Signal Processing for Advanced Storage Media

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Securing our World through Technology

Advanced Media for Storage

- Blu-ray disc
- Patterned media
- Holographic storage
- Two-dimensional optical storage

Two-Dimensional Intersymbol Interference

Existing schemes, like the Viterbi algorithm, cannot deal with 2D ISI due to complexity considerations

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Outline

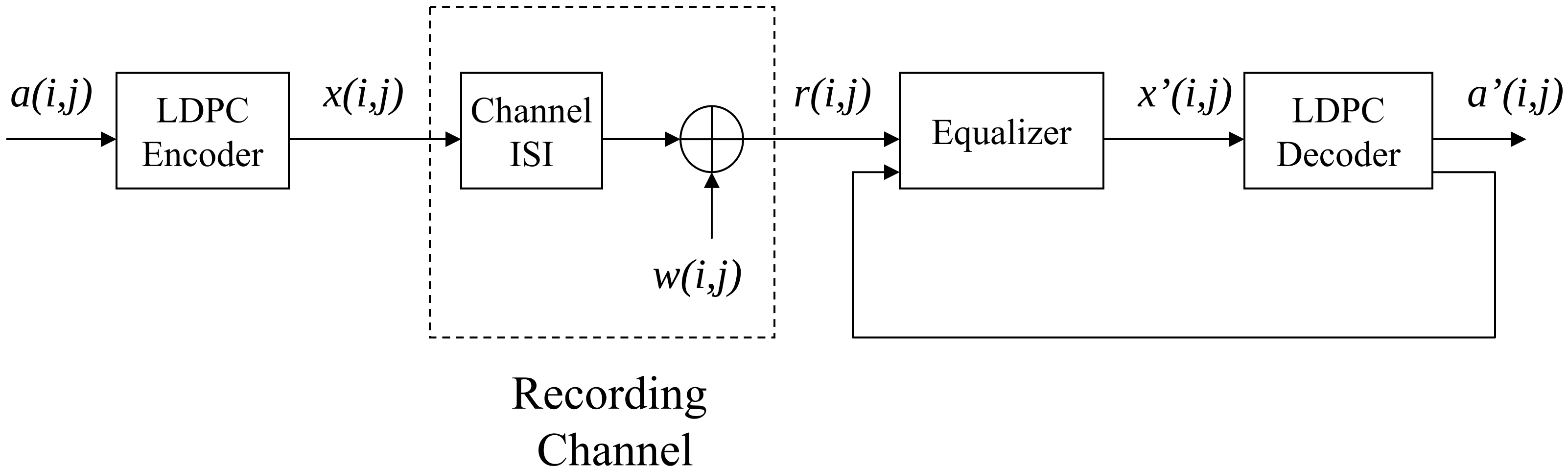
- Joint Equalization and Decoding: Linear 2D ISI
  - MMSE Equalization
  - Full Graph Decoding
  - Modified Full Graph Decoding
  - Separable Channel Models
- 2D Optical Data Storage: Nonlinear ISI
  - Full Graph Decoding
  - Density Evolution for Threshold Behavior
- Conclusions

Joint Equalization and Decoding Schemes for 2D ISI

- Performance can be improved dramatically by combining error control coding with equalization
  - Base on existing equalization schemes
  - Jointly model channel ISI and parity check matrix for error control code—three level graph
  - Employ novel message-passing algorithms that take advantage of the 2D dependence

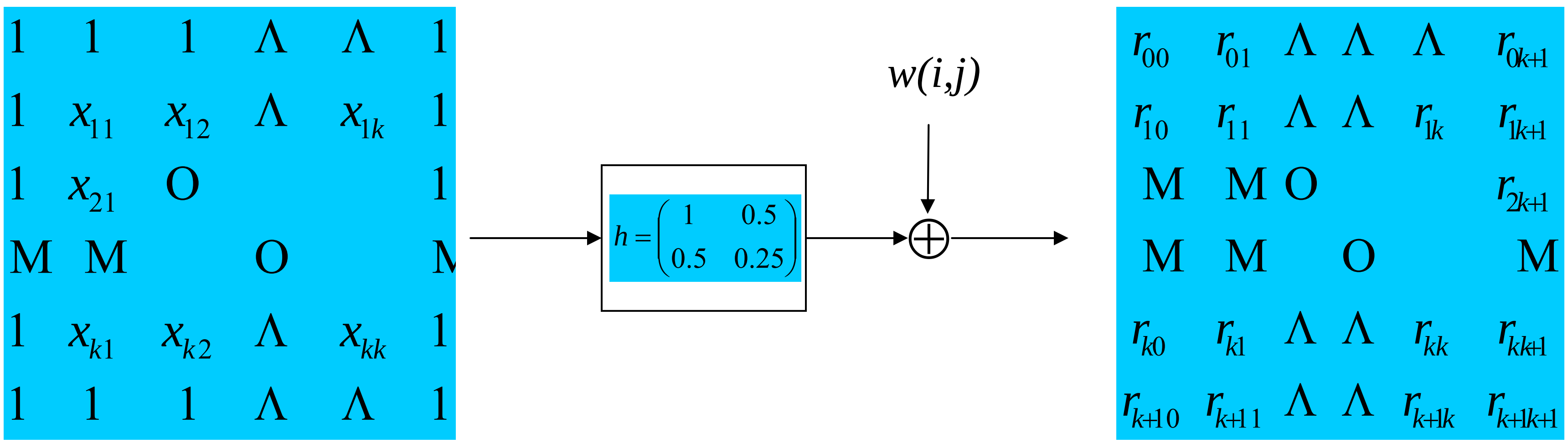
Singla *et al.*, "Iterative decoding and equalization for 2-D recording channels," *IEEE Trans. Magn.*, Sept. 2002.

Channel Model



- Low-density parity-check codes used for error correction
- $x(i,j) \in \{+1,-1\}$
- Channel ISI is 2D
- Noise is assumed to be AWGN

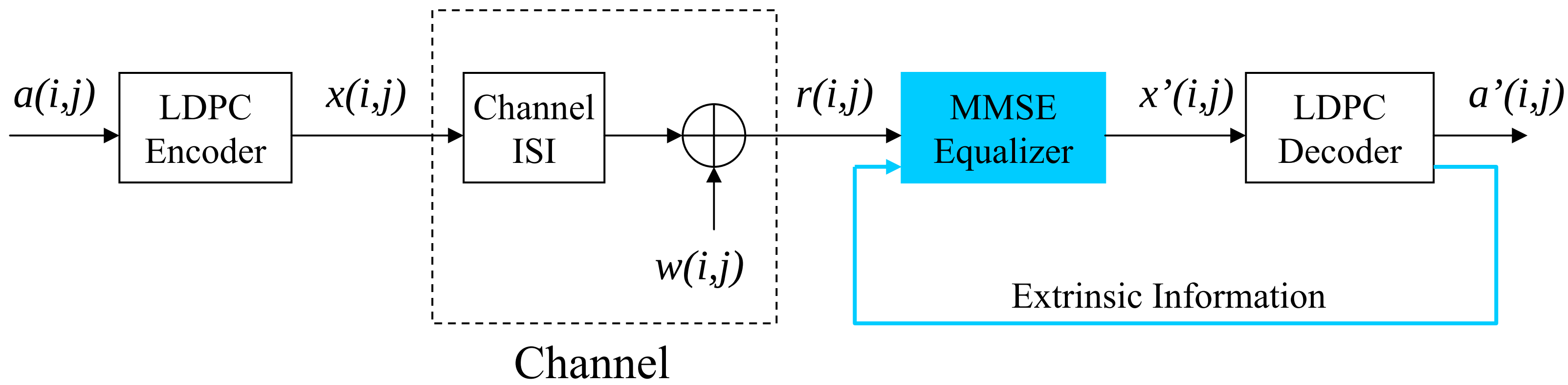
2D Linear Intersymbol Interference



Guard Band

$$r_{i,j} = x_{i,j} + 0.5x_{i-1,j} + 0.5x_{i,j-1} + 0.25x_{i-1,j-1} + w_{i,j}$$

MMSE Equalization



- Equalizer may or may not iterate with the LDPC decoder
- Soft information, estimated mean of the codeword, passed from LDPC decoder to equalizer

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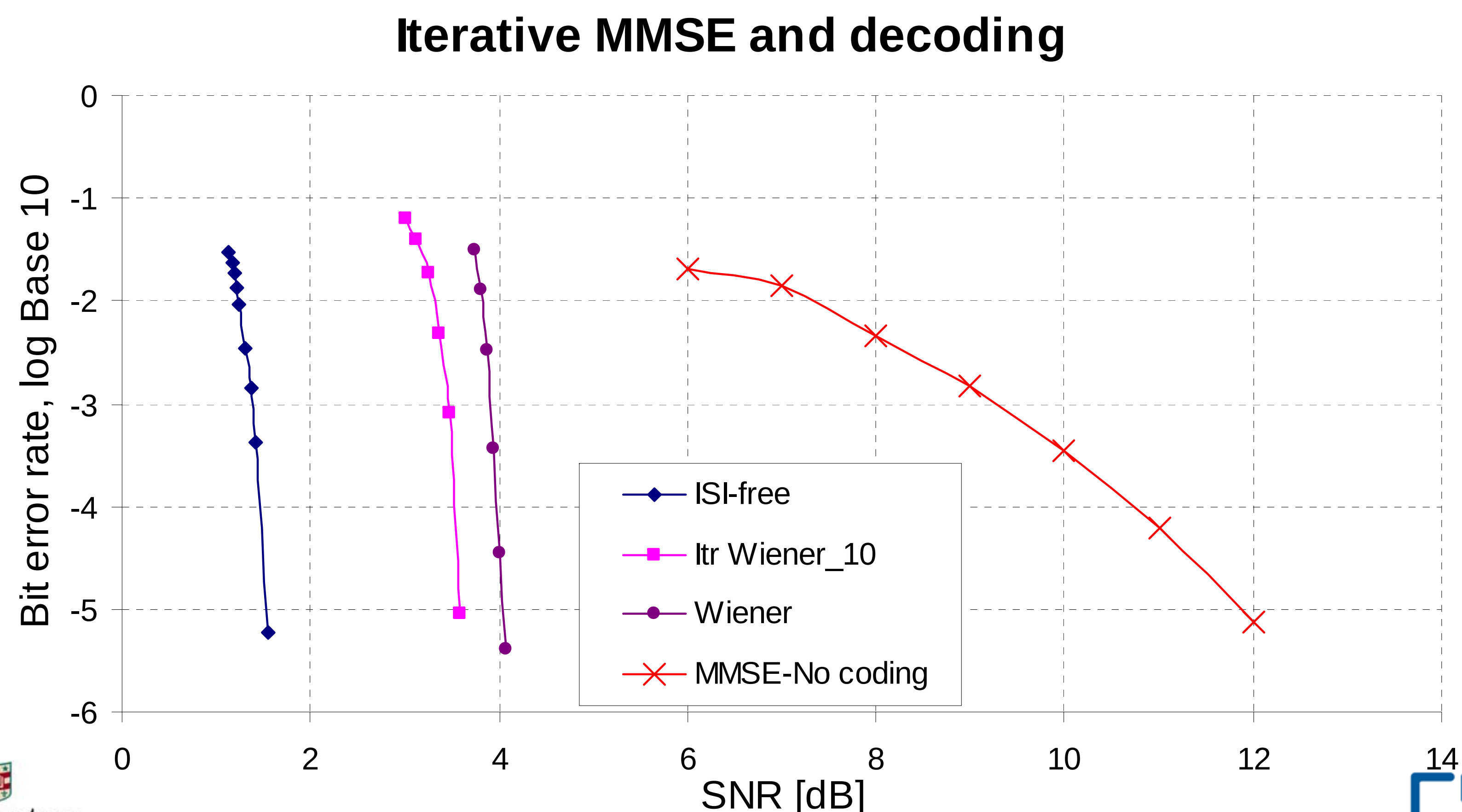
Performance

Block length 10000, regular, rate-0.5, LDPC code



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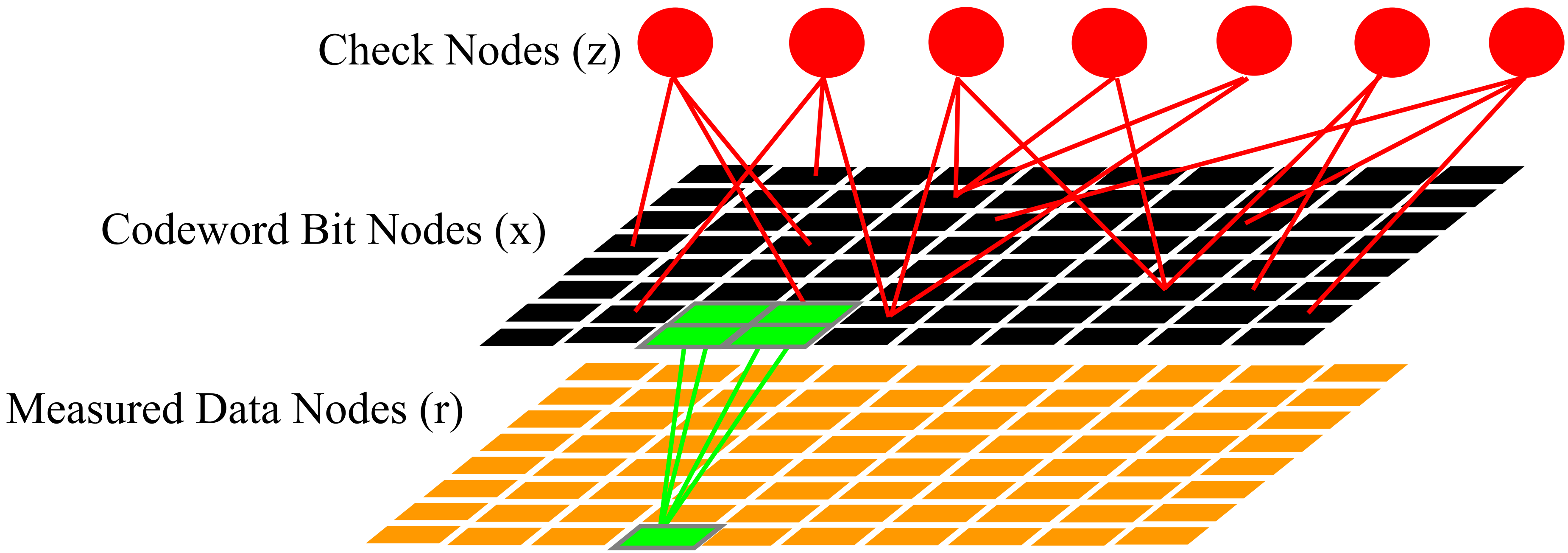
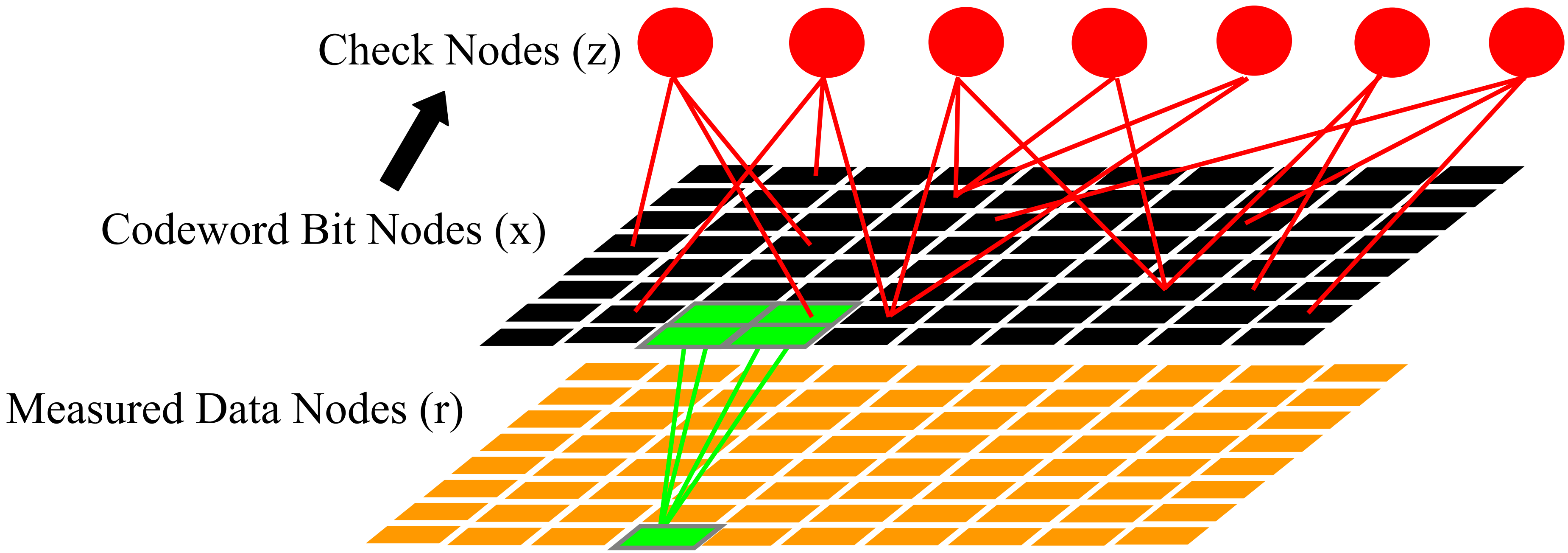
Full Graph Message-Passing



$$h = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 0.25 \end{pmatrix} \qquad r_{i,j} = x_{i,j} + 0.5x_{i-1,j} + 0.5x_{i,j-1} + 0.25x_{i-1,j-1} + w_{i,j}$$

Full Graph Message-Passing



$$L_{x\to z}^{(l)} = \sum_{m\in N(x)} L_{m\to x}^{(l-1)} + \sum_{z'\in N(x)\setminus z} L_{z'\to x}^{(l-1)}$$

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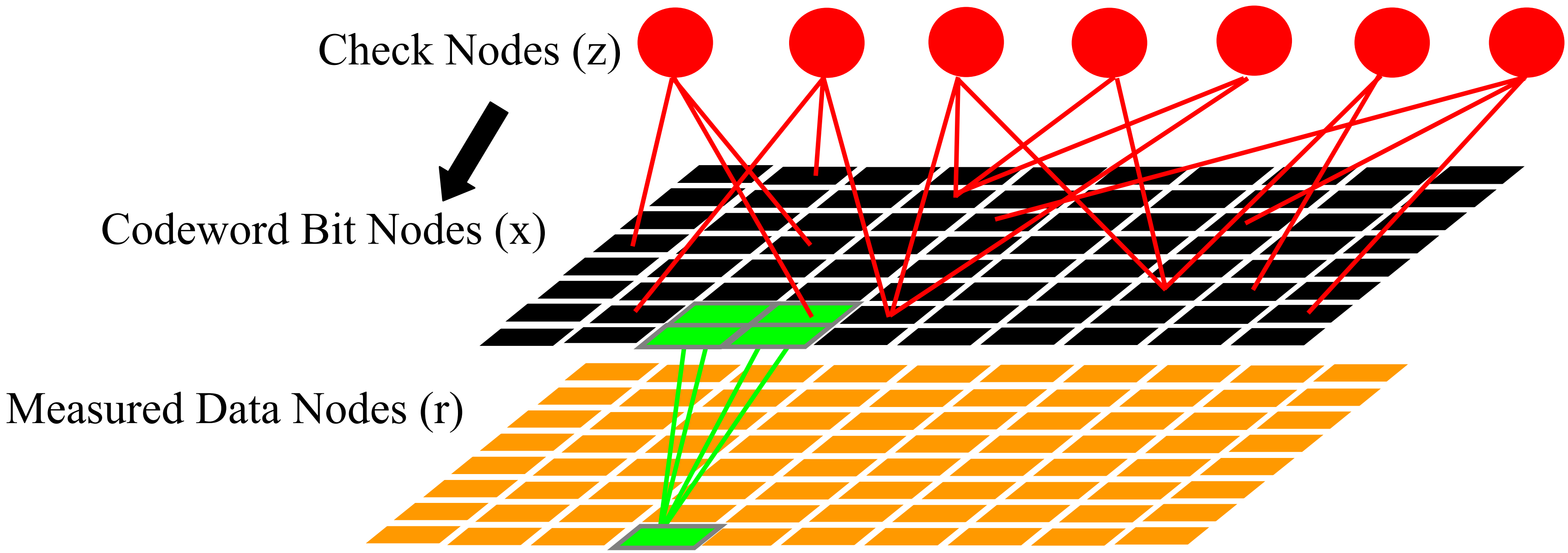
Full Graph Message-Passing



$$\tanh \frac{L_{z \to x}^{(l)}}{2} = (-1)^{z} \prod_{x' \in N(z) \setminus x} \tanh \frac{L_{x' \to z}^{(l-1)}}{2}$$

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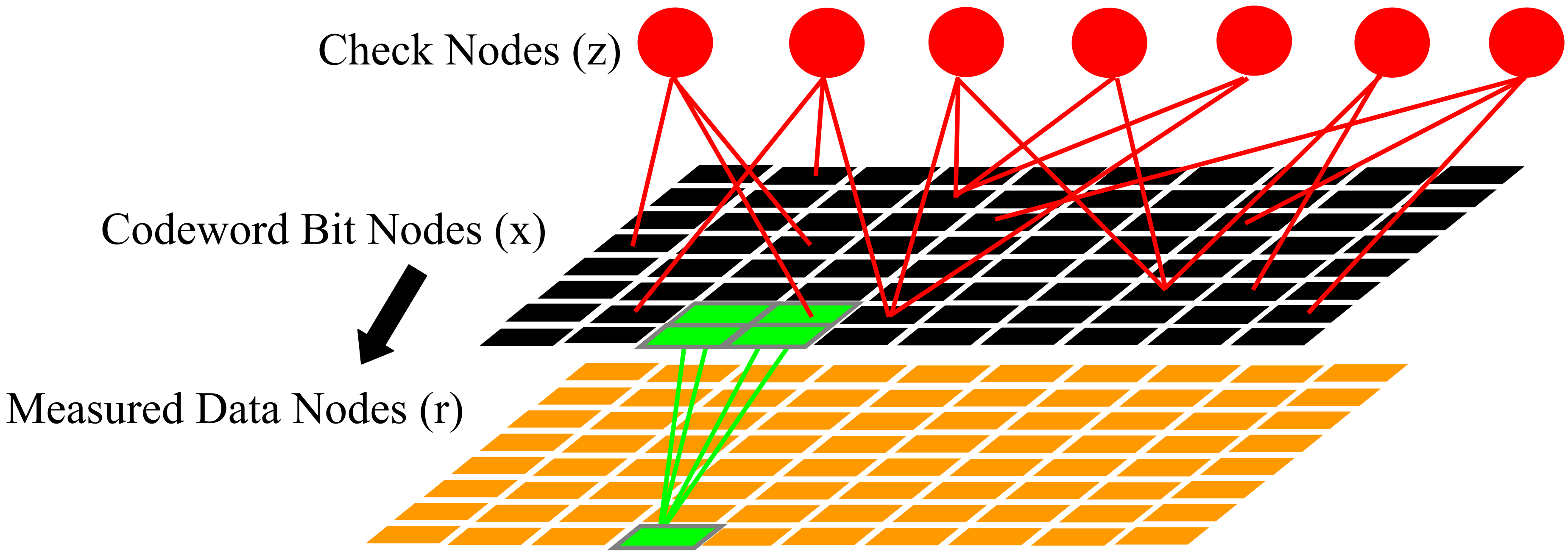
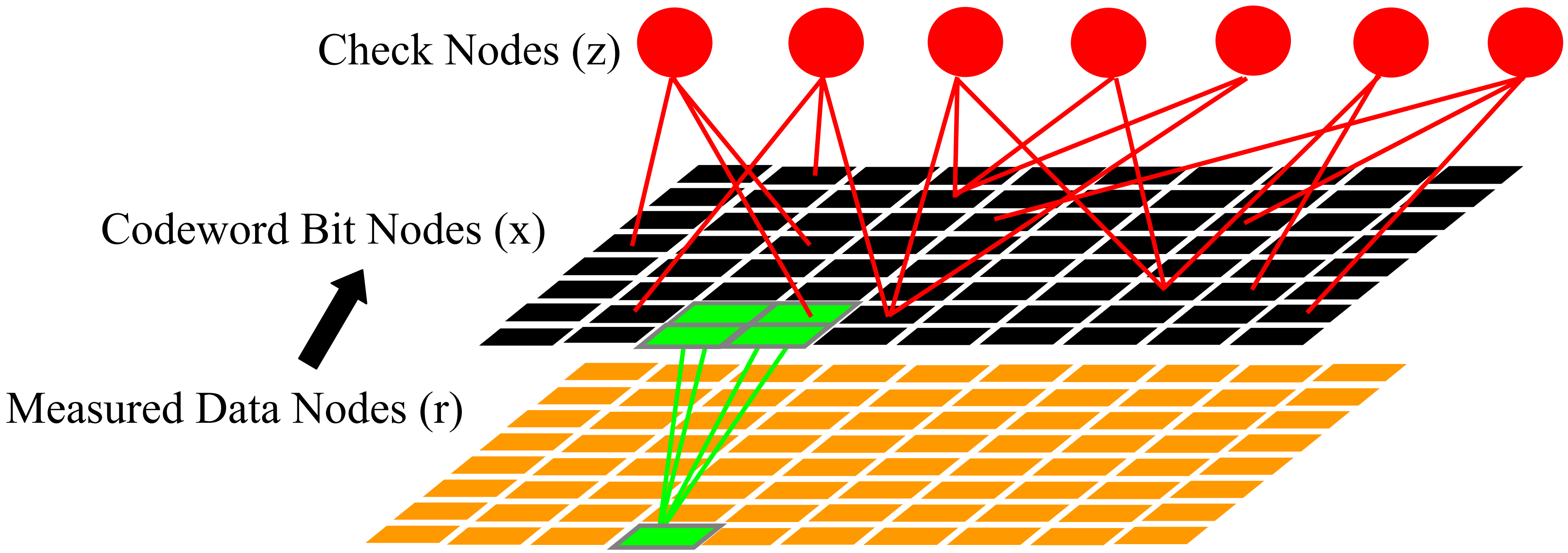
Full Graph Message-Passing



$$L_{x\to m}^{(l)} = \sum_{m'\in N(x)\setminus m} L_{m'\to x}^{(l-1)} + \sum_{z\in N(x)} L_{z\to x}^{(l)}$$

Full Graph Message-Passing



$$L_{m\to x}^{(l)} = f\left(\{L_{x'\to m}^{(l)} : x' \in N(m)\setminus x\}\right)$$

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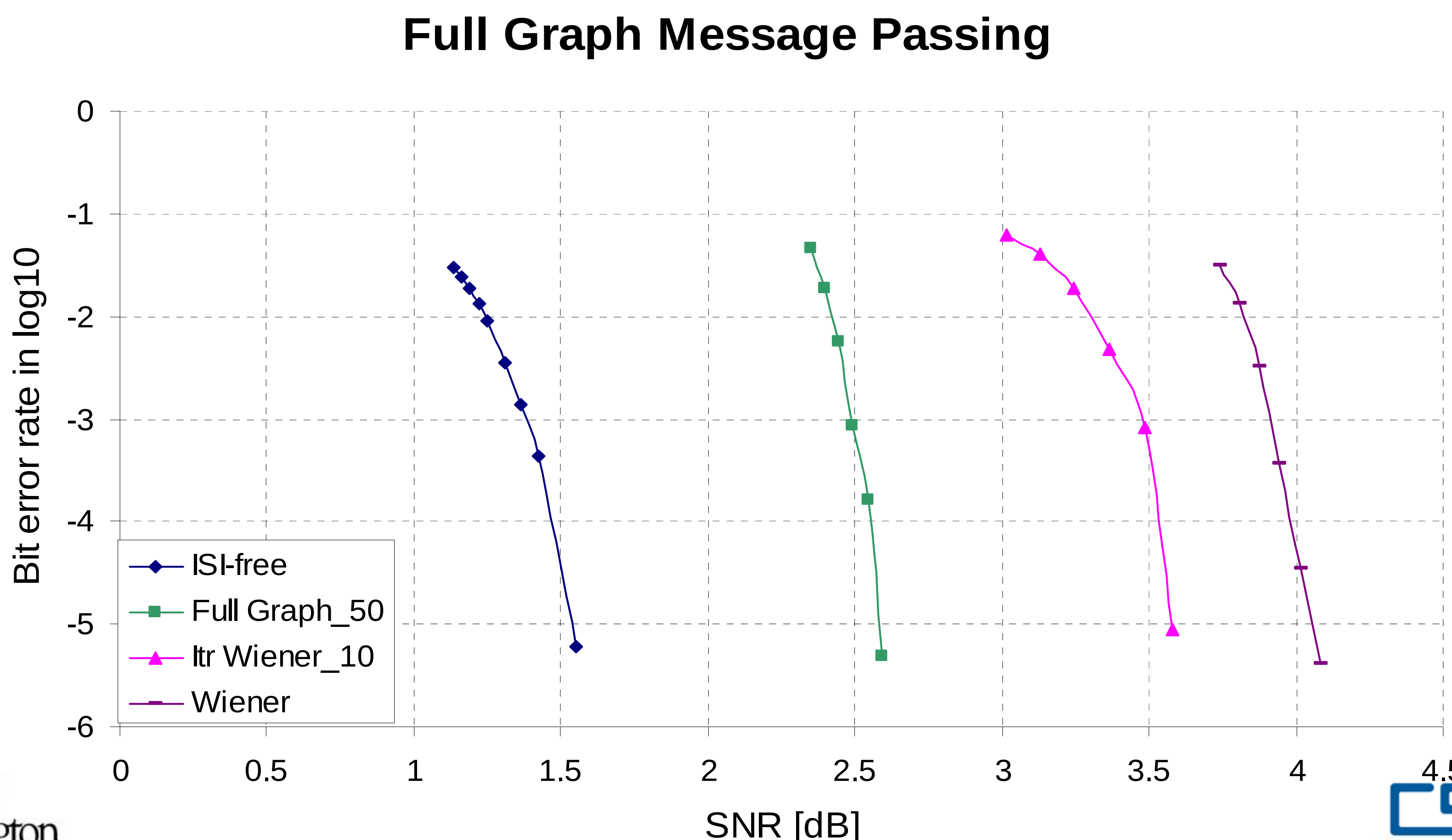
Performance

Block length 10000, regular, rate-0.5, LDPC code



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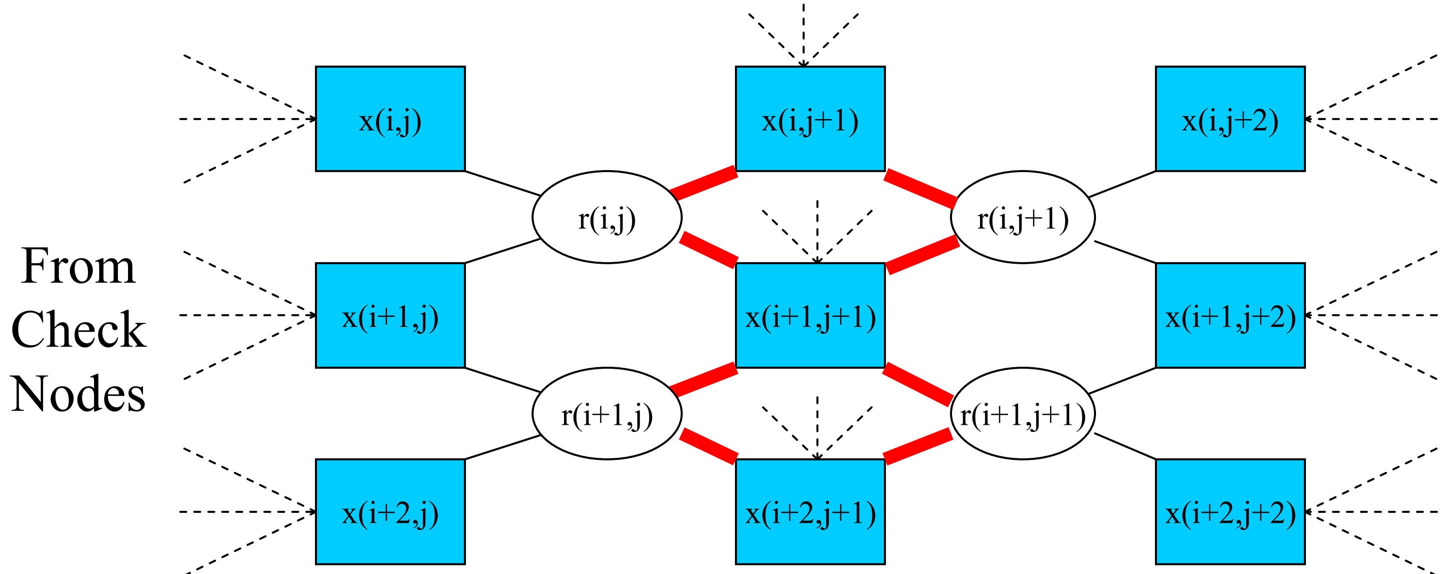
Full Graph Analysis

- Length 4 cycles present which degrade performance of message-passing algorithm



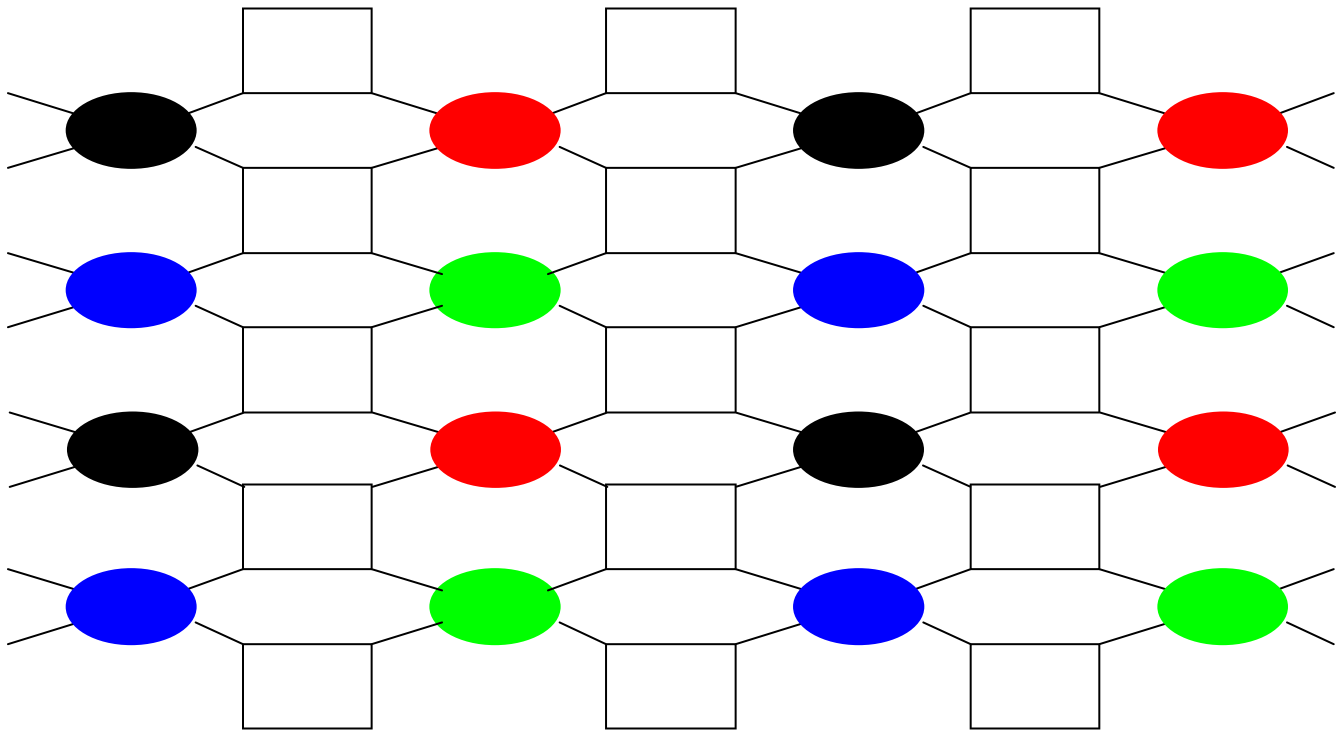
Kschischang *et al.*, "Factor graphs and the sum-product algorithm," *IEEE Trans. Inform. Theory*, Feb. 2001.

Modified Full Graph Message-Passing

- From Imaging – Data set is grouped into subsets to increase rate of convergence
- For Decoding – Measured data is grouped into subsets and a modified schedule is employed: results in increase in girth of full graph

H. M. Hudson, and R. S. Larkin, "Accelerated image reconstruction using ordered subsets of projection data," *IEEE Trans. Med. Img.*, Dec. 1994

Grouped ISI Graph

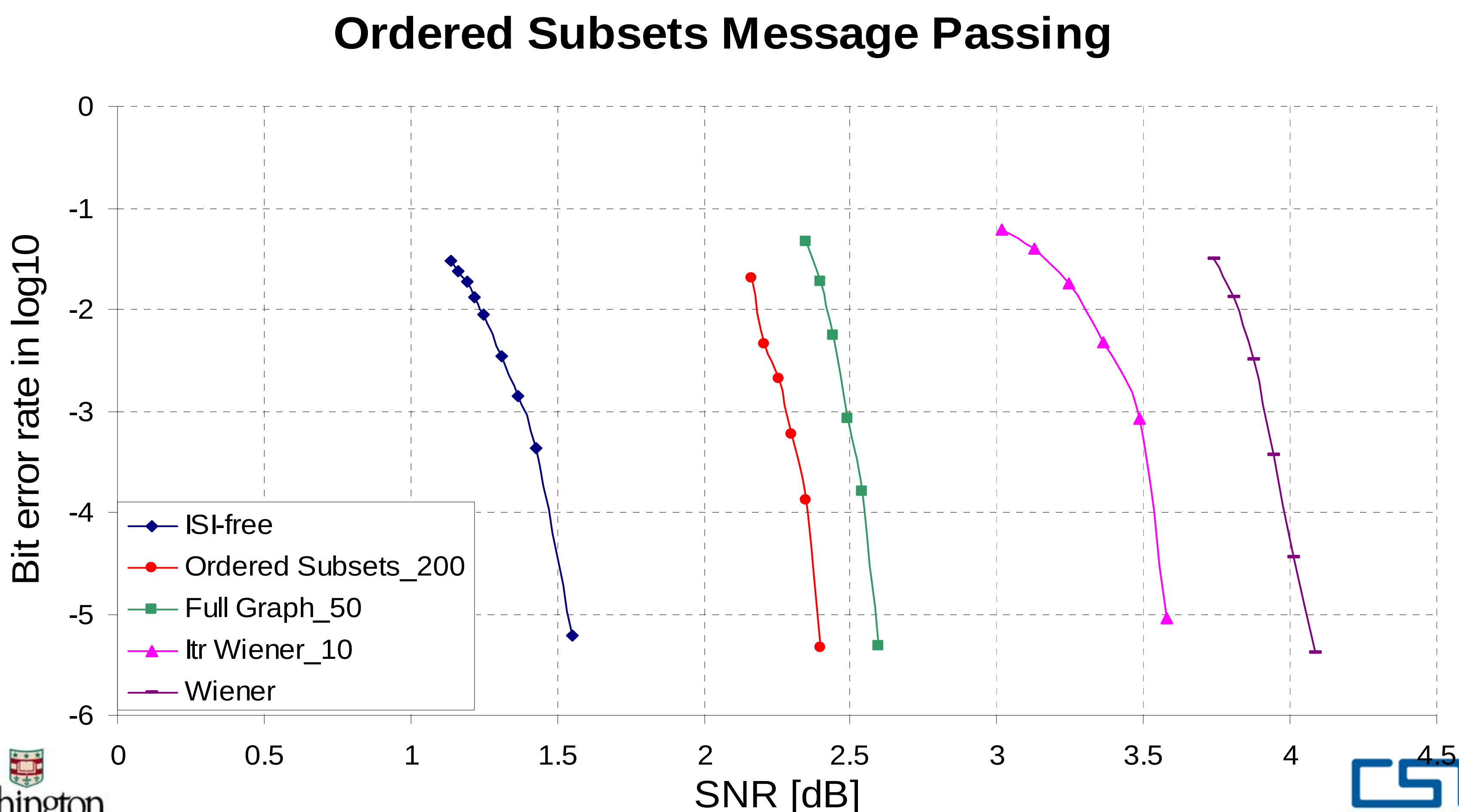


- ❑ Labeling of data nodes into 4 subsets
- ❑ For each iteration use data nodes of one label only

J. A. O'Sullivan, and N. Singla, "Ordered subsets message-passing," *Int'l Symp. Inform. Theory,* Yokohama, Japan 2003.

Performance

Block length 10000, regular, rate-0.5, LDPC code



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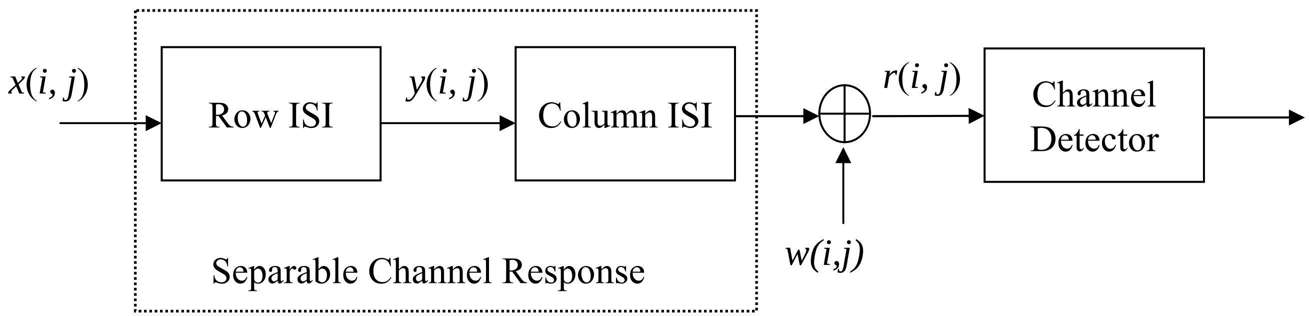
A Separable 2D ISI

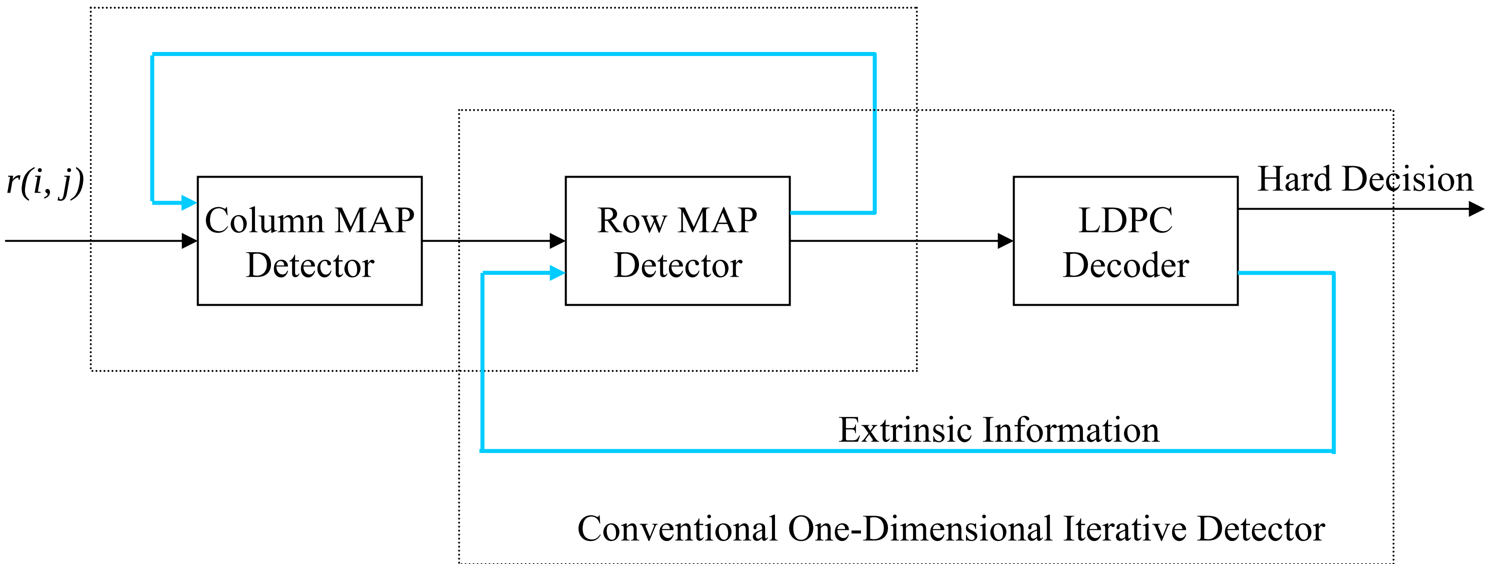


$$h = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 0.25 \end{pmatrix} = \begin{pmatrix} 1 \\ 0.5 \end{pmatrix} \begin{pmatrix} 1 & 0.5 \end{pmatrix}$$

- Advantages of Separable 2D ISI
  - Apply existing one-dimensional equalization methods
  - Reduced Detector Complexity

Tüchler *et al.*, "Turbo equalization: principles and new results," *IEEE Trans. On Comm.*, May 2002.

Row-Column Decoder Diagram

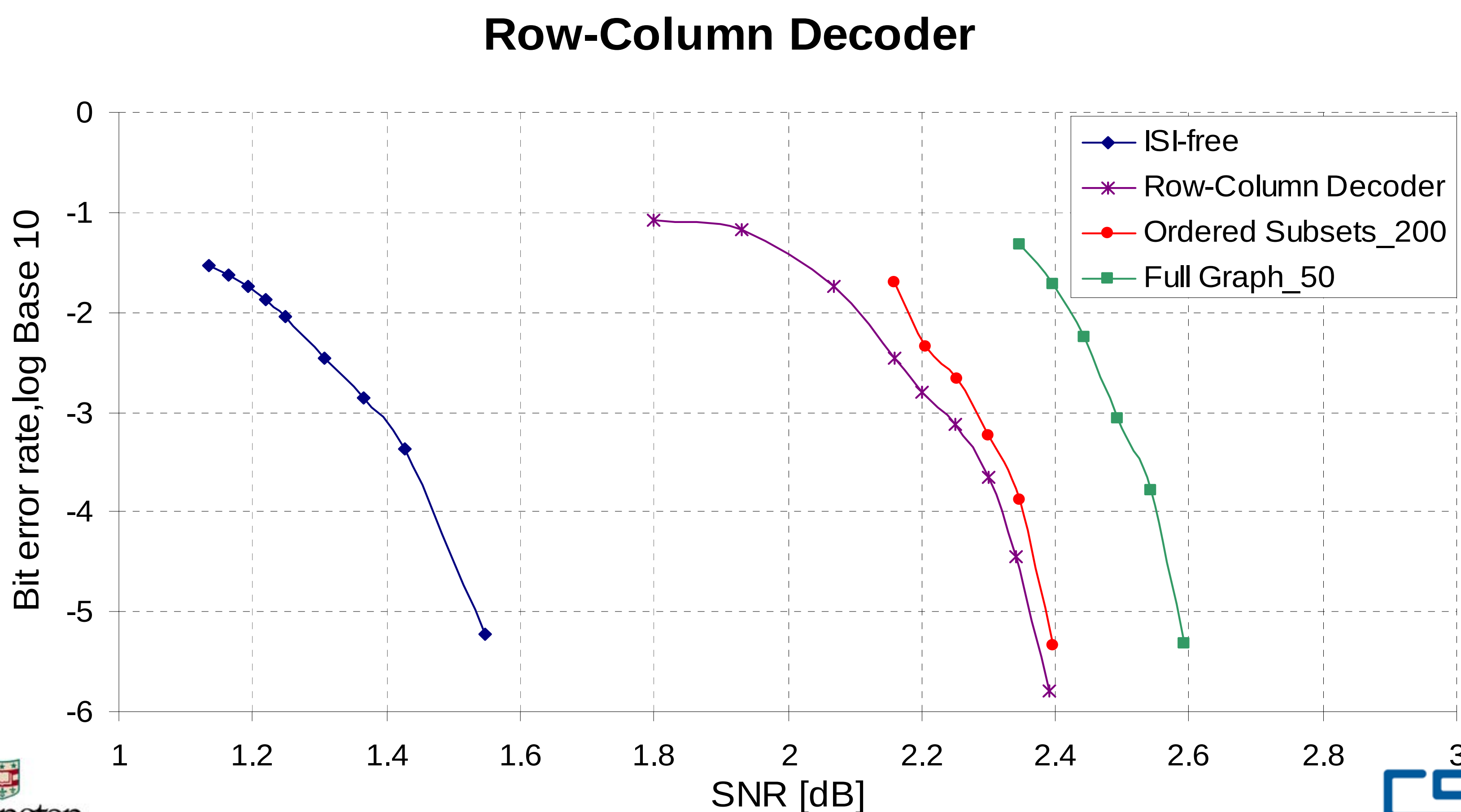


- Inputs to column detector are not binary

Wu *et al.*, "Iterative detection and decoding for separable two-dimensional intersymbol interference" *IEEE Trans. Magn.*, July 2003.

Performance

Block length 10000, regular, rate-0.5, LDPC code



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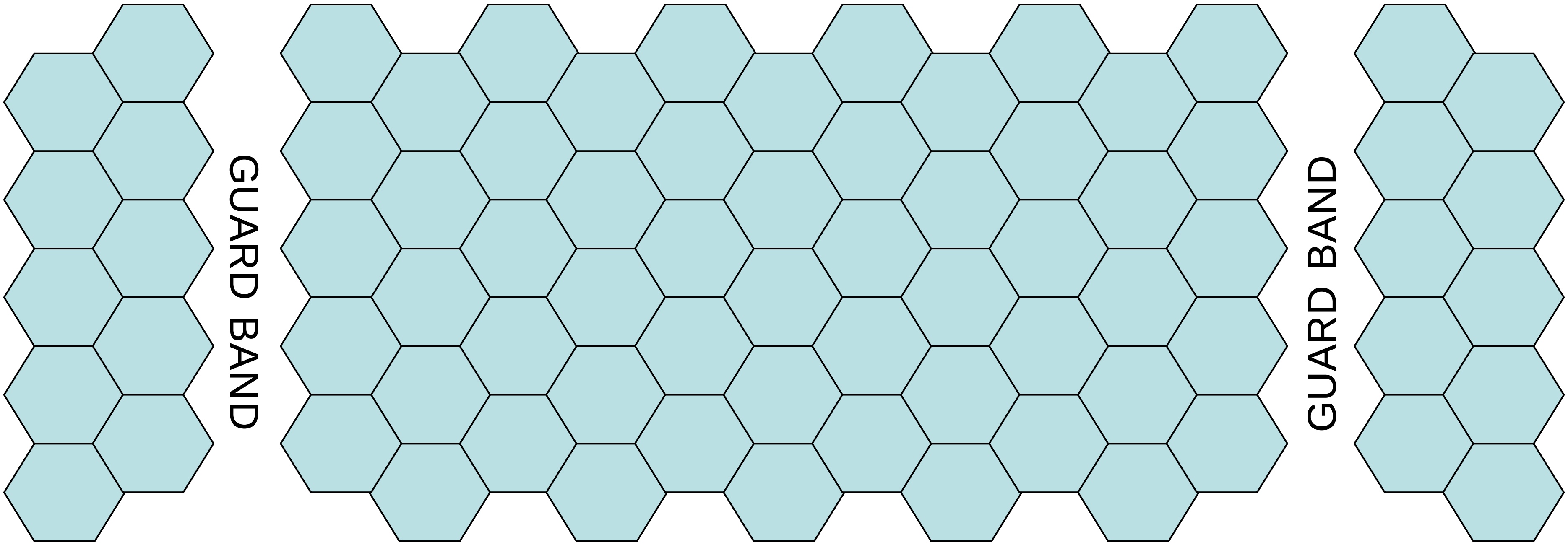
Two-Dimensional Optical Storage



- 11 rows of hexagonal bit-cells stacked together
- Guard band separates adjacent stacks

A. H. J. Immink et al., "Signal processing and coding for two-dimensional optical storage," GLOBECOM 2003.

2D ISI Model

$$I(\mathbf{R}) = 1 - \sum_{j} c_j u_j + \sum_{j \neq k} d_{j,k} u_j u_k \qquad j,k \in \mathrm{N}(\mathbf{R})$$

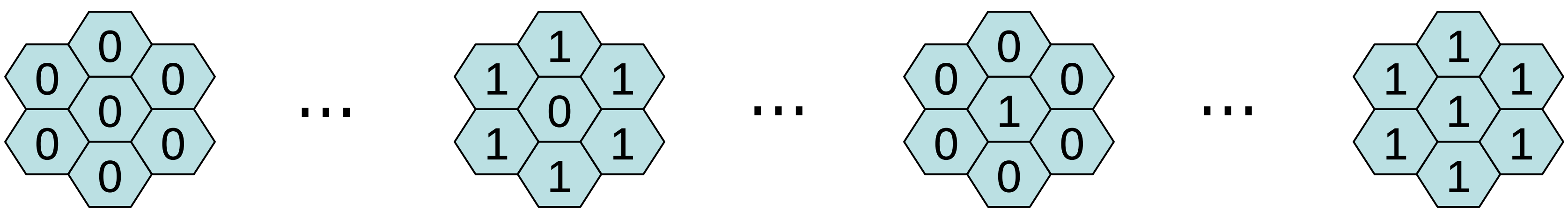
$I(\mathbf{R})$: received intensity at location **R**
c_j : coefficients of linear ISI
$d_{j,k}$: coefficients of nonlinear ISI
u_j : binary data written on disc

- Based on scalar diffraction model proposed by Wim Coene
- Nonlinear ISI

W. M. J. Coene, "Nonlinear signal-processing model for scalar diffraction in optical recording," *App. Opt.*, Nov. 2003.

2D ISI Model

- ISI coefficients calculated using recording specifics
- For simplicity use only nearest neighbors: 14 configurations



2D ISI: Signal Levels

Signal levels using nearest neighbors only

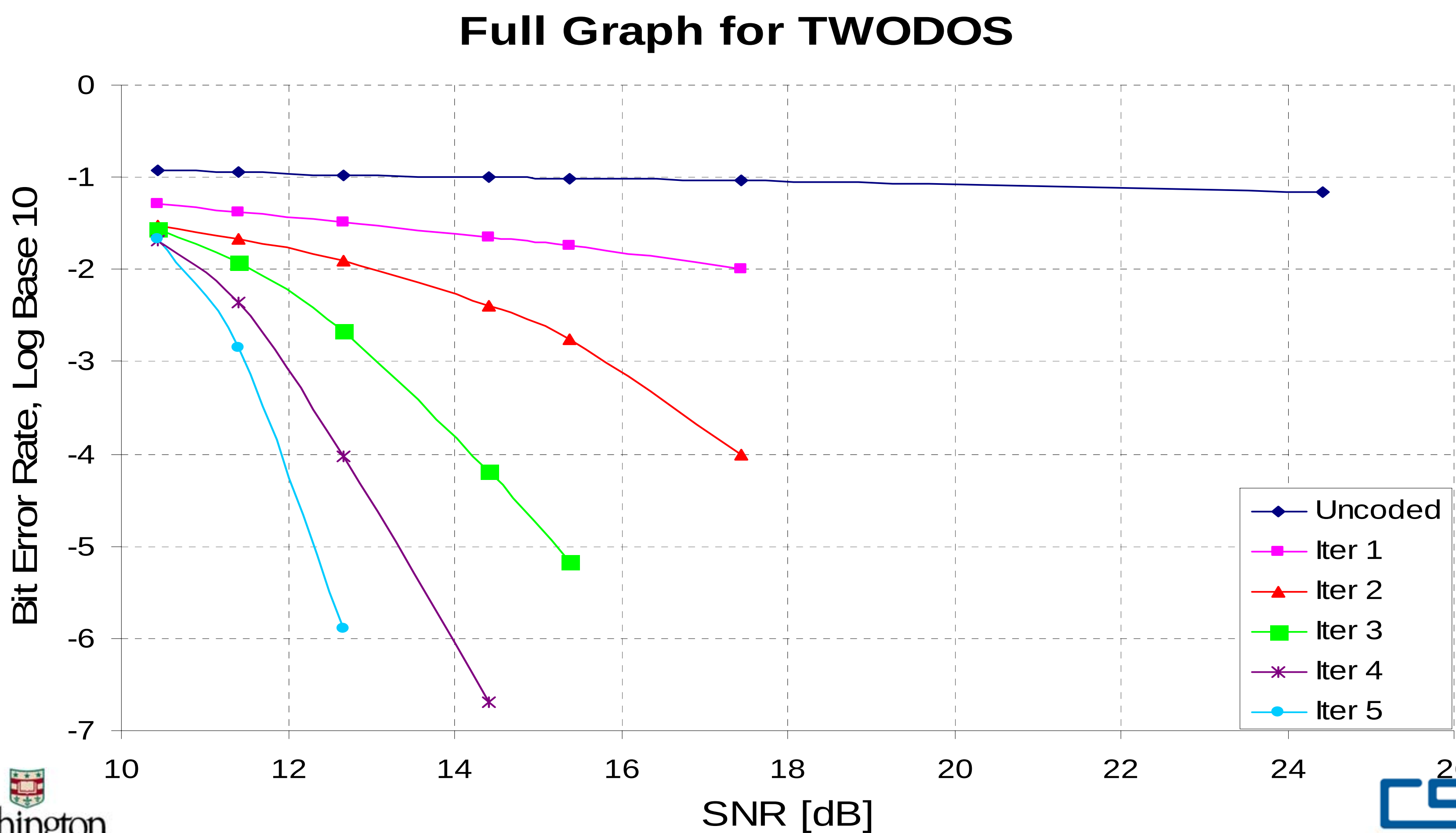
Nonzero neighbors	Central bit=0	Central bit=1
0	0.95	0.50
1	0.80	0.35
2	0.70	0.30
3	0.55	0.20
4	0.45	0.15
5	0.35	0.10
6	0.25	0.05

- Range of signal when central bit is 0 is greater than when central bit is 1: asymmetry due to nonlinear ISI

A. H. J. Immink *et al.*, “Signal processing and coding for two-dimensional optical storage,” *GLOBECOM* 2003.

Full Graph Performance Results

Block length 10000, regular, rate-0.9, LDPC code



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Density Evolution

- Assume messages are i.i.d. random variables
- Evolve message densities through the message maps
- If densities converge to desired density, then error-free transmission possible otherwise not
- Gives lower bound on performance of message-passing scheme

T. Richardson and R. Urbanke, "Capacity of low-density parity-check Codes under message-passing decoding," *IEEE Trans. Inform. Theory*, Feb. 2001

Density Evolution for Full Graph Message-Passing

- Codeword bit nodes to check nodes

$$L_{x\to z}^{(l)} = \sum_{m\in N(x)} L_{m\to x}^{(l-1)} + \sum_{z'\in N(x)\backslash z} L_{z'\to x}^{(l-1)} \qquad \text{CONVOLUTION}$$

- Check nodes to codeword bit nodes

$$\tanh\frac{L_{z\to x}^{(l)}}{2} = (-1)^{z} \prod_{x'\in N(z)\backslash x} \tanh\frac{L_{x'\to z}^{(l-1)}}{2} \qquad \text{LOOKUP TABLE}$$

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Density Evolution…

- Codeword bit nodes to measured data nodes

$$L_{x\to m}^{(l)} = \sum_{m'\in N(x)\backslash m} L_{m'\to x}^{(l-1)} + \sum_{z\in N(x)} L_{z\to x}^{(l)} \quad \text{CONVOLUTION}$$

- Measured data nodes to codeword bit nodes

$$L_{m\to x}^{(l)} = f(\{L_{x'\to m}^{(l)} : x'\in N(m)\setminus x\})$$

MONTE CARLO SIMULATION

Density Evolution Results

Full graph algorithm for TWODOS

Code Parameters (d_v,d_c)	Rate	Threshold Full Graph (σ^2)	SNR [dB]
(3,4)	0.25	0.0206	6.846
(3,6)	0.50	0.0071	8.462
(3,30)	0.90	0.0025	10.443

Complexity Considerations

- LDPC code complexity per iteration is linear in block length
- At every iteration the number of computations on the channel ISI graph are proportional to number of edges in the channel ISI graph
- Messages are floating point precision: fixed point implementation needed

Conclusions

- Joint Decoding and Equalization
  - Prior simulations for magnetic media
  - Current simulations for optical hexagonal storage—account for nonlinearity
- Two-Dimensional Approach
- Potential SNR Improvement

Washington University Team

- Professor Ronald S. Indeck
  Expertise in magnetics, optics, experimental design, system integration
  - President of the IEEE Magnetics Society
  - Founder of Magnetics Information Systems Center at Washington University
  - Founding Director of the Center for Security Technologies at Washington University
  - Numerous national and international advisory appointments
- Professor Joseph A. O'Sullivan
  Expertise in information theory, imaging systems design and analysis, signal and image processing
  - Chair of the Washington University Faculty Senate
  - Director of the Electronic Systems and Signals Research Laboratory at Washington University
  - Associate Director of the Center for Security Technologies at Washington University
  - Past associate editor and publications editor for the IEEE Transactions on Information Theory
- Team of graduate students including Naveen Singla
- Synergistic research activities at Washington University include reconfigurable hardware for high speed computations

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