Data Detection for Two-Dimensional ISI Channels

Joseph A. O'Sullivan, Naveen Singla, Yunxiang Wu, Ronald S. Indeck

Department of Electrical Engineering, Washington University, St. Louis, MO 63130.
e-mail:{jao, singla, ywu, rsi}@ee.wustl.edu

Introduction

Two-dimensional (2-D) inter symbol interference (ISI) arises during detection in 2-D magnetic recording systems and page oriented optical memories. Various detection algorithms for 2-D media have been proposed in [1]-[3]. We investigate the performance of some detection algorithms used together with low-density parity-check (LDPC) codes as an error correcting code (ECC) for 2-D ISI channels. The motivation for considering 2-D recording is two fold; firstly, as 1-D recording density nears saturation, 2-D emerges as a natural extension for increasing storage capacity. Secondly, data in 2-D may be stored in sectors bigger than in 1-D, and LDPC and turbo codes which are being studied for their applicability as ECC on recording media as discussed in [4], [5], show good performance for large enough block lengths.

In this paper we refer to the part of our algorithm that mitigates the effects of ISI as detection. The iterations, message passing and decisions directly associated with the LDPC codes are referred to as decoding.

Channel Model

We assume the channel is a discrete 2-D ISI channel. The input data, *x*, is an $l \times l$ matrix with elements $x(i, j) \in \{\pm 1\}$. The noise is assumed to be additive white Gaussian noise (AWGN) with variance σ^2. The output of the channel is,

$$r(i, j) = \sum_{k_1, k_2 = 0}^{L} x(i - k_1, j - k_2) h(k_1, k_2) + w(i, j), \tag{1}$$

where *w* is AWGN. The channel point spread function (channel response) we used for our simulations is,

$$h = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 0.25 \end{pmatrix} \tag{2}$$

In (1), L+1 represents the number of elements over which the ISI extends in each dimension. With the above assumptions, the recording system can be represented by a discrete-time, baseband communication system shown in Figure1.

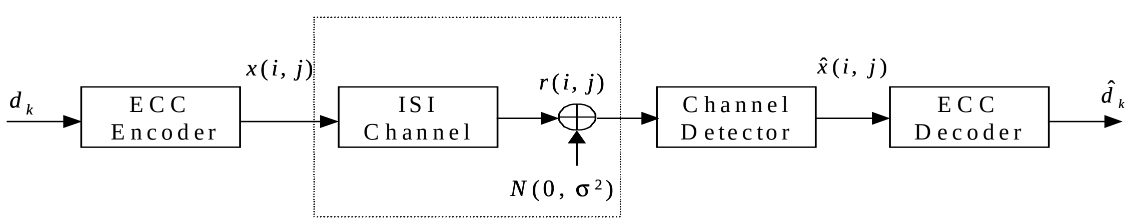

Figure 1. Discrete-time system.

For error correction we use LDPC coset codes [6] with the code graph drawn from the ensemble of graphs with degree polynomials λ(x) and ρ(x) [7]. Prior to transmission over the channel a guard band of all 1's is added around the codeword matrix. The reason for using the guard band is to isolate sectors in 2-D and also to provide trellis termination states for a convolutional code that may be used for error correction.

Soft Detection Algorithms for Uncoded 2-D ISI Channels

Recently, the message-passing algorithm has generated a lot of interest. A number of "very good" detection/decoding algorithms can be categorized into this algorithm, such as the well-known turbo decoding [8] algorithm. Although message-passing algorithms provide a systematic way to design soft detection algorithms for 2-D ISI channels, their application here is impaired due to the presence of loops in the graph topology as discussed in [9]. We empirically study the application of soft decoding algorithms for detection on 2-D ISI channels. The proposed algorithms provide approximate *a posteriori* probabilities (APP) of the input symbols. Some iterative equalization techniques are discussed in [10].

A. Decision Feedback MAP Algorithm (DFMAP)

This detection algorithm is named after the decision feedback Viterbi algorithm [2]. Here we generalized the BCJR [11] algorithm to a multi-in/single-out detector. For the considered channel response, during the process of the detection of row i, the channel input for column j is $[x_{i-1,j}, x_{i,j}]^T$ and the observed data is $r_{i,j}$. The trellis for this channel response is shown in Figure 2.

In Figure 2, the symbol φ signifies the fact that the trellis state does not depend on the input symbol of row i-1. The DFMAP applies the BCJR algorithm on this trellis with a modified transition probability γ

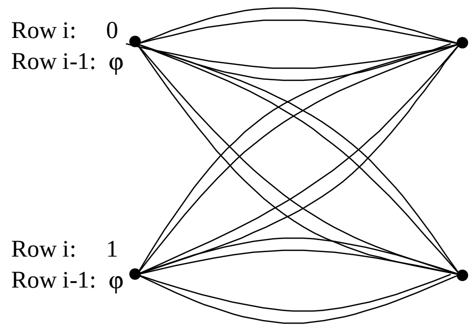

Figure 2. The states are independent of row i-1 input.

$$\begin{aligned}\gamma_{i,j}(s', s) &= P(\{r_{i,j} \cap s\} \mid s') \\ &= \sum_{all\text{ branches } B_i} P(\{r_{i,j} \cap s \cap B_i\} \mid s') \\ &= \sum_{all\text{ branches } B_i} P(r_{i,j} \mid \{s \cap B_i \cap s'\}) \cdot P(\{s \cap B_i\} \mid s')\end{aligned} \quad , \qquad (3)$$

where *s'* and *s* are the trellis states for column j-1 and j, respectively. The probability $P(\{s \cap B_i\} \mid s')$ is the *a priori* probability of the current encoder input bits. For the considered channel response

$$\begin{aligned}P(\{s \cap B_i\} \mid s') &= P(x_{i,j} \cap x_{i-1,j}) \\ &= P(x_{i,j}) \cdot P(x_{i-1,j})\end{aligned} \quad . \qquad (4)$$

The last equation follows from the assumption of independent channel input bits. In DFMAP, $P(x_{i-1,j})$ is the APP value determined from the detection of the previous row.

B. Minimum Mean Squared Error (MMSE) Detection

Before partial-response maximum likelihood (PRML) detection was used, equalization and then thresholding were the detection technology for magnetic recording channels. In [12], 2-D MMSE equalization was shown to be "very effective" for detection on some 2-D ISI channels. We use a 2-D MMSE equalizer for detection that is designed subject to the input power constraint and assuming the input to be Gaussian.

C. A Row-by-Row Detection Algorithm

In general, equalization causes noise correlation, which may degrade detector performance. The row-by-row detector is a combination of an equalizer and a MAP detector. In this paper a zero-forcing criterion was used for equalization that is used to reduce the ISI in the column dimension only. There are other possible choices for equalization, such as MMSE equalization. The remaining ISI is taken into consideration by the MAP detector. A 1-D MAP algorithm can be found in [5], [13], [14] and has been well studied.

D. A Column-and-Row Detection Algorithm

This algorithm is designed for the case when the channel response matrix is separable. It is very suitable for iterative decoding schemes. For a non separable channel response matrix, equalization might be considered to equalize the channel response to a nearby separable matrix. A separable channel response is analogous to the 2-D encoder for product codes [15] as shown in Figure 3. The column and row ISI, viewed as encoders, are not systematic, so decoding algorithms developed for product codes are not suitable.

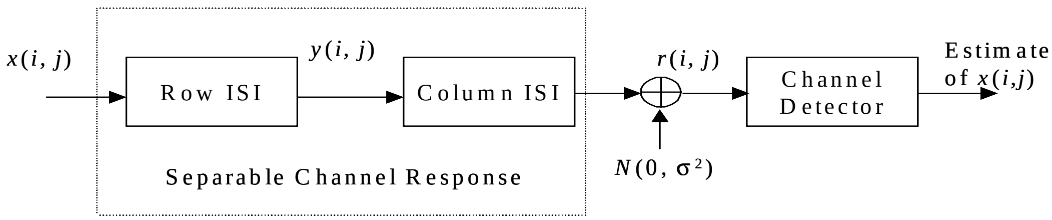

Figure 3. Illustration of the 2-D ISI process for a separable channel response

The BCJR algorithm for non systematic convolutional codes is used for column detection. During column detection, the approximate APP of each column of the matrix **Y,** $y_{i,j} \in \{-1.5, -0.5, 0.5, 1.5\}$, are calculated using the corresponding column of the observed data matrix **R.** The APP of the column detector are used as the transition probabilities for the branches of the row detector trellis. Then applying the MAP algorithm, approximate APP of the input data $x_{i,j}$ can be calculated. The performance of the four detection algorithms is shown in Figure 4.

Iterative Decoding for Coded 2-D ISI Channels

In the previous section, we considered the detection of uncoded 2-D channels. In this part, we compare the performances of four iterative decoding schemes for the considered channel. The ECC used was a rate ½ regular LDPC code.

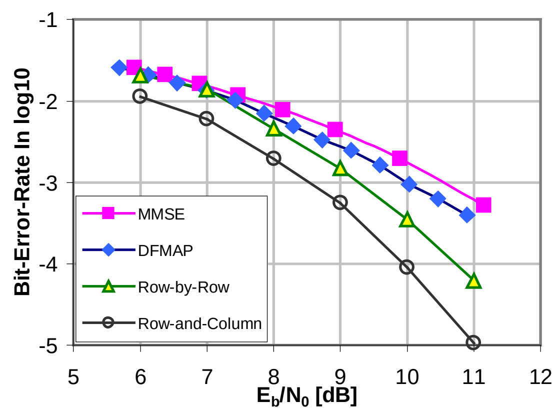

Figure 4 Performance comparison for detection algorithms.

A. Iterative Decoding with Wiener Filtering

The Wiener filter was used in conjunction with the LDPC code. Filtering was done iteratively if decoding failed after a single application of the filter with posteriors of the codeword bits being passed from the LDPC decoder to the Wiener filter.

B. A Full Message-Passing Algorithm

The second approach was to perform message passing on a three level graph of the observed data, the codeword bits and the check bits of the LDPC code. The upper two levels represent the LDPC code bipartite graph whereas the lower two levels represent the channel ISI graph. A section of the graph is shown in Fig. 5. Message passing was done on this 'full graph' using the schedule, $\mathbf{x} \to \mathbf{c} \to \mathbf{x} \to \mathbf{r} \to \mathbf{x}$, where **x** are the variable nodes, **c** the parity check nodes and **r** the data nodes. Initialization of the posteriors $P(x_{ij}|r)$ is done using the form of the ISI and assuming the code bits to be i.i.d. with x_{ij} having equal probability of being {+1,-1}. Message passing is then done between the variable nodes and check nodes for a fixed number of iterations and if decoding fails then message passing is done on the full graph, again for a fixed number of iterations.

The message passing for the variable node, check node graph is the same as described in [6] where for the variable to check messages we also have to consider the messages sent to the variable nodes from the data nodes.

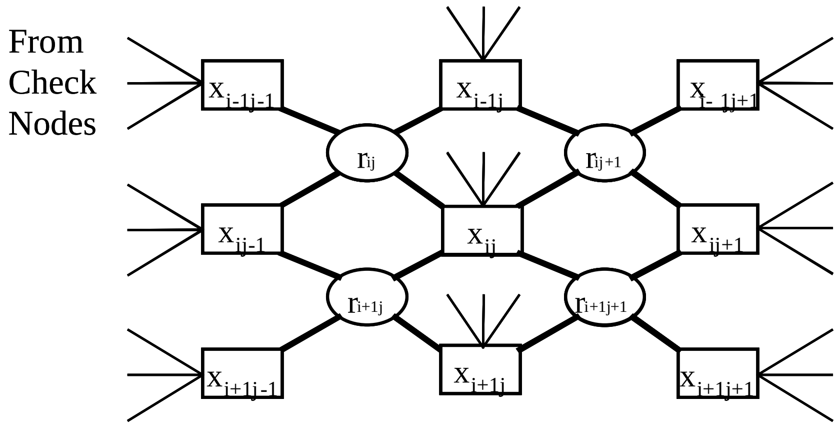

Figure 5. The full graph includes edges connecting variables nodes (x_{ij}) to the check nodes (not shown) and to the data nodes (r_{ij}). The 2-D ISI yields many length 4 cycles in this graph for eg. $r_{ij} \to x_{ij} \to r_{ij+1} \to x_{i-1j} \to r_{ij}$.

The observed data matrix has elements:

$$r_{ij} = x_{ij} + 0.5x_{ij-1} + 0.5x_{i-1j} + 0.25x_{i-1j-1} + w_{ij}\,. \qquad (5)$$

At each iteration the messages passed on the ISI graph for x={+1, -1} are:
1) Variable nodes to data nodes

$$v_{ij}(x) = \alpha_{ij}\, p_{ij}(x) \prod_{kl \in N(ij)} \mu_{kl}(x)\,, \qquad (6)$$

$p_{ij}(x)$ is the product of the messages arriving at the variable node from the check nodes it is connected to and α_{ij} is a normalizing constant to make $v_{ij}(x)$ a probability.
2) Data nodes to variable nodes

$$\mu_{ij}(x) = \sum_{x_{kl}:kl \in N(ij)} p(r_{ij} \mid x_{kl}, x_{ij} = x) \prod_{kl \in N(ij)} v_{kl}(x_{kl})\,. \qquad (7)$$

Using (5) the conditional probabilities above are Gaussian with mean $x_{ij} + 0.5x_{ij-1} + 0.5x_{i-1j} + 0.25x_{i-1j-1}$ and variance σ^2.
$N(ij)$ is the neighborhood of the (ij)th bit defined as all nodes connected to the (ij)th node except the node connected on the edge carrying the outgoing message. Thus we are using the well known 'turbo principle' [16]. Again, while considering the neighborhood of the variable nodes we need to take into account the messages passed by the check nodes also.

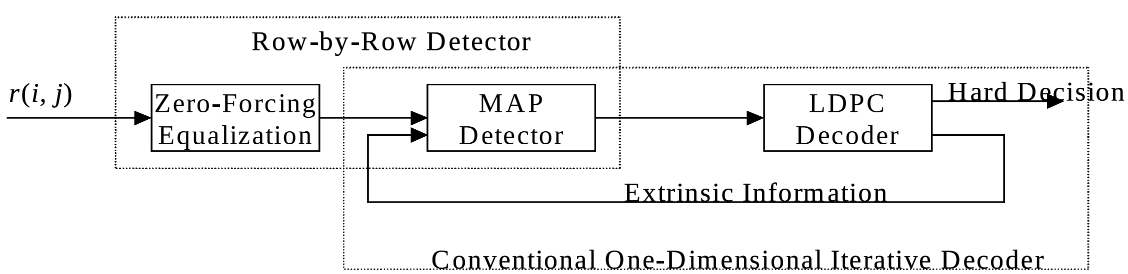

Figure 6 Iterative decoding using the row-by-row algorithm.

C. Using the Row-by-Row Detection Algorithm
In the third method, iteration is done between the row-by-row detection algorithm and the LDPC decoder. The equalizer of the row-by-row detection algorithm does not join the iteration process. Figure 6 shows a block diagram for this method. The equalizer may also join the iteration process.

D. Using the Column-and-Row Detection Algorithm

In this method iterations are performed between the column-and-row detection algorithm and the LDPC decoder. Since the number of input symbols to the column ISI increases with ISI length, computations performed by the column detector are much more than those of the row detector. Column detection is done once and then the row detector and LDPC decoder exchange extrinsic information. Therefore, after one column detection, we treat 2-D detection the same as 1-D ISI channel detection.

Results and Discussions

The results for the proposed iterative decoding schemes are shown in Figures 7 and 8. The performance of the full graph algorithm is degraded due to the presence of length four cycles in the channel ISI graph. As noted in the literature [9], the presence of many short cycles, as we have, deteriorates the performance of the message-passing algorithm. The ECC used a block length 10000, rate ½ regular (3,6) LDPC code.

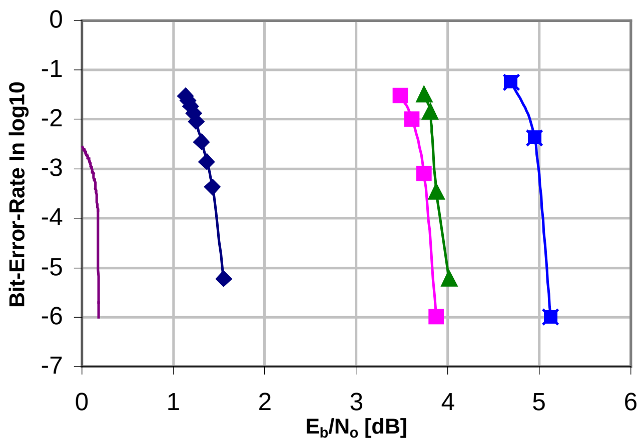

Figure 7. Performance curves, from left to right, capacity of an AWGN channel for rate=1/2, LDPC code on an ISI free channel, Wiener filter after 9 iterations, Wiener filter applied once on the data, full graph message passing with 9 iterations on the full graph.

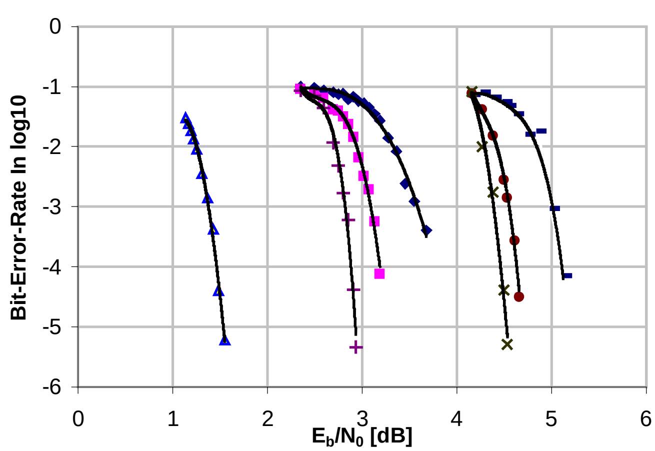

Figure 8. Performance curves, from left to right, LDPC code on an ISI-free AWGN channel, column-and-row detection with LDPC (iterations three, two, and one) and row-by-row detection with LDPC (iterations five, two, and one) on the 2-D ISI channel.

The performance of the iterative Wiener filter is encouraging. For the case when the channel point spread function is separable into 1-D ISI in each dimension the column-and-row detection algorithm combined with the LDPC code outperforms all other algorithms. There is a 1dB improvement in the performance using this algorithm over iterative Wiener filtering. We are also looking at other schedules for message passing on the full graph to improve the performance.

References

1. X. Chen, K. M. Chugg, and M. A. Neifeld, "Near-Optimal Parallel Distributed Data Detection for Page-Oriented Optical Memories," *IEEE Journal of Selected Topics in Quantum Electronics*," vol. 4, No. 5, pp. 866-879, Sept./Oct. 1998.
2. R. Krishnamoorthi, "Two-dimensional Viterbi like algorithms," M.S. thesis, University of Illinois at Urbana-Champaign, 1998.
3. William Weeks IV, "Full Surface Data Storage," Ph.D. thesis, University of Illinois at Urbana-Champaign, 2000.
4. H. Song, R. M. Todd, and J. R. Cruz, "Performance of low density parity check codes on magnetic recording channels," in *Proc. 2nd Int. Symp. Turbo Codes and Related Topics*, 2000, pp. 395-398.
5. W. Ryan, "Performance of high rate turbo codes on a PR4-equalized magnetic recording channel," in *Proc. IEEE Int. Conf. Commun.*, 1998, pp. 947-951.
6. A. Kavcic, X. Ma, and M. Mitzenmacher, "Binary Intersymbol Interference Channels: Gallager Codes, Density Evolution and Code Performance Bounds," submitted to *IEEE Trans. Inform. Theory*.
7. T. Richardson, A. Shokrollahi, and R. Urbanke, "The Capacity of Low-Density Parity Check Codes under Message-Passing Decoding," to appear in *IEEE Trans. Inform. Theory*.
8. C. Berrou, A. Glavieux, and P. Thitimajshima, "Near Shannon limit error correcting coding and decoding: Turbo-codes," in *Proc. IEEE Int. Conf. On Communications*, pp. 1064-1070, May 1993.
9. Frank R. Kschischang, Brendan J. Frey and Hans-Andrea Loeliger, "Factor graphs and the sum-product algorithm," *IEEE Trans. Inform. Theory*, vol. 47, No. 2, pp. 498-519, Feb. 2001.
10. M. Tuchler, R. Koetter, A. Singer, "Turbo Equalization: principles and new results," submitted to *IEEE Transactions on Communications,* Aug. 2000.
11. L.R. Bahl, J. Cocke, F. Jelinek, and J. Raviv, "Optimal decoding of linear codes for minimizing symbol error rate," *IEEE Trans. Inform. Theory*, vol. 20, pp. 284-287, Sept. 1974.
12. K. M. Chugg, X. Chen, and M. A. Neifeld, "Tow-dimensional equalization in coherent and incoherent page-oriented optical memory," *J. Opt. Soc. Am. A*, vol. 16, No. 3, pp. 549-562, Mar. 1999.
13. Y. Wu, and J. R. Cruz, "Performance of serially concatenated convolutional codes for magnetic recording," *IEEE Trans. Magn.*, vol. 37, pp. , Sept. 2000.
14. Y. Wu, and J. R. Cruz, "Noise predictive turbo systems," *IEEE Trans. Magn.*, vol. 37, pp. 742-747, Mar. 2001.
15. J. Hagenauer, E. Offer, and L. Papke, "Iterative Decoding of Binary Block and Convolutional Codes," *IEEE Trans. Inform. Theory*, vol. 42, No. 2, Mar. 1996.
16. J. Hagenauer, "The turbo principle: Tutorial introduction and state of the art," *International Symposium on Turbo Codes*, pp. 1-11, Sept. 1997.