Near field imaging with negative dielectric constant lenses

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A recent analysis by Pendry [J. B. Pendry, Phys. Rev. Lett. 85, 3966 (2000)] shows that a dielectric slab with a dielectric constant $\varepsilon = -1$ can produce a perfect image in the electrostatic limit ($c \to \infty $), independent of the permeability $\mu$, and therefore is a perfect lens. Here we include retardation effects and show how distance and dissipation make the lens no longer perfect. Nevertheless we conclude that very significant improvements over conventional near field imaging may be obtained in the microwave regime. © 2002 American Institute of Physics. [DOI: 10.1063/1.1471933]

There is a great deal of interest in near field imaging, and the recovery of subwavelength information. One means of achieving this goal is the realization of perfect lenses. Such objects would ideally reconstruct the image of a source perfectly. Because imaging in all frequency ranges is so common and so important in many technologically diverse areas, it would be enormously important to improve resolution in a simple and useful fashion at any frequency.

It has been shown by Veselago\textsuperscript{1} that a parallel-sided slab with dielectric constant $\varepsilon$ and permeability $\mu$ simultaneously equal to $-1$ acts a perfect lens, in the sense that if the thickness of the slab is $d$ and a source is located at $d_1$ from the slab, all the light transmitted through the slab would be focused on the other side of the slab, at distance $d-d_1$. It is because the refractive index $n$ of this material is $-1$, and therefore from Snell’s law, $\sin \theta_i = n \sin \theta_f$, each incoming light ray bends to the opposite side of the normal at each boundary ($\theta_i = - \theta_f$). Two such bends clearly refocus all the incoming light rays. The Maxwell’s equations inside the material are identical to that in vacuum, and it is easy to show that by matching at the boundaries to a field in a normal medium with $\varepsilon' = 1$, $\mu = 1$, only the advanced solution of Green’s functions should be used inside the material with $\varepsilon = -1$, $\mu = -1$. This means that the slab reverses the direction of time (Ref. 2) so that all decaying solutions increase exponentially inside the slab. Then they decay in the vacuum and refocus at a distance $d-d_1$.

Unfortunately, there is no known material with $\mu(\omega) = -1$ at optical frequencies, although Smith \textit{et al.} have made meta materials with $\mu = -1$ at rf frequencies.\textsuperscript{3} A negative dielectric constant, on the other hand, is much more common. Many materials, including simple metals, doped semiconductors, and gas plasmas, have a long wavelength dielectric constant which is accurately given by:

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i \gamma)}.$$  \hfill (1)

where $\omega_p$ is effective plasma frequency, and $\gamma$ represents the losses present. Typical values for $\omega_p$ are in the order of several eV, and $\gamma$ is small.\textsuperscript{4} It, therefore, from a practical point of view would be very interesting to know if materials with $\mu = 1$, $\varepsilon = -1$ with small losses are still like perfect lenses.

Recently Pendry showed that slabs made of such materials are perfect lenses if the electrostatic limit is taken, that is, retardation effects are neglected.\textsuperscript{5} In this letter, we will show explicitly how retardation effects make a perfect lens imperfect and our conclusion is that even though they are imperfect, lenses made from slabs of metallic plasma materials can be very important and practical imaging devices.

The general problem we wish to consider consists of solving Maxwell’s equation for a negative dielectric slab in the presence of an arbitrary spatial current distribution at frequency $\omega$. Assuming no variation in the $z$ direction, i.e., the direction parallel to the slab (Fig. 1), the general problem reduces to finding the response to a localized current source

$$j_x = p \delta(x) \delta(y), \quad j_y = j_z = 0.$$ \hfill (2)

This is a line dipole of strength $p$ (amp meter) per unit length (meter) along $z$. Each dipole is oscillating at the $x$ direction with frequency $\omega$. This special choice of the current density source makes the generated electromagnetic field purely $P$ wave, i.e., its polarization is in the $x$-$y$ plane, which is, as we shall see, important. Because the current source $j_x$ is singular, we choose to image the field at a distance $0.1d$ from the origin (see Fig. 1). In this case we put $j_x$ at a distance $d/2 + 0.1d$ from the slab of width $d$. This displaced source procedure gives us all the current physics and is absolutely unnecessary when a real extended source is present. Any realistic current source translational invariant in the $z$ direction can be expressed as superposition of Eq. (2). This "two dimensional" (2D) geometry is similar to the one chosen by Pendry.\textsuperscript{5} He showed that the focal plane of such a lens was at $d/2$ on the other side of the slab.

Without the slab, the vector potential for the sinusoidal current source (Eq. 2) in vacuum is (in Lorentz gauge)

$$A(r, \omega) = \int d\mathbf{r'} G(r - r', \omega) \frac{1}{c} j(r', \omega).$$ \hfill (3)

The retarded Green’s function

$$G(r, \omega) = 4 \pi \int \frac{d^3 k}{(2\pi)^3} \frac{e^{i k \cdot r}}{k^2 - (\omega/c + i 0^+)^2},$$ \hfill (4)

where the $i 0^+$ ensures the waves propagate outward from the source. For our $j(r') = p \delta(x') \delta(y') \delta [r' = (x', y', 0)]$ pointing in the $x$ direction, only $A_x$ is nonzero and after doing the integral over $k_y$, $A_x$ is
The line dipole current source is placed at \( x = y = 0 \) and extended along the \( z \) axis, with the polarization oscillating in the \( x \) direction. The field on the plane at a distance \( d/2 \) right of the slab, the image plane, is scanned and compared to the field on the plane at a distance \( d/2 \) left of the slab, the source plane. The line dipole current source is placed a little bit away from the source plane \((0.1d)\) to avoid the divergence of the field in the source plane.

\[
A_z(r, \omega) = \frac{4\pi}{c} \int_{-\infty}^{\infty} \frac{dk_y}{2\pi} e^{ik_yz} e^{i(k_x x + k_y y)} \frac{1}{2k_y^2 - (\omega/c)^2 + c^2},
\]

(5)

where

\[
k_y^2 = \frac{\sqrt{(\omega/c)^2 - k_x^2}}{i}, \quad \text{if} \quad k_x < \omega/c;
\]

\[
k_y^2 = \frac{\sqrt{k_x^2 - (\omega/c)^2}}{i}, \quad \text{if} \quad k_x > \omega/c.
\]

(6)

That is, \( A_z \) is simply a sum of plane waves.

The fields then are found from Maxwell’s equations: \( \mathbf{B} = \nabla \times \mathbf{A} \) and \( \nabla \times \mathbf{B} = i\omega \mathbf{E} \). If \( k_x \gg \omega/c \), then \( k^2_y \) is imaginary and this plane wave component exponentially decays in the \( y \) direction. It is just the loss of these Fourier components which limits the resolution of conventional lenses.

The physics for slab imaging can clearly be described by a set of complex polarization transmission coefficients for these \( k_x \gg \omega/c \) plane waves. When one of these plane waves passes through the slab, it would pick up a “phase change” and “amplitude change.” The transmission coefficients, furthermore, depend on the polarization of the incident plane waves. When the incident waves are \( P \) polarized (electric field parallel to the plane of incidence), the projection of the electric field has a component perpendicular to the surface of the slab and excitation of longitudinal surface plasmons is possible. For the so-called \( S \) waves (electric field perpendicular to the plane of incidence) hence parallel to the surface of the slab, they can only excite transverse waves. In the slab, it is clear from the work presented in Ref. 5 that it is the longitudinal surface plasmons which help to correct the phase change, i.e., amplify the amplitude, giving an effect similar to the advanced Green’s function for the \( \mu = -1, \epsilon = -1 \) material. Since only \( P \) waves effectively couple to the surface plasmons, we expect that only those \( P \)-polarized source fields could possibly be perfectly reconstructed.
If the slab were to be close to “perfect lens,” the amplitude gain (and phase change) of the waves acquired from passing through the slab, \( T_p \), \( T_s \) (\( p, s \) denotes the polarization of the wave), should compensate the amplitude decay (and phase change) from traveling in vacuum. For evanescent wave, the amplitude decay in vacuum for the geometry in Fig. 1 is
\[
e^{-\sqrt{k^2 - k_0^2}d/2} \times e^{-\sqrt{k^2 - k_0^2}d/2} = e^{-\sqrt{k^2 - k_0^2}d} = \phi(d).
\]
(7)
For propagating waves, the phase change \( \phi(d) \) is \( e^{i\sqrt{k^2 - k_0^2}d} \). Here \( k_x \) is the wave vector parallel to the slab, and \( k_0 \) is the wave vector in the vacuum. Therefore, if both \( T_p \phi, T_s \phi \) are 1, for a wide range \( \Delta k_x \) of \( k_x \), then the slab will act like a perfect lens, with resolution roughly given by \( (\Delta k_x)^{-1} \).

The plane wave transmission coefficients are given in Ref. 5. Everything is a function of the 2D dimensionless quantities of \( \alpha = k_x d \) and \( u = k_x d \). Neglecting the retardation effect is the same as taking \( \alpha = 0 \). In this case, \( T_p \phi = 1, \ T_s \phi = e^{-2\pi u}, \) for all \( u \). This shows why the slab is a perfect lens when the electrostatic limit is taken, since electrostatics has the nature of \( P \)-polarized fields.

Plots of \( T_p \phi, T_s \phi \), for real \( \epsilon = -1 \), as well as the simulation results of the reconstructed image, are shown in Figs. 2 and 3 for several different values of \( \alpha \). We see that for all values of \( \alpha, T_p \phi \) dies off exponentially, while \( T_s \phi \) has a number of interesting features. For \( \alpha = 0.05 \), which corresponds to a “near field” regime \( (\alpha \ll 126) \), \( T_p \phi \) is practically flat and equal to 1 within a finite window in \( u \), which extends from 0 to 10. For \( u \) beyond the window \( T_p \phi \) exponentially drops off to 0. In addition there are extremely high jumps at both ends of the window. The jumps occur at the zeros of the denominator of \( T_p \). They relate to the strong coupling of the incident polarized electromagnetic wave with a mode of the slab. Since their position depends on \( \alpha \), we might call them slab polaritons. When \( \alpha \) is larger than some critical value close to 1, there is no longer any zero in the denominator of \( T_p \) and the nature of \( T_p \) is completely changed (see Fig. 2).

The finite window where \( T_p = 1 \) means the slab acts like a \( P \)-polarized low-pass filter which effectively amplifies the evanescent modes only for those \( k_x d \) within this window. For \( k_0 d = 0.05 \), the relative resolution (relative to a conventional lens) of the \( \epsilon = -1 \) slab is roughly \( R_p = (k_x)_{\max} d/(2k_0 d) = 10/0.1 = 100 \). It greatly surpasses conventional lenses. Near field imaging, i.e., no slab or lens present, would give a \( R_p \approx 10 \), since \( (k_x)_{\max} d \approx 1 \), so that in this geometry we are almost a factor of 10 better than a conventional near field imaging. From Fig. 3, we see the intensity and contrast of the reconstructed image is somewhat reduced and the image has a wiggly pattern associated with it. The wiggles in the image are due to the sharp cutoff which is present in the finite transmission window. As expected, the near field image (no slab) compared to the image reconstructed by the slab, is much flatter and weaker even at such a short distance, due to the exponential decay of evanescent wave.

As \( \alpha \) increases, the flat window becomes narrower and \( T_p \phi \) becomes larger than 1, i.e., it gradually loses its flatness. \( T_p \phi \) is always larger than 1, an overshoot effect, because the amplitude is amplified in the slab \( (\epsilon = -1) \) at a rate \( \sqrt{k_x^2 - k_0^2} \), per unit length, which is larger than the rate that a perfect lens requires, i.e., \( \sqrt{k_x^2 - k_0^2} \), the decay rate in vacuum. The value of \( \alpha \) corresponding to the numerical results presented in Ref. 5 is 0.7. From Figs. 2 and 3, we readily see that such a lens is even in the absence of losses not better than a conventional lens.

Realistic materials always have losses, which are not extremely small, although \( \gamma/\omega \approx 10 \) for Ag and \( \gamma/\omega \approx 0.01 \) for gas plasmas in the few GHz range are not unrealistic numbers. When losses are present, \( T_p \phi \) becomes complex. If the losses are small, the real part of \( T_p \phi \) still has the feature of being flat and close to 1 within a finite but narrower and smoother window. In order to investigate the effects of losses when losses are present, we numerically constructed an image of our line dipole with two values of \( \alpha = 0.05, 0.25 \) and \( \gamma/\omega = 0.005 \), i.e., \( \epsilon = -1 + 0.01i \). The images are shown in Fig. 3 as the green curves. For \( \alpha = 0.05 \) the small damping has a strikingly large effect. It weakens the image and broadens it by at least a factor of 4. For \( \alpha = 0.25 \) the effect is significant but less dramatic. The reason for the sensitivity to loss is obvious as one realizes that \( T_p \), the transmission coefficient for moderately large \( k_x d \) is
\[
T_p \approx \frac{e^{-k_x d}}{i \text{Im}\epsilon/2 + e^{-2k_x d}}.
\]
(8)
In order that \( T_p \approx e^{-k_x d}, \) i.e., \( T_p \phi \approx 1, \) \( \text{Im}\epsilon \ll e^{-2k_x d}, \) which can be a very small number for the higher Fourier components.

The more interesting case of a point dipole pointing in an arbitrary direction, can also be analyzed in exactly the same manner.\(^6\) Clearly there would be a decrease in intensity due to the missing \( S \) components but, more importantly, the elimination of the evanescent \( S \)-wave component from the source field could further blur the reconstructed image, because for arbitrary direction of the radiating dipole the amount of \( S \) wave depends in a complicated way on the projection of the field of each plane wave component. As an example, Fig. 4 shows images of a point dipole pointing in different directions, for \( \alpha = 0.25, \) \( \epsilon = -1 \). In Fig. 4(a), the point dipole is perpendicular to the slab in the \( y \) direction, and the component plane waves are pure \( P \) waves. In Fig. 4(b), the point dipole is in the \( z \) direction, and the component plane waves are mixtures of \( P \) and \( S \) waves. The differences in resolution and intensity are significant. The pure \( P \) wave case is better.

We have shown the dielectric slab with \( \epsilon = -1, \) \( \text{Im}\epsilon \ll 1, \mu = 1 \) can achieve excellent imaging under suitable conditions. The resolution attained by this mechanism far exceeds that of the traditional lenses and even near field imaging. Plasma slabs may be a very practical realization in the microwave regime.

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\(^1\) V. G. Veselago, Sov. Phys. Usp. 10, 509 (1968).
\(^6\) I. T. Shen and P. M. Platzman (unpublished).