Decoding for Magnetic Recording Media with Overlapping Tracks

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***Abstract*— Increasing recording track density by allowing overlap of adjacent tracks can lead to substantial increase in storage density for magnetic recording. However, track overlap may cause severe inter-track interference (ITI) and result in loss of performance. Sophisticated signal processing techniques must then be used to recover this loss. We study the use of joint equalization and decoding for magnetic recording with overlapping tracks. We present results for a scheme that uses minimum mean-squared-error (MMSE) equalization in conjunction with error correction coding using low-density parity-check (LDPC) codes. The recording process is simulated using a micromagnetic model for longitudinal magnetic recording. We use a three track system to study the track overlap. The outer two tracks are allowed to overlap on the middle track to simulate ITI. Bit error rate simulations show that the MMSE-LDPC decoding scheme incurs negligible loss when each of the outer tracks overlap 10% on the middle track. By varying the recording parameters, the trade-off between storage density and performance is also studied. We show that by a judicious choice of LDPC codes a recording with track overlap can have better performance than when there is no overlap. Hence, a higher storage density can be obtained without loss in performance.**

***Index Terms*— Multi-track recording, minimum mean-squared-error, low-density parity-check codes, inter-track interference, two-dimensional intersymbol interference.**

I. INTRODUCTION

THIS PAPER addresses the decoding of the magnetic recording channel subject to both intersymbol interference (ISI) and inter-track interference (ITI). Increasing track density, by squeezing the tracks closer or further, allowing overlap of adjacent tracks, leads to increased storage density. However, this increase comes at the expense of increased ITI, which leads to loss in performance, especially in perpendicular recording with side read. The use of sophisticated signal processing techniques then becomes essential to counteract this increase in ITI. The multihead recording system approach, championed by Barbosa [1], has proved successful in this respect. This idea uses an array of heads for simultaneously reading multiple tracks followed by joint detection across the tracks. Soljanin and Georghiades [2] used minimum distance analysis and showed the benefit of using multihead recording systems. Singla, *et al.,* proposed various joint equalization and decoding schemes for channels with two-dimensional ISI [3]-[5]. Two-dimensional modulation codes that impose constraints down the track as well as across tracks have also been proposed [6] for ITI channels. Roh, *et al.*, have proposed PRML decoding for multi-track recording using a single head [7].

In this paper the performance of using joint equalization and decoding for magnetic recording with overlapping tracks is studied. A micromagnetic model for longitudinal magnetic recording, developed by Porter [8], is used to simulate the recording process. It is assumed that writing and reading is done using a single head and that each track receives interference only from its two neighboring tracks. Thus, the system under consideration is a three track system. The channel model and the recording process are described in Section II. A scheme that performs two-dimensional minimum mean-squared-error (MMSE) equalization followed by error correction using low-density parity-check (LDPC) codes is used for decoding. A brief description of this scheme is given in Section III. Bit error rate simulations and noise tolerance thresholds are used to study the performance of the MMSE-LDPC decoding scheme. By simulating recording for different parameters, the trade-off between performance and storage density is also studied. These results and discussions are presented in Section IV. Section V concludes the paper.

II. CHANNEL MODEL

The channel is modeled as a discrete-time channel;

$$y(i,j) = \sum_{k_1=0}^{L_1} \sum_{k_2=0}^{L_2} h(k_1,k_2) x(i-k_1, j-k_2) + w(i,j); \quad (1)$$

where $y(i,j)$ are the readback data; $x(i,j)$ are the recorded data; and $w(i,j)$ are the noise samples. The recorded data are obtained by encoding the user data with a low-density parity-check (LDPC) code [11]. The user data and the recorded data are assumed to be binary. The recorded data symbols are in the alphabet $\{\pm 1\}$. The noise is assumed to be additive white Gaussian with zero mean and variance σ^2. The matrix $\mathbf{h} = \{h(k_1,k_2) : 0 \le k_1 \le L_1, 0 \le k_2 \le L_2\}$, referred to as the spreading function, represents the ISI and ITI. The ISI spreads over L_1 recorded bits whereas the ITI spreads across L_2 tracks.

The head sensitivity function determines the spreading function; we use the micromagnetic simulation to estimate it. To this end, a simulation of the recording process was run; a known data pattern was written on 3 neighboring tracks. By analyzing the resulting simulated readback signal from the middle track, the spreading function is estimated

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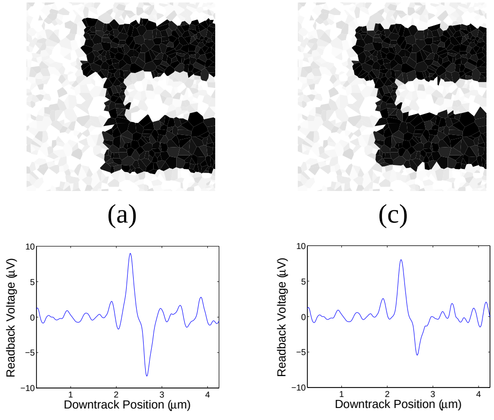
assuming that the readback process is linear. This estimated spreading function is then used in the MMSE-LDPC scheme to determine its performance. Let W denote the width of a track when there is no overlap. The read width is fixed to W. To simulate ITI, the outer tracks are allowed to overlap on the middle track. Fig. 1 shows the Voronoi pattern and the corresponding readback waveform for writing a simple data pattern across the 3 tracks. Figs. 1(a) & (b) show the recording for 10% track overlap and Figs. 1(c) & (d) for 20% overlap. A track overlap of 10% means that each of the outer tracks overlap 10% on the middle track. The parameter α denotes the width of the middle track that is not overwritten by the outer tracks, normalized by W. Thus α takes on the values 1, 0.8, 0.6 for no overlap, 10% overlap, and 20% overlap, respectively.



(a) (c)

(b) (d)

Fig. 1. Voronoi pattern and corresponding readback waveform for (a) & (b) recording with 10% overlap ($\alpha = 0.8$) and (c) & (d) recording with 20% overlap ($\alpha = 0.6$). The white and black regions in (a) & (c) correspond to the two directions of magnetization. The Voronoi regions show a $5\mu m \times 5\mu m$ area of the medium. The average grain area is approximately 0.01 μm^2.

III. MMSE-LDPC DECODING

Chugg, *et al.*, [9] showed the potential of MMSE equalization for two-dimensional ISI, arising during readback in page-oriented optical memories. They did not, however, employ any error correction coding. Singla, *et al.*, showed the benefit of using error correction coding with MMSE equalization for two-dimensional ISI channels [3].

The decoding scheme used here performs MMSE equalization followed by LDPC decoding. This equalizer was proposed by Tüchler, *et al.*, [10] for one-dimensional ISI channels and was extended by Singla and O'Sullivan [4] for use on two-dimensional ISI channels. The equalizer calculates the linear MMSE estimate of each recorded bit given an $N \times N$ block of the readback data. Typically, N^2 is much smaller than n, the block length of the LDPC code. The coefficients of the equalizer are calculated using the statistics of the readback data and the recorded data, and using the estimated spreading function. The output of the equalizer, a vector of recorded bit probabilities, is passed to the LDPC decoder. The decoder uses this information as *a priori* information and then performs sum-product message-passing [11] on the LDPC code bipartite graph. After a fixed number of iterations the decoder passes its *extrinsic* information to the equalizer. The equalizer uses this extrinsic information to recalculate its coefficients and the MMSE estimate, then passes its extrinsic information to the LDPC decoder. This process is continued until the decoder converges or a fixed number of iterations is exhausted. For each bit, the equalizer performs computations on the order of N^6. N is chosen so that the $N \times N$ block of the readback data, used for calculating the MMSE estimate of a recorded bit, has size of the order of the interference neighborhood size $L_1 \times L_2$. Thus the equalizer's per bit complexity is cubic in the size of the interference neighborhood. The complexity of the LDPC decoder is linear in the block length of the code. Hence, the overall complexity of the MMSE-LDPC decoder is $O(nN^6)$, where n is the block length of the code. The complexity of the equalizer can be reduced by using a recursive algorithm to update the coefficients.

IV. RESULTS AND DISCUSSION

Fig. 2 shows the simulation results for the MMSE-LDPC scheme. The LDPC code used is a block length 10000, regular (3,6) code [11] which is a rate $1/2$ code. The MMSE-LDPC decoder performs a maximum of 5 iterations and 20 iterations of LDPC decoding are performed after every equalization. The results are shown for two values of the minimum feature length (bit length) in the downtrack direction: 0.4 μm (solid curves) and 0.2 μm (dashed curves). Henceforth, the minimum feature length is denoted by l. The read width, W, is fixed to 1 μm, hence, the track width with overlap, as defined in Section II, is α μm. Three recording scenarios are considered for either case; no track overlap, 10% track overlap, and 20% track overlap giving rise to 0%, 11%, and 33% estimated ITI, respectively. The interference spreads across 4 bits in the downtrack direction. The simulated recording medium has high anisotropy energy and low exchange energy. The bit-aspect ratio is defined as the ratio of track width to bit length. The bit-aspect ratio is 2.5, 2, and 1.5 for the three recordings with l=0.4 μm and 5, 4, and 3 for l=0.2 μm. The signal-to-noise ratio (SNR) is defined as

$$SNR = 10 \cdot \log\left(\frac{E_b E_h}{2r\sigma^2}\right), \tag{2}$$

where E_b is the energy per bit at the user side (1 in this case), $E_h = \sum_{k_1=0}^{L_1} \sum_{k_2=0}^{L_2} h^2(k_1, k_2)$ is the energy in the spreading function estimated using the micromagnetic simulation, r is the rate of the LDPC code.

The results show that the MMSE-LDPC decoding scheme can tolerate a significant amount of ITI; for the recording with bit-aspect ratio of 1.5 a bit-error rate of 10^{-5} is obtained at an SNR of about 5.5 dB. Also, the loss in SNR is very small when 10% track overlap is allowed. The loss is about 0.5 dB for l=0.4 μm and less than 0.2 dB for l=0.2 μm. However, SNR loss of about 2.5 dB is incurred when track overlap is increased to 20%.

All other parameters remaining fixed, the storage density is directly proportional to the rate r of the LDPC code, and inversely proportional to l, and α. Table I compares for

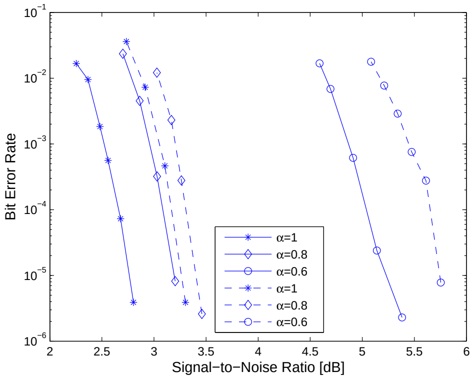


Fig. 2. Results for using the MMSE-LDPC decoding scheme for magnetic recording with overlapping tracks.

recordings with different l, r, and α, the SNR threshold for the MMSE-LDPC decoding scheme. The SNR threshold is defined as the minimum SNR required by the decoder to ensure that the bit error rate goes to zero as the LDPC code block length goes to infinity. This threshold can be computed using a density evolution algorithm [4]. The relative density refers to the density of a recording normalized to the recording with l=0.4 μm, r=0.5, and α=1 μm. This recording is henceforth referred to as the “base recording.”

TABLE I

STORAGE DENSITY AND PERFORMANCE COMPARISON

(l, α, r)	Relative Density	Threshold SNR [dB]	Relative SNR [dB]	SNR Loss [dB]
(0.4, 1.0, 0.50)	1.000	2.753	0.000	0.000
(0.4, 0.8, 0.50)	1.250	2.854	-0.101	-0.769
(0.4, 0.8, 0.40)	1.000	2.011	0.742	-0.769
(0.4, 0.6, 0.50)	1.667	3.613	-0.860	-1.320
(0.4, 0.6, 0.40)	1.333	2.755	-0.002	-1.320
(0.2, 1.0, 0.50)	2.000	2.726	0.027	-0.314
(0.2, 0.8, 0.50)	2.500	2.805	-0.052	-1.090
(0.2, 0.8, 0.40)	2.000	1.950	0.803	-1.090
(0.2, 0.8, 0.25)	1.250	1.344	1.409	-1.090
(0.2, 0.6, 0.50)	3.333	3.570	-0.835	-1.624
(0.2, 0.6, 0.40)	2.667	2.738	0.015	-1.624

The relative SNR for a recording is defined as the SNR threshold for the base recording minus the SNR threshold for that particular recording. Thus, a positive relative SNR implies that that recording has a lower (hence better) SNR threshold than the base recording. Fig. 2 shows that using a block length 10000 LDPC code the MMSE-LDPC decoding scheme incurs a loss of 0.2 dB when recording parameters are changed from (0.2, 1.0, 0.50) to (0.2, 0.8, 0.50) while Table I shows that asymptotically, as the block length goes to infinity, there is a loss of 0.05 dB going from the former recording to the latter. As track overlap increases the storage density also increases, thus one can use a lower rate LDPC code to obtain better performance whilst maintaining a higher storage density. This can be seen, for example, from the recording with parameters l=0.4 μm and α=0.8 for which using a rate 0.4 code gives a lower SNR threshold than the base recording and the storage density remains the same.

To judge whether track overlap is beneficial or not, the SNR loss due to the narrowing of the tracks has to be accounted for. Relative to the base recording, the SNR loss for a recording is defined as $10\log_{10}E_h/E_{h_b}$, where E_h and E_{h_b} are the energy in the estimated spreading function for that recording and the base recording, respectively. The SNR loss values are given in the last column of the table. As can be seen from the table, the loss in SNR loss outweighs the gain achieved by using lower rate LDPC codes, except for the recording with l=0.2 μm, $\alpha = 0.8$, and using a rate 0.25 LDPC code. For this case, the improvement in performance using a lower rate LDPC code recovers the loss due to the increased ITI, and more. Also the storage density in this case is higher than the base recording. Thus, the MMSE-LDPC decoding scheme allows for increased storage density without compromising performance.

V. CONCLUSION

Joint equalization and decoding for magnetic recording channels having both ISI and ITI was studied. A micromagnetic model for longitudinal magnetic recording was used to simulate the recording process. A three track system was considered and ITI was simulated by allowing the outer tracks to overlap on the middle track. A decoding scheme that uses minimum mean-squared-error equalization in conjunction with low-density parity-check decoding was used for this system. Simulations show that the decoding scheme can tolerate track overlap without significant loss in performance. By comparing recordings with different parameters it was shown that storage density can be improved by allowing track overlap without compromising on the performance when the MMSE-LDPC scheme is used for decoding. We are working to extend our results to using experimental signals read off of a magnetic hard disk.

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