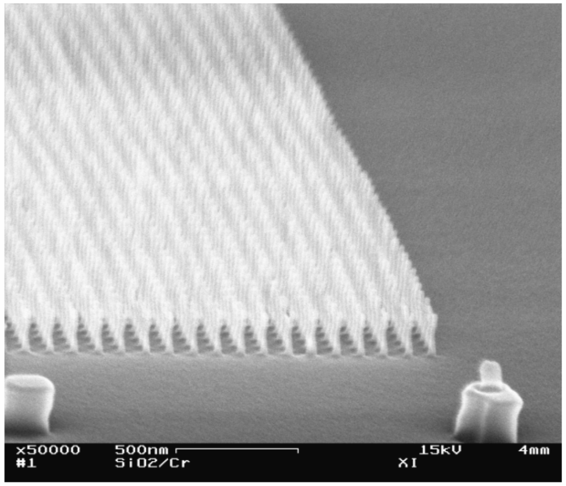

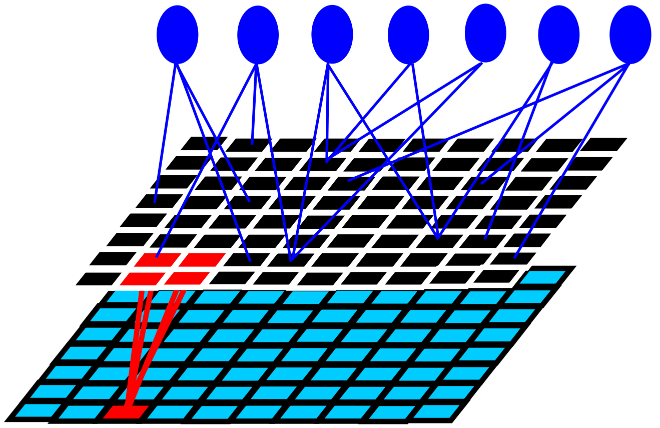

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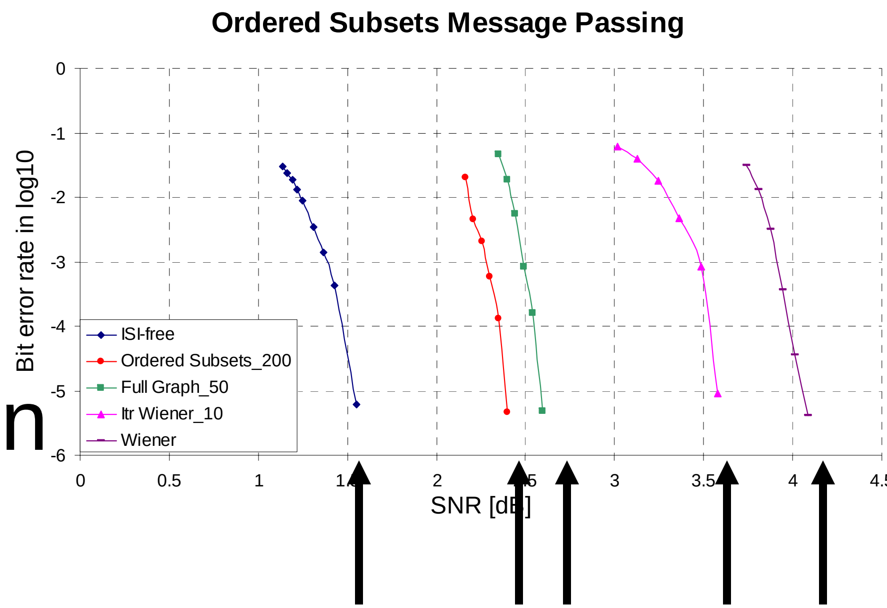
Performance Prediction by Analysis



- Predict Thresholds
- Avoid Simulations
- Basis for Design
- Thresholds Depend on
  - Code Family
  - ISI
  - Equalization and Decoding Scheme
- Approach: Pass Density Functions as Messages → *Density Evolution*

T. Richardson and R. Urbanke, "Capacity of low-density parity-check Codes under message-passing decoding," *IEEE Trans. Inform. Theory*, Feb. 2001

Density Evolution Results

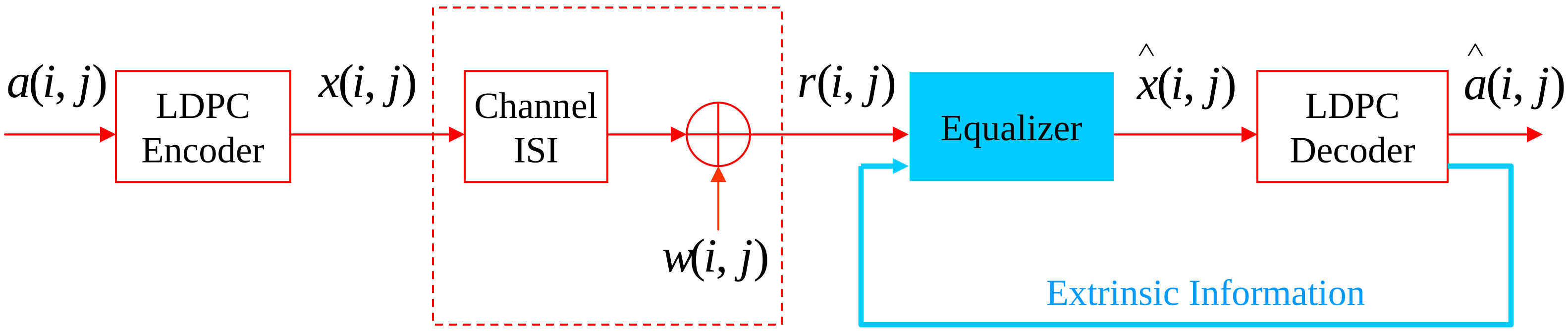
Thresholds for Full graph algorithms

Code Parameters (d_v,d_c)	Rate	Threshold SNR [dB] Full Graph	Threshold SNR [dB] Modified Full Graph
(3,4)	0.25	1.86	1.69
(3,6)	0.50	1.91	1.73
(3,30)	0.90	4.41	4.21

Conclusions

- Advanced recording schemes may give rise to 2D ISI
- Use joint detection and decoding for 2D ISI
  - MMSE equalization and decoding
    - good performance, moderate complexity
  - Message passing algorithms
    - full graph algorithm performance deteriorated due to short cycles
    - ordered subsets message-passing gives best performance for general 2D ISI
  - Separable ISI decoding
    - best performance
    - low complexity
    - approximate channel response by separable response

Iterative MMSE Equalization and Decoding

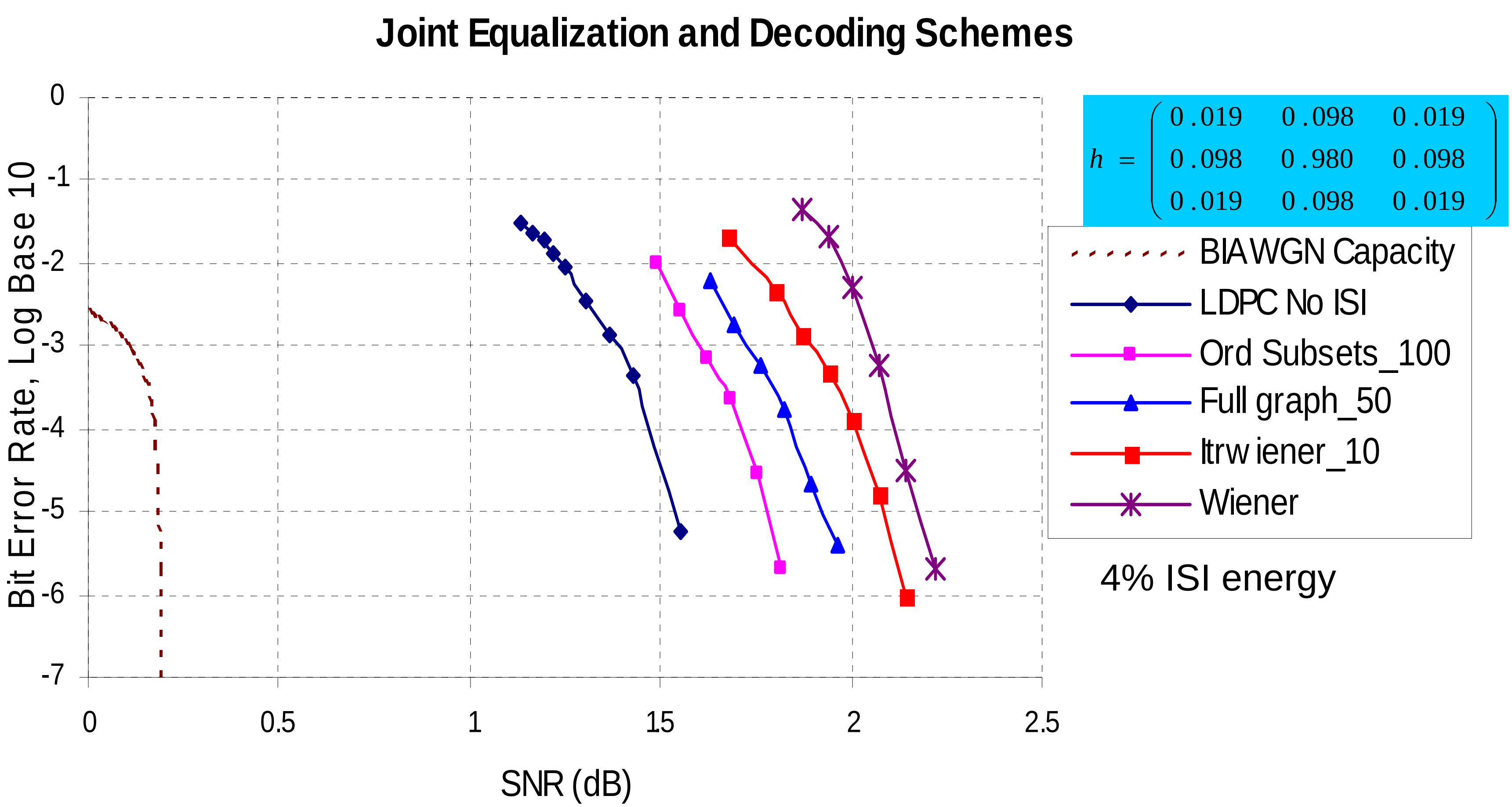


- Soft information, estimated mean of the codeword, passed from LDPC decoder to equalizer

$$E[x] = \Pr(x=1) - \Pr(x=-1)$$

$$\hat{x} = E[x] + W**[r - h**E[x]]$$

Performance



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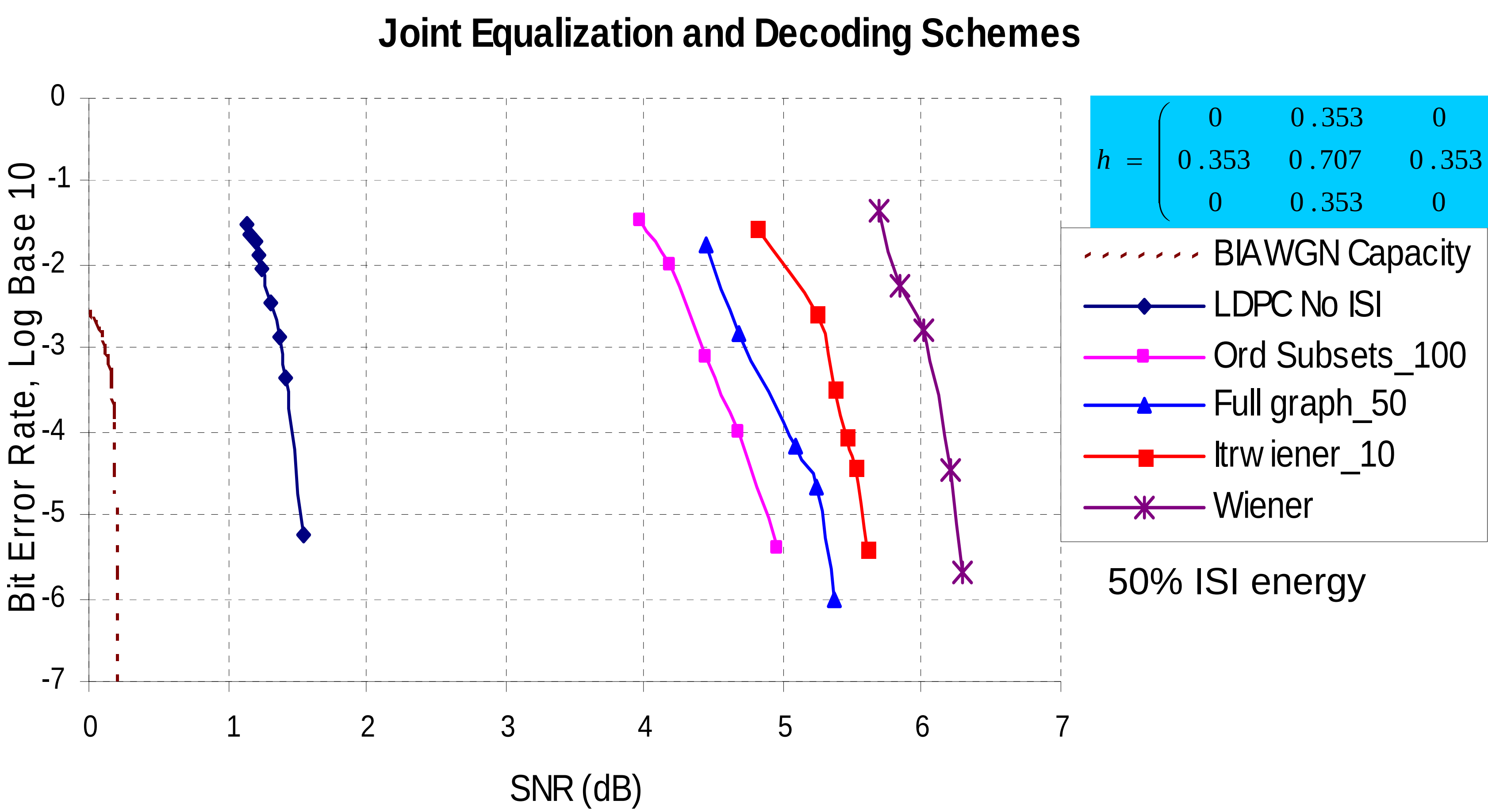
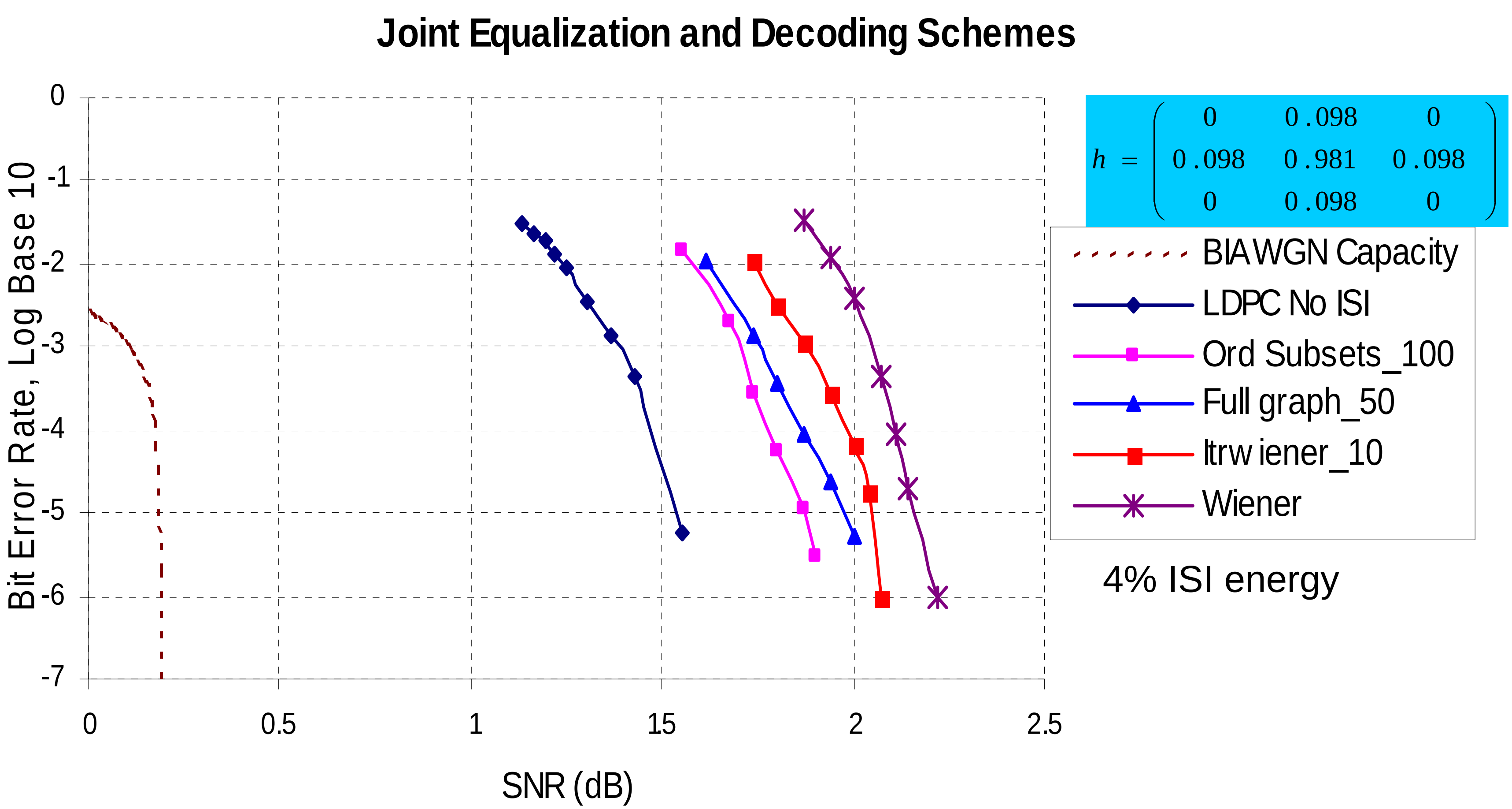
Performance



$$h = \begin{pmatrix} 0 & 0.353 & 0 \\ 0.353 & 0.707 & 0.353 \\ 0 & 0.353 & 0 \end{pmatrix}$$

50% ISI energy

Performance



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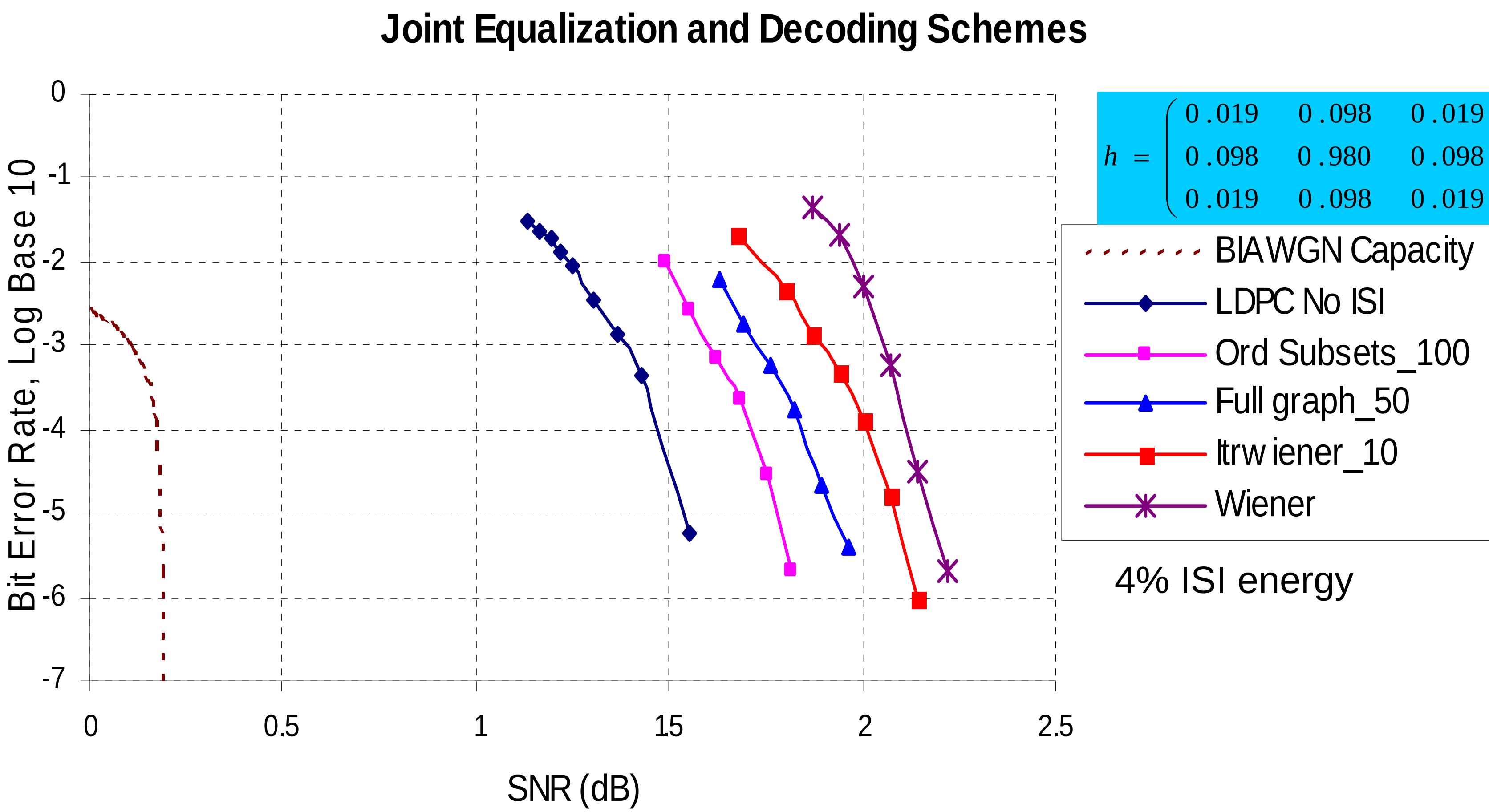
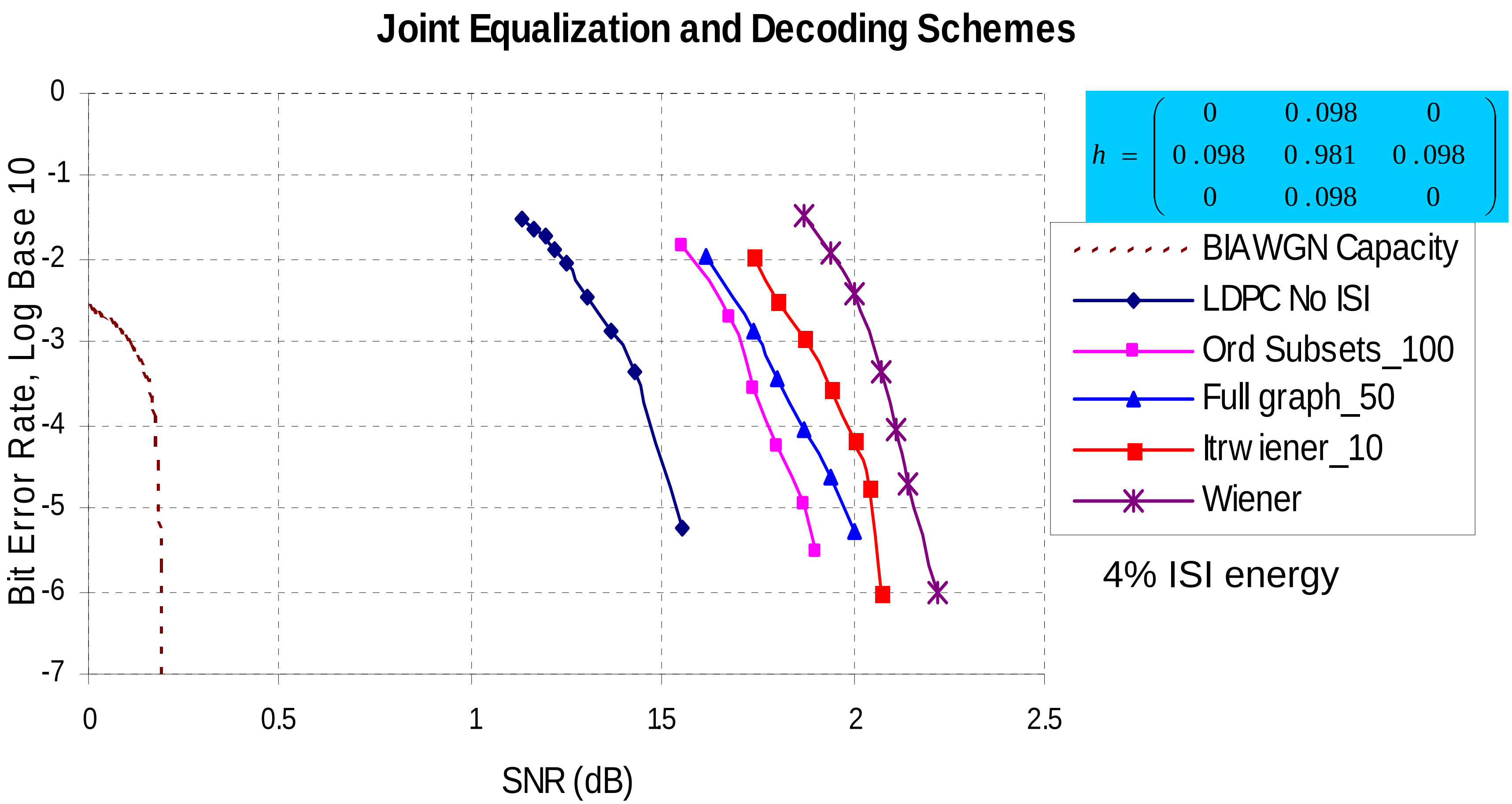
Performance



$$h = \begin{pmatrix} 0.019 & 0.098 & 0.019 \\ 0.098 & 0.980 & 0.098 \\ 0.019 & 0.098 & 0.019 \end{pmatrix}$$

4% ISI energy

Performance



Performance Prediction by Density Evolution

- Assume messages are i.i.d. random variables
- Evolve message densities through the message maps
- If densities converge to desired density, then error-free transmission possible otherwise not
- Gives lower bound on performance of message-passing scheme

T. Richardson and R. Urbanke, "Capacity of low-density parity-check Codes under message-passing decoding," *IEEE Trans. Inform. Theory*, Feb. 2001

Density Evolution for Full Graph Message-Passing

- Codeword bit nodes to check nodes

$$L_{x\to z}^{(l)} = \sum_{m\in N(x)} L_{m\to x}^{(l-1)} + \sum_{z'\in N(x)\backslash z} L_{z'\to x}^{(l-1)} \quad \text{CONVOLUTION}$$

- Check nodes to codeword bit nodes

$$\tanh\frac{L_{z\to x}^{(l)}}{2} = (-1)^z \prod_{x'\in N(z)\backslash x} \tanh\frac{L_{x'\to z}^{(l-1)}}{2} \quad \text{LOOKUP TABLE}$$

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Density Evolution…

- Codeword bit nodes to measured data nodes

$$L_{x\to m}^{(l)} = \sum_{m'\in N(x)\backslash m} L_{m'\to x}^{(l-1)} + \sum_{z\in N(x)} L_{z\to x}^{(l)}$$

CONVOLUTION

- Measured data nodes to codeword bit nodes

$$L_{m\to x}^{(l)} = f(\{L_{x'\to m}^{(l)} : x' \in N(m)\setminus x\})$$

MONTE CARLO SIMULATION

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