Influence of Pit-Shape Variation on the Decoding Performance for Two-Dimensional Optical Storage (TwoDOS)

Naveen Singla and Joseph A. O'Sullivan
Electronic Systems & Signals Research Laboratory
Department of Electrical & Systems Engineering
Washington University
St. Louis, MO 63130, USA
Email: [singla, jao]@wustl.edu

***Abstract*— We study the influence of the variation in the shape of the pits for the two-dimensional optical storage (TwoDOS) paradigm vis-à-vis the readback signal and decoding performance. The algorithm used for decoding is a message-passing based joint equalization and decoding algorithm proposed by Singla and O'Sullivan (Intl. Symp. Info. Theory, Sept. 2005). The performance dependence of the algorithm on pit shape is studied by comparing the noise-tolerance thresholds for the algorithm. The TwoDOS model under consideration also includes two forms of media noise: pit-position noise, arising due to the deviation in the location of the pits from their intended positions and pit-size noise, arising due to variations in the radii of the pits. It is shown that as the shape of the pit changes from the ideal cylindrical shape towards a conical shape there is a sharp decrease in the range of the signal intensity. This decrease in the range leads to increased signal folding which is detrimental for the decoding algorithm. However, we show that when the ratio of the area of the pit to the area of the hexagonal bit cell is large (∼1) then the extent of signal folding for a conical profile is actually less than that of a cylindrical profile. Then, in this case, the performance of the decoder for a conical profile is better than for a cylindrical profile.**

I. Introduction

Optical storage has successfully filled the niche for removable media over the past three decades. In keeping with the increasing demand for storage capacity, optical storage has evolved from the CD (700 MB) to the DVD (4.7 GB for a single layer/single sided disc) to the blu-ray disc (BD, 25 GB). Although the underlying medium in each case is a planar disc, the storage, however, is quasi two-dimensional. Data are stored in a spiral starting from the center of the disc moving outwards, but with successive revolutions of the spiral well-separated. This is done to reduce the interference from adjacent tracks during the readback of a particular track; an interference commonly referred to as inter-track interference (ITI). However, adjacent bits within a track do interfere; this is referred to as intersymbol interference (ISI). Numerous efficient signal-processing techniques exist to detect the data reliably in the presence of ISI (ignoring the ITI). Examples of such schemes are the Viterbi algorithm and the BCJR algorithm which are used for maximum-likelihood sequence detection and bit-by-bit maximum *a posteriori* detection, respectively.

Two-dimensional optical storage (TwoDOS) [1], purported to be the successor of BD, is a novel storage technology that is a first step towards truly two-dimensional storage. Data are still stored in a spiral but instead of containing only one row of bits, as is done conventionally, TwoDOS allows for multiple rows of bits to be stacked together in each revolution of the spiral. Successive revolutions of this "broad" spiral are separated by a guard band of one empty row of bits. In addition to reducing the wasteful inter-track spacing, TwoDOS stores data in hexagonal bit cells, as opposed to rectangular in conventional recording, leading to denser packing and a further increase in storage density. Furthermore, read-out is performed in parallel across multiple rows using an array of lasers, leading to improved data rates. Using the same optics as BD (same wavelength laser and objective with same numerical aperture), TwoDOS has achieved a twofold increase in storage capacity and tenfold increase in data rate over BD.

A channel model for TwoDOS was proposed by Coene [2]. This model uses scalar diffraction theory, proposed by Hopkins [3], to represent the measured intensity as a function of the stored data. The intensity is a nonlinear function of the stored data containing both linear and bilinear contributions from the stored data bits. This model was extended by Moinian *et al.* [4] to include the effects of media noise. They considered two sources of media noise: pit-position noise, arising due to the deviation in the location of the pits from their intended positions; and pit-size noise, arising due to variations in the radii of the pits. However, neither Coene nor Moinian *et al.* considered the effect of the variation of the shape of the pits on the measured intensity. They assumed that the pits have an ideal cylindrical shape so that the phase difference between the light reflected from the pit and from the land is a constant. While this assumption is a good starting point, a more sophisticated model of the channel would account for the variation in the shape of the pits. To that end, in this paper we consider the TwoDOS channel model with the aforementioned sources of media noise and varying pit shapes. In particular, we study the change in the signal levels as the pit shape changes from the ideal cylindrical shape to a conical shape. This phenomenon was first studied by Hopkins and Chung [3]. It is shown that as the shape of the pit changes from the ideal cylindrical shape towards a conical shape, there is a decrease in the range of the signal intensity. The effect of this decrease in range may be beneficial or not, depending on the ratio of the area of the pit with respect to the area of the bit cell. When this ratio is small (∼0.5) a conical shape increases the amount of signal folding. When the ratio is large (∼1) then the extent of signal folding for a conical profile is actually less than that of a cylindrical profile.

As mentioned previously, various low-complexity schemes exist for detecting data in the presence of ISI (ignoring ITI). However, for TwoDOS the ITI is the same as the ISI and this gives rise to two-dimensional (2D) ISI during readback. 2D ISI is not as easy to deal with as one-dimensional ISI, primarily because the schemes that work for the latter do not extend for the former, at least not without an exponential increase in complexity. Recently Ordentlich and Roth have shown that the problem of maximum-likelihood sequence detection for two-dimensional ISI channels is NP-complete [5]. This has spurred a lot of research in the past few years for developing schemes to mitigate 2D ISI. Currently, many low-complexity schemes, none of them provably optimal, exist. They have had mixed success for 2D ISI channels [6]-[14]. One of these schemes, proposed by Singla *et al.* [6], is a joint equalization and decoding scheme. This scheme, termed the full graph algorithm, uses sum-product message-passing to decode data on 2D ISI channels. Singla and O'Sullivan [7] extended the full graph algorithm for application to the nonlinear channel model of TwoDOS. Therein they also proposed a density evolution algorithm to compute the noise-tolerance thresholds for the full graph algorithm and showed, via simulations, that the computed thresholds represent lower bounds on the asymptotic performance of the full graph algorithm for the TwoDOS channel model. In this paper noise-tolerance thresholds are computed for the full graph algorithm for varying degrees of media noise and different pit shapes. The noise-tolerance thresholds are used as a measure of performance for the full graph algorithm. The noise-tolerance thresholds corroborate the trends observed in the signal levels: when the area of a pit is small compared to the area of a bit cell, the performance of the full graph algorithm degrades when the pit shape changes from cylindrical towards conical. However, the opposite trend is observed when the ratio of areas is large. This is a direct consequence of the extent of signal folding.

The rest of the paper is organized as follows. Section 2 discusses the channel model for TwoDOS and describes the mathematical model for the noise sources. Section 3 describes the pit shape variation considered in this paper and shows the resulting read-out signals for the TwoDOS channel model. Section 4 briefly describes the full graph algorithm, the associated density evolution algorithm, and presents the noise-tolerance thresholds for the TwoDOS channel model with media noise and pit shape variation. Section 5 concludes the paper.

II. Channel Model

In the TwoDOS paradigm data, are stored in a broad spiral and the bit cells are hexagonal. As in conventional optical recording, a 0/1 is represented physically as a land/pit region, however, successive pits are not allowed to overlap. The data bit 1 is stored as an isolated pit in a bit cell. Two parameters specify the geometry of the recording: the lattice parameter, denoted a_H, is the center-to-center distance between adjacent bit cells; and the pit radius, denoted ρ_0. The optics is assumed to be the same as for BD, that is, a blue-violet laser of wavelength (λ) 405 nm and an objective with numerical aperture (NA) 0.85 are used for read and write. In what follows we describe the scalar diffraction based channel model for TwoDOS proposed by Coene [2]. We will follow the notation used by Coene.

The scalar diffraction model considers the complex-valued optical wave front of the scanning laser spot, denoted $p(\mathbf{R}-\mathbf{R}_p)$, that is incident at a position $\mathbf{R}_p$ in the plane of the disc. The optical wave front is subsequently diffracted by the pits and lands on the disk, after which it propagates back through the objective lens toward the photodetector. The surface of the disc is represented by a reflection function, denoted $r(\mathbf{R})$. Following Coene [2] we define $r(\mathbf{R})$ as follows:

$$r(\mathbf{R}) = 1 + \sum_m a_m(\mathbf{R}-\mathbf{R}_m)W(\mathbf{R}-\mathbf{R}_m), \qquad (1)$$

where the "window function" $W(\mathbf{R}-\mathbf{R}_m)$ for a pit centered at $\mathbf{R}_m$ is 1 inside the pit area and 0 outside the pit area; $a_m(\mathbf{R}-\mathbf{R}_m)$ is the term that accounts for the phase difference between a land and a pit bit and is defined as

$$a_m(\mathbf{R}-\mathbf{R}_m) = u_m[\exp(j\phi(\mathbf{R}-\mathbf{R}_m)) - 1], \qquad (2)$$

where $u_m \in \{0,1\}$ denotes the value of the stored bit and $\phi(\mathbf{R}-\mathbf{R}_m)$ denotes the position-dependent phase change. Using scalar diffraction theory Hopkins showed that the complex-valued wave function in the exit pupil of the objective can be obtained as follows;

$$|\psi\rangle = \mathrm{FT}_{\mathbf{R}\to\mathbf{\Omega}}[p(\mathbf{R}-\mathbf{R}_p)r(\mathbf{R})], \qquad (3)$$

where $\mathrm{FT}_{\mathbf{R}\to\Omega}$ denotes the Fourier transform from the disc plane to the plane of the exit pupil of the objective lens. The detected signal (intensity) can be obtained as the squared norm of the wave function:

$$\begin{aligned} I &= \langle\psi|\psi\rangle, \\ &= \int_{(CA)} \psi^*(\mathbf{\Omega})\psi(\mathbf{\Omega})\mathrm{d}\mathbf{\Omega}, \end{aligned} \qquad (4)$$

where CA denotes the central aperture of the objective lens. After straightforward manipulations the detected signal can be written as

$$I = 1 + \sum_m \langle\psi_L|\psi_m\rangle + \sum_m \langle\psi_m|\psi_L\rangle + \sum_{m,n} \langle\psi_m|\psi_n\rangle, \qquad (5)$$

where

$$|\psi_L\rangle = \text{FT}_{\mathbf{R}\to\mathbf{\Omega}}[p(\mathbf{R}-\mathbf{R}_p)], \quad (6)$$

and

$$|\psi_m\rangle = \text{FT}_{\mathbf{R}\to\mathbf{\Omega}}[p(\mathbf{R}-\mathbf{R}_p)a_m(\mathbf{R}-\mathbf{R}_m)W(\mathbf{R}-\mathbf{R}_m)]. \quad (7)$$

The signal intensity is normalized so that the "all-land" term $\langle\psi_L|\psi_L\rangle = 1$. At this point one can compute the signal levels given the geometry of the recording and the parameters of the optics.

As mentioned previously, two sources of media noise are considered in this paper: pit-size noise and pit-position noise. Moinian *et al.* [4] have used these noise sources for multilevel TwoDOS. In what follows we briefly describe how the media noise is modeled mathematically.

1) **Pit-Size Noise**: Coene [2] showed that in order to reduce signal-folding during readback the pits must occupy only a small area of the bit cells. Through simulations he concluded that a pit area equal to half the area of the bit cell reduces the signal folding significantly. This gives an "optimum" area of $\sqrt{3}a_H^2/4$ for the pit, the corresponding radius is denoted ρ_{opt}. It is very difficult to ensure that the area of all the pits is equal to the optimum area, there is always going to be some deviation from the optimum area. This variation in the area of the pits is modeled as a Gaussian with mean given by the optimum area and standard deviation σ_{ps}, which will be assumed to be a fraction of the optimum area.
2) **Pit-Position Noise**: The pit-position noise accounts for random variations in the position of a pit from its intended location which is at the center of a bit cell. The variation in the position has two terms: the displacement from the center of the bit cell and the direction of the displacement. The displacement is modeled as a Gaussian with zero mean and standard deviation σ_{pp} and the direction of displacement is derived from a uniform distribution ranging from 0 to 2π. The standard deviation is assumed to be a fraction of the lattice parameter.

To incorporate media noise, the channel model given by (5) needs to be modified slightly: firstly, the center of the window function $W(\mathbf{R}-\mathbf{R}_m)$ is not exactly at $\mathbf{R}_m$ but is perturbed by an amount determined by the pit-position noise; secondly, the size of the window is larger or smaller than the nominal size depending on the pit-size noise. This modifies the computation of the wave function $|\psi\rangle$ in (3). In addition to media noise we also assume that the data are corrupted by additive white Gaussian noise (AWGN) with zero mean and variance given by σ_w^2.

III. Pit Shape Variation

The pits on an optical disc are produced using a photoresist technique, hence the height profile is inevitably rounded; that is, the walls of the pit are not infinitely steep. Hopkins and

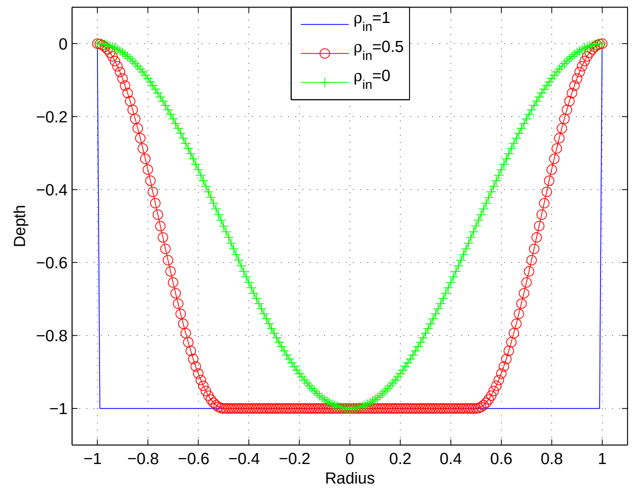

Fig. 1. Three different pit depth profiles corresponding to cylindrical, frustoconical, and conical shapes.

Chung [3] studied the effect of the variation of the depth profile of the pits on the readback signal. The depth profile they used, and the one used in this paper, described as a function of the radial distance from the center of a pit is

$$d(\rho) = \begin{cases} d & \rho \le \rho_{in} \\ d\sin^2\left(\frac{\pi(\rho_0-\rho)}{2(\rho_0-\rho_{in})}\right) & \rho_{in}\le\rho\le\rho_0 \end{cases} \quad (8)$$

where $d = \lambda/4$ is the depth which gives rise to a phase difference of π; ρ_{in} is the distance up to which the pit bottom is flat and ρ_0 is the radius of the pit at the surface of the disc. The phase change corresponding to a given depth profile is $\phi(\rho) = 2d(\rho)\cdot 2\pi/\lambda$. Fig. 1 shows the pit shapes for three different scenarios: $\rho_{in} = \rho_0$, corresponding to a cylindrical pit; $\rho_{in} = 0$, corresponding to an (almost) conical pit; and $\rho_{in} = 0.5\rho_0$, corresponding to an intermediate pit shape (christened "frustoconical" by Hopkins and Chung because it looks like the frustum of a cone). In the figure, ρ_0 and d are assumed to be 1.

In what follows the radius of the pits refers to the radius of the pits at the surface of the disc which is the same for all three pit shapes.

A. Signal Levels

The extent of the interference in TwoDOS (or in any optical disc storage for that matter) is limited by the spot size of the laser used for reading. The intensity pattern obtained when a plane wave-front is diffracted by a lens is the Airy disc [15] which falls off fast enough that the contributions from the side lobes can be neglected without significant loss in accuracy. For BD optical parameters the diameter of the main lobe of the airy disc turns out to be 580 nm. For TwoDOS when the storage densities are low-to-moderate, this leads to interference from the nearest neighbors only, but for high storage densities (twice that of BD) the interference from bits in the other "shells" also becomes significant [2].

In this paper we only consider the case where a_H=165 nm, corresponding to a 1.4 fold increase in storage density over BD (without taking into consideration the loss due to coding redundancy). For this case it suffices to assume that the interference is constrained to the first shell only. Using a nearest neighbor interference model, the signal intensity depends on the data bit stored in the central bit cell and the 6 neighboring bit cells. It was shown by Coene [2] that the signal levels for clusters with the same central bit and same number of nonzero neighbors have very similar values. Hence, the signal intensity can be considered as a function of the central bit and the number of nonzero neighbors only. This approximation gives a total number of 14 clusters. Fig. 2 shows four of these 14 clusters.

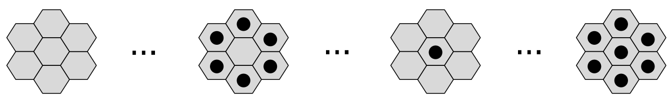

Fig. 2. Four of the possible 14 nearest neighbor configurations for TwoDOS. The dark circles in the cells depict a pit corresponding to a stored 1. Absence of a pit indicates a stored 0. In this case the pits cover only about half the area of the hexagonal bit cells. This is done to reduce signal folding [2].

Fig. 3 shows the signal levels for the 14 cluster types for the three different pit shapes. The cluster type is indexed as $7u_c + \sum_{j=1}^{6} u_j$, where u_c is the value of the central bit and u_j are the values of the neighboring bits. Thus, for example, the cluster types of the four clusters in Fig. 2 are, respectively, 0, 6, 7, and 13. The signal values are calculated for BD parameters, i.e., λ=405 nm, NA=0.85. The pit radius is chosen so that the pit covers half the area of the bit cell which corresponds to a pit radius of 60 nm.

Fig. 3 shows that as the pit shape changes from cylindrical to conical the signal value corresponding to a particular cluster

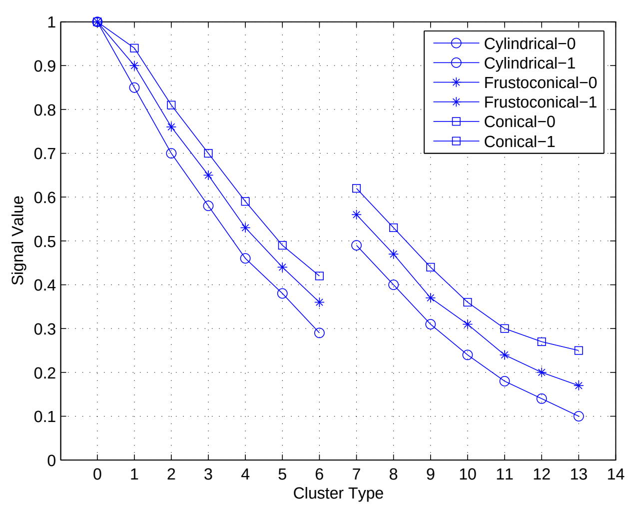

Fig. 3. The signal levels for TwoDOS for three different pit depth profiles corresponding to cylindrical, frustoconical, and conical shapes. The curves correspond to a hexagonal lattice parameter of 165 nm and pit radius of 60 nm.

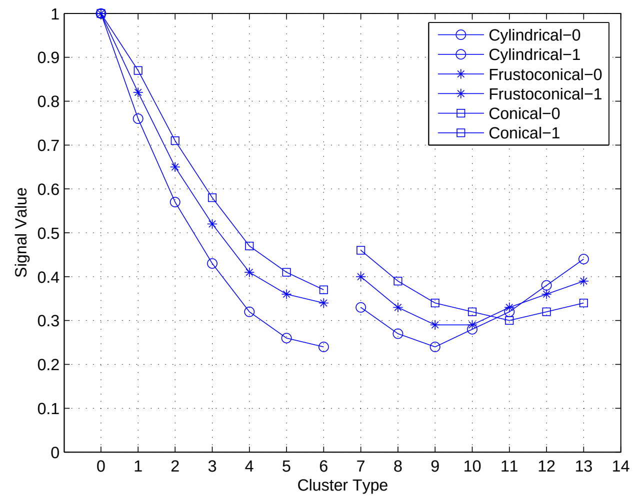

Fig. 4. The signal levels for TwoDOS for three different pit depth profiles corresponding to cylindrical, frustoconical, and conical shapes. The curves correspond to a pit radius of 82.5 nm.

type increases. This is to be expected since for a cylindrical pit the light reflected from any point is out of phase with the incident light (recall that the pit depth is chosen so as correspond to a phase change of π). On the other hand, when the pit shape changes so that the walls are no longer infinitely steep, the reflected light will have a continuous phase change from 0 to π leading to less reduction in intensity. As a consequence of this there is a marked decrease in the modulation range of the signal. For the cylindrical shape the signal levels range from 0.1 to 1, whereas for a conical profile the range is reduced to 0.27 to 1. Also note that the overlap between the signal values when the central bits are 0 and 1 is almost the same for the three depth profiles. This results in increased signal folding and is expected to degrade the performance for detection algorithms since now it becomes more difficult to distinguish between the different cluster types.

The results of Fig. 3 are intuitively satisfying, the pit radius is chosen so as to maximize the modulation range of the signal for a cylindrical pit shape. If the pit covers half the area of a bit cell then one would expect the signal level to be almost zero when the cluster type is 13. However, as shown by Coene [2], departure from this “optimum” pit radius can give rise to signal folding. In particular, when pits cover more than half the area of the bit cell then as the cluster type index increases the disc surface looks more and more like a perfectly reflecting mirror, giving rise to anomalous signal values. In that case it is reasonable to expect that changing the pit shape from cylindrical to conical will reduce signal folding since again the conical pit will not diffract the light as much as a cylindrical pit. Simulations confirm this, as is shown next.

Fig. 4 shows the signal levels when the pit radius is increased from 60 nm to 82.5 nm, the maximum that the pits can have without overlapping. The figure shows significant signal folding for the cylindrical pit shape and that signal folding is reduced markedly when the pit profile changes to frustoconical and ultimately conical. This enables more reliable detection of the stored data as is confirmed by the results in the next section.

IV. Noise-Tolerance Thresholds

The full graph algorithm, proposed by Singla *et al.* [6], is a message-passing based algorithm that is used for joint equalization and decoding for linear 2D ISI channels. Singla *et al.* focus on advanced storage technologies and assume that prior to storage the data is encoded using a low-density parity-check (LDPC) code. The algorithm performs sum-product message-passing on the joint factor graph [18] of the channel and the LDPC code to compute the *a-posteriori* probabilities of the stored bits given the observed data. Following is a brief description of the full graph algorithm. The joint code/channel graph has three types of nodes: parity-check nodes, codeword bit nodes, and observed data nodes corresponding, respectively, to the parity-check equations of the LDPC code, the stored data bits, and the observed data. The parity-check nodes are connected to the codeword bit nodes through the parity-check matrix of the LDPC code and the codeword bit nodes are connected to the observed data nodes through the 2D ISI. Sum-product message-passing is performed on this graph using the following schedule: messages are passed from the codeword bit nodes to the measured data nodes and back, then from the codeword bit nodes to the check nodes and back. This completes one iteration. The process is repeated until the decoder converges or a maximum number of iterations have been performed. Due to the 2D ISI the channel part of the graph has many short cycles so that the computed *a-posteriori* probabilities are only approximate. Singla and O'Sullivan [7] extended the use of the full graph algorithm to nonlinear 2D ISI channels. They showed that for the TwoDOS channel model with only AWGN, the full graph algorithm outperforms the multi-track Viterbi algorithm, proposed by Immink *et al.* [14] for detection in TwoDOS, by more than 6 dB. The LDPC code used was a regular (3,30) code which has a rate of 0.9.

For many channels and decoders of interest, LDPC codes exhibit a threshold phenomenon [16]; there exists a critical value of the channel parameter (e.g., variance of an AWGN or crossover probability of a BSC), called the noise-tolerance threshold, such that an arbitrarily small bit error probability can be achieved if the channel's noise level is smaller than the noise-tolerance threshold. On the other hand, when the channel's noise level is larger than the noise-tolerance threshold, the probability of bit error is larger than a positive constant. Richardson and Urbanke [16] developed an algorithm called *density evolution* for iteratively calculating message densities, enabling the determination of the aforementioned threshold. Kavčić *et al.*, [17] extended the work of Richardson and Urbanke to compute noise-tolerance thresholds for one-dimensional ISI channels. Singla and O'Sullivan [7] presented a density evolution algorithm for computing noise-tolerance thresholds for the full graph algorithm applied to TwoDOS. They also showed, via simulations, that the computed thresholds represent lower bounds on the asymptotic performance of the full graph algorithm for the TwoDOS channel model. The density evolution algorithm tracks the evolution of the density of the messages that are passed in the message-passing algorithm. For a fixed channel noise level the message densities are evolved iteratively. If the probability of bit error tends to zero as the iterations progress then the noise level is incremented and again the densities are evolved. This process is continued until a certain noise level is obtained such that the probability of bit error does not go to zero as the iterations progress, this then is the noise-tolerance threshold for that LDPC code on that channel. In what follows we present noise-tolerance thresholds of the full graph algorithm for the TwoDOS channel model incorporating the media noise described in Section II and AWGN and for different pit shapes.

The thresholds are computed for a_H=165 nm, hence, the interference is restricted to the first shell only. Again, three pit shapes are considered: cylindrical, frustoconical, and conical. For each pit shape varying degrees of pit-size and pit-position noise are considered. The LDPC code in every case is a randomly constructed, regular (3,30) code. The SNR (E_b/N_0) is defined as the average signal energy divided by the noise power;

$$SNR = 10\cdot\log_{10}\frac{\left(\sum_{n=0}^{6}\binom{6}{n}(s_{n_0}^2+s_{n_1}^2)\right)/2^7}{R\cdot(2\sigma_w^2)}, \quad (9)$$

where s_{n_0}/s_{n_1} is the signal level given that the central bit is a 0/1 and has n nonzero neighbors and R is the LDPC code rate.

Table I, Table II, and Table III, show the computed thresholds when the pit shapes are cylindrical, frustoconical, and conical, respectively, for a pit radius of 60 nm. The SNR for the cases when the AWGN variance is below 0.0001 is set to ∞. In each case it is observed that as the media noise increases the associated threshold becomes higher. Moreover, pit-size noise degrades the threshold much more than pit-position noise. Thus the recording is much more sensitive to variations in the pit size than it is to the position of the pits within the bit cells. This is in agreement with the results of Moinian *et al.* [4].

TABLE I
Thresholds for the full graph algorithm for TwoDOS with cylindrical pits and varying media noise.

Pit-Size Noise (σ_{ps})	Pit-Position Noise (σ_{pp})			
	0	1	2	5
0	12.4720	12.8533	13.4333	15.1173
1	13.1336	14.0132	15.1173	16.7654
2	14.0132	15.8637	17.1130	20.9152
3	15.6004	17.4909	20.9152	27.9049

The tables also show that as the pit shape changes from cylindrical towards conical there is a degradation in the threshold. This is in agreement with the results of the previous section; signal folding increases as the pit shape changes from cylindrical to conical thus it becomes harder for the decoding algorithm to distinguish between different clusters leading to degradation in performance.

TABLE II
THRESHOLDS FOR THE FULL GRAPH ALGORITHM FOR TWODOS WITH FRUSTOCONICAL PITS AND VARYING MEDIA NOISE.

Pit-Size Noise (σ_{ps})	Pit-Position Noise (σ_{pp}) 0	1	2	5
0	12.9912	13.5115	14.2876	16.9358
1	13.8395	14.8946	16.4436	19.4539
2	15.0045	17.1132	19.7757	27.9049
3	16.7654	19.4539	∞	∞

TABLE III
THRESHOLDS FOR THE FULL GRAPH ALGORITHM FOR TWODOS WITH CONICAL PITS AND VARYING MEDIA NOISE.

Pit-Size Noise (σ_{ps})	Pit-Position Noise (σ_{pp}) 0	1	2	5
0	13.5115	14.1028	15.0045	18.3624
1	14.3830	15.7300	17.4909	20.9152
2	15.7300	17.9049	24.8946	∞
3	18.8740	24.8946	∞	∞

The thresholds for the case when the pit radius is 82.5 nm (that is, the pits almost cover the bit cells) are, respectively, ∞, 25.88 dB, and 23.85 dB when the pit shapes are cylindrical, frustoconical, and conical. In this case no media noise is considered. Again, the results are in agreement with those of the previous section; when the pits cover almost the entire bit cell the performance of the full graph algorithm improves as the pit profile varies from cylindrical to conical.

Throughout this paper we have assumed that the radius of the differently shaped pits is the same at the surface of the disc. There are other choices for the definition of pit radius; the pits could have radii (at the surface) such that they have equal volumes or such that equal average radii (averaged over the entire depth). In either case the radii at the surface will be different for the three pit shapes and the trends in decoding could be markedly different from what is presented in this paper. These alternate cases are the subject of future research.

V. CONCLUSION

In this paper we studied the influence of the pit shape on the signal levels and the decoding for the TwoDOS paradigm. It was shown that as the shape of the pit changes from the ideal cylindrical shape towards a conical shape, there is a sharp decrease in the range of the signal intensity. This can lead to an increase or decrease in the amount of signal folding depending on whether the pits occupy a small or large area of the bit cell. The decoding performance was studied using the full graph algorithm. The channel model for TwoDOS takes into account two sources of media noise: pit-size noise, variation in the size of the pits; and pit-position noise, variation in the location of the pits. Using density evolution, noise-tolerance thresholds were computed for the full graph algorithm for the TwoDOS paradigm taking into account the media noise. The noise tolerance thresholds support the conclusions drawn from observing the signal levels; when pit shape changes from cylindrical towards conical the performance degrades when the pit areas are small compared to the area of the bit cell whereas the performance improves when the pits almost cover the bit cell.

VI. ACKNOWLEDGMENT

The authors would like to thank the reviewers for their helpful comments and suggestions. This work was supported by the Office of Naval Research under Award N00014-03-1-0110.

REFERENCES

[1] W. M. J. Coene, "Two-dimensional optical storage," *Optical Data Storage 2003*, Proc. SPIE 5069, pp. 90-92, 2003.
[2] W. M. J. Coene, "Nonlinear signal-processing model for scalar diffraction in optical recording," *Appl. Opt.*, vol. 42, pp. 6525-6535, Nov. 2003.
[3] H. H. Hopkins and C. S. Chung, "Influence on the read-out signal of the height profile of the pits (or bumps) on optical discs," *J. Mod. Opt.*, vol. 42, pp. 57-83, Jan. 1995.
[4] A. Moinian, L. Fagoonee, and B. Honary, "Symbol detection for multilevel two-dimensional optical storage using a non-linear channel model," *Intl. Conf. Comm.*, Seoul, South Korea, May 2005.
[5] E. Ordentlich and R. M. Roth, "On the computational complexity of 2D maximum-likelihood sequence detection," *Info. Theory and Applications Workshop*, San Diego, California, Feb. 2006.
[6] N. Singla, J. A. O'Sullivan, R. S. Indeck, and Y. Wu, "Iterative decoding and equalization for 2-D recording channels," *IEEE Trans. Magn.*, vol. 38, pp. 2328-2330, Sept. 2002.
[7] N. Singla and J. A. O'Sullivan "Joint equalization and decoding for nonlinear two-dimensional intersymbol interference channels," *Intl. Symp. Info. Theory*, Adelaide, Australia, Sept. 2005.
[8] Y. Wu, J. A. O'Sullivan, R. S. Indeck, and N. Singla. "Iterative detection and decoding for separable two-dimensional intersymbol interference," *IEEE Trans. on Magn.*, vol. 39, pp. 2115-2120, July 2003.
[9] P. S. Kumar and S. Roy, "Two-dimensional equalization: theory and application to high density magnetic recording," *IEEE Trans. Comm.*, vol. 42, pp. 386-395, Feb. 1994.
[10] K. M. Chugg, X. Chen, and M. A. Neifeld, "Two-dimensional equalization in coherent and incoherent page-oriented optical memory," *J. Opt. Soc. Amer. A*, vol. 16, pp. 549-562, Mar. 1999.
[11] M. Marrow and J. K. Wolf, "Iterative detection of 2-dimensional channels," *IEEE Info. Theory Workshop*, Paris, France, Mar. 2003.
[12] O. Shental, A. J. Weiss, N. Shental, and Y. Weiss, "Generalized belief propagation receiver for near-optimal detection of two-dimensional channels with memory," *IEEE Info. Theory Workshop*, San Antonio, Texas, Oct. 2004.
[13] W. Weeks IV, "Full surface data storage," Ph.D. thesis, University of Illinois at Urbana-Champaign, 1996.
[14] A. H. J. Immink *et al.*, "Signal processing and coding for two-dimensional optical storage," *Proc. GlobeComm.*, Piscataway, New Jersey, 2003.
[15] J. W. Goodman, *Introduction to Fourier Optics*. New York: McGraw Hill, 1996.
[16] T. Richardson and R. Urbanke, "The capacity of low-density parity-check codes under message-passing decoding," *IEEE Trans. on Inform. Theory*, vol. 47, pp. 599-618, Feb. 2001.
[17] A. Kavčić, X. Ma, and M. Mitzenmacher, "Binary intersymbol interference channels: Gallager codes, density evolution and code performance bounds," *IEEE Trans. Inform. Theory*, vol. 49, pp. 1636-1652, July 2003.
[18] F. R. Kschischang, B. J. Frey, and H. A. Loeliger, "Factor graphs and the sum-product algorithm," *IEEE Trans. Inform. Theory*, vol. 47, pp. 498-519, Feb. 2001.