Photon antibunching and bunching in a ring-resonator waveguide quantum electrodynamics system

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We numerically investigate the photonic state generation and its nonclassical correlations in a ring-resonator waveguide quantum electrodynamics system. Specifically, we discuss photon antibunching and bunching in various scenarios, including the imperfect resonator with backscattering and dissipations. Our numerical results indicate that an imperfect ring resonator with backscattering can enhance the quality of antibunching. In addition, we also identify the quantum photonic field phenomenon in the photon scattering dynamics and the shoulder effect in the second-order correlation function.

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First observed in the resonance fluorescence of sodium atoms [1], photon antibunching manifests a genuine quantum correlation of sub-Poissonian photon statistics. Photon antibunching is a key requirement in many applications for quantum information science. In recent years, much attention has been devoted to generations of quantum antibunching in solid-state platforms. For instance, molecules [2] and quantum dots [3–7] have been demonstrated to generate photon antibunching. Recent developments based on real space dynamics have been developed to investigate cQED systems [14–16]. At present, although the system of a ring resonator (whispering-gallery-mode resonator) has been experimentally realized [17,18], the detailed numerical investigation has yet to be carried out. In this Letter, we provide a detailed numerical investigation for this case.

The ring-resonator waveguide quantum electrodynamics (QED) system studied in this Letter is shown schematically in Fig. 1. Specifically, it consists of a single-mode waveguide, a two-level atom of transition frequency $\omega_a$, and a ring resonator. For brevity, we shall call it a waveguide-atom-ring circuit (WARC). An ideal ring resonator supports two degenerate counter-propagating modes, clockwise mode $a$ and counter-clockwise mode $b$, both of eigenfrequency $\omega_c$. Examples include a micro-disk [19], a micro-sphere [20], or a micro-toroid [21]. The ring-waveguide coupling strength is $\Gamma$, and the ring-atom coupling strength is $g$. Manufacturing imperfections of various natures exist. These imperfections include minor contaminations, structural defects [22], and surface roughness [23], all of which contribute to the backscattering. Backscattering in the ring resonator interconverts mode $a$ and $b$, which is characterized by the intermode coupling strength $b$ [24]. In addition, intrinsic dissipation rates, which take into account the leakage of photons from the waveguided modes, are denoted by $\gamma_a$ for the atom and $\gamma_c$ for the ring resonator. The WARC Hamiltonian is given by

$$\frac{\mathcal{H}}{\hbar} = \int_{-\infty}^{\infty} dx \left( -i \hbar \frac{\partial}{\partial x} \right) \left[ -i \hbar \frac{\partial}{\partial x} \right] c_a(x) + \int_{-\infty}^{\infty} dx \left( i \hbar \frac{\partial}{\partial x} \right) c_b(x) + \int_{-\infty}^{\infty} dx V(x) [c_a(x)a + a^\dagger c_b(x) + c_b(x)b + b^\dagger c_a(x)] + \omega_a a^\dagger a + \omega_c b^\dagger b + \omega_a a^\dagger b^\dagger + g(\sigma_a a + \sigma_c b^\dagger)$$

Fig. 1. Schematic of the ring-resonator waveguide QED system. A ring resonator of eigenfrequency $\omega_c$ is side-coupled to a waveguide. An atom of transition frequency $\omega_a$ is coupled to the ring resonator. The ring-waveguide and ring-atom coupling strength are given by $\Gamma$ and $g$, respectively. The ring resonator supports counter-clockwise mode $a$ and clockwise mode $b$. The on-resonance case ($\omega_a = \omega_c$) is considered throughout. (The off-resonance case performs inferiorly compared to the on-resonance case regarding the quality of photonic correlations.)
where $\Gamma = V^2/2\nu_g$, and the ground state of the atom has been chosen as the energy reference point. $e^i(x)$ and $e^s(x)$ denote the creation and annihilation operators, which create a photon at spatial point $x$, with the subscripts $R$ and $L$ denoting the right- and left-moving photonic branches, respectively. $a^d$ and $b^d$ denote the creation operator in the ring resonator for mode $a$ and mode $b$, respectively. $\sigma_+$ and $\sigma_-$ denote the raising and lowering operator of the atomic transition, respectively. $\nu_g$ denotes the group velocity of the wave packet. In addition, the general two-photon state is given by

$$|\Phi(t)\rangle = \left( \int_{-\infty}^{\infty} dx_1 dx_2 \left[ \phi_{RR}(x_1, x_2, t) \frac{1}{\sqrt{2}} e^{i\omega t} a^d(x_1) e^s_R(x_2) \\
+ \phi_{RL}(x_1, x_2, t) \frac{1}{\sqrt{2}} e^{i\omega t} a^d(x_1) e^s_L(x_2) \\
+ \phi_{LR}(x_1, x_2, t) \frac{1}{\sqrt{2}} e^{i\omega t} b^d(x_1) e^s_R(x_2) \\
+ \phi_{LL}(x_1, x_2, t) \frac{1}{\sqrt{2}} e^{i\omega t} b^d(x_1) e^s_L(x_2) \right] \\
+ \int_{-\infty}^{\infty} dx \left[ \epsilon_{RR}(x, t) e^{-i\omega t} a^d(x) a^t + \epsilon_{RL}(x, t) e^{-i\omega t} a^d(x) b^t \right] \\
+ \int_{-\infty}^{\infty} dx \left[ \epsilon_{LR}(x, t) e^{-i\omega t} b^d(x) a^t + \epsilon_{LL}(x, t) e^{-i\omega t} b^d(x) b^t \right] \\
+ \int_{-\infty}^{\infty} dx \left[ \epsilon_{RA}(x, t) e^{-i\omega t} c^d_R(x) \sigma_+ \\
+ \epsilon_{LA}(x, t) e^{-i\omega t} c^d_L(x) \sigma_+ \right] \\
+ \epsilon_{aA}(t) e^{-i\omega_a t} a^d t a^t + \epsilon_{aA}(t) e^{-i\omega_a t} a^d b^t \\
+ \epsilon_{bA}(t) e^{-i\omega_b t} b^d t b^t \\
+ \epsilon_{aA}(t) e^{-i(\omega_a + \omega_b) t} a^d \sigma_+ + \epsilon_{bA}(t) e^{-i(\omega_a + \omega_b) t} b^d \sigma_+ \right) |0, -\rangle,$$

(2)

where $|0, -\rangle$ denotes the vacuum state. $\phi(x_1, x_2, t)$ denotes the probability amplitude corresponding to the case where two photons are in the waveguide. The subscript specifies the moving branch of each photon. $e(x, t)$ denotes the probability amplitude where one photon is in the waveguide branch (specified by the subscript $R$ or $L$, respectively), and the other is in the excited atom, the ring mode $a$, or mode $b$ (specified by the subscript $A$, $a$, or $b$, respectively). $\epsilon(t)$ denotes the probability amplitude corresponding to the case where both photons are not in the waveguide.

The information about two-photon correlations is encoded in the second-order correlation function: $g^{(2)}(\tau)$. It has been shown in Ref. [14] that a weak coherent state can yield the same second-order correlation function $g^{(2)}(\tau)$ as a two-photon Fock state does. Here, we adopt the same approach by using a two-photon Fock state input from the left (i.e., initially $\phi_{RR} = \phi_{in}$) and evolve the system in time numerically. The scattered wave function is then recorded to compute numerically $g^{(2)}(\tau)$ to demonstrate photon antibunching and bunching [14].

The photon antibunching and bunching are the phenomena of photon-photon correlation that go beyond the single-photon picture. Such an inter-photon correlation indicates a quantum nonlinearity at the level of individual photons [25]. Nonetheless, an understanding of the single-photon dynamics of the system could help to unveil the frequency range wherein the two-photon correlation is significant. Specifically, each photon in the two-photon wave packet operates at the frequency identified from the single-photon dynamics.

Single-photon spectra can be straightforwardly obtained by analyzing single-photon equations of motion [24,26]. We plot single-photon reflection spectra for both the ideal case [Fig. 2(a), where $\Gamma/g = 0.6$, $b = 0$, $\gamma_a = \gamma_r = \gamma = 0$] and the backscattering case [Fig. 2(b), where $\Gamma/g = 0.6$, $h/g = 0.4$, $\gamma_a = \gamma_r = \gamma = 0$]. In the ideal case, the reflection spectrum is symmetric with respect to $\omega_\gamma$, and three resonance peaks attain unity. In contrast, in the backscattering case, the spectrum is asymmetric. Such an asymmetry can be understood as follows. The intermode coupling, $h$, further dresses up the dressed modes $a$ and $b$, thereby altering the eigenfrequencies to $\omega_a - b$ and $\omega_a + 1/2(b \pm \sqrt{8g^2 + h^2})$, respectively.

To demonstrate, numerically, the unitary reflection at all reflection peaks, we find that the reflection is larger than 99.9% for a resonant single-photon Gaussian wave packet with a bandwidth $g/40$ and a center frequency $\omega_k$ tuned at each reflection peak frequency.

To investigate the two-photon correlation, we send in a two-photon Gaussian wave packet. The two-photon Gaussian state is a product state (i.e., uncorrelated) of two identical single-photon Gaussian wave packets, each with the same spatial width and the same center frequency as those in the preceding single-photon case throughout this Letter.

In the ideal case, we numerically investigate the two-photon dynamics at the following two frequencies: $\omega_{k_1} = \omega_a$ and $\omega_{k_2} \approx \omega_a - 1.281g$, which correspond to the single-photon reflection center peak and the left peak, respectively, in Fig. 2(a). At $\omega_{k_1}$, we find that two photons are essentially reflected [Fig. 3(a)], with a two-photon reflection larger than 99.65%, which is roughly equal to the joint probability of two photons being reflected independently. There is no two-photon correlation occurring. Here, the two-photon reflection is defined as the probability of two photons both being reflected. After scattering, in the graphic representation [Figs. 3(a) and 3(b)], the two-photon wave function in the LL branch (quadrant III) indicates two reflected photons; the wave function in the RR branch (quadrant I) indicates two transmitted photons; and the wave function in the RL (quadrant II) and LR (quadrant IV) branches indicates one transmitted and one reflected photon.

At $\omega_{k_2}$, the reflection is, however, only 90.1%. A significant portion (≈2.53%) of the two-photon wave function is now transmitted, indicating a two-photon correlation. After scattering, the reflected two-photon wave function is depleted along the diagonal ($x_1 = x_2$). The depletion indicates a low

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**Fig. 2.** Single-photon spectra. The black dashed lines refer to the reflection spectra (left axis), with arrows indicating the left resonance peak. The colored solid lines refer to normalized atomic and ring-mode excitations (right axis). (a) Ideal case: $\Gamma/g = 0.6$, $b = 0$, $\gamma_a = \gamma_r = \gamma = 0$. (b) Backscattering case: $\Gamma/g = 0.6$, $h/g = 0.4$, $\gamma_a = \gamma_r = \gamma = 0$. 

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probability of detecting two reflected photons at the same spatial point, indicating photon antibunching. In contrast, the two transmitted photons in the RR branch emerging along the diagonal \(x_1 = x_2\), indicate photon bunching. Numerically, we also find roughly 3.69\% of the wave function emerges in the RL and LR branches.

We plot \(g^{(2)}(\tau)\) in Figs. 3(c)–3(e), \(g_{\text{RR}}^{(2)}(\tau)\) exhibits a sharp peak at \(\tau = 0\) \(g_{\text{RR}}^{(2)}(0) \approx 4.5\) [Fig. 3(c)], and \(g_{\text{LL}}^{(2)}(\tau)\) has a dip at \(\tau = 0\) \(g_{\text{LL}}^{(2)}(0) \approx 9.76 \times 10^{-2}\) [Fig. 3(e)]. Numerically, we also find the same \(g^{(2)}(\tau)\) for the case when the two-photon wave packet operates at the right reflection peak in Fig. 2(a). \(g_{\text{RL}}^{(2)}(\tau)\) is plotted in Fig. 3(d) (red line). We notice that \(g_{\text{RL}}^{(2)}(\tau)\) is asymmetric and peaks at some nonzero value, \(g \tau \approx 0.4\), indicating a time lag introduced by the WARC between the two counter-propagating photons. Furthermore, the wave function in the RL branch is above the diagonal \(x_1 = -x_2\). Numerically, we find that the transmitted photon propagates 0.4/g ahead of the reflected photon. This time lag is attributed to the fact that the reflected photon undergoes an atomic absorption and spontaneous decay whereas the transmitted photon does not.

We now numerically investigate the effects of the backscattering \(b\). Specifically, we investigate the case of \(b = 0.4g\). When the two-photon wave packet operates at the leftmost reflection peak \(\omega_k \approx \omega_L - g\), Fig. 2(b)], we find numerically that \(g_{\text{RR}}^{(2)}(\tau)\) and \(g_{\text{LL}}^{(2)}(\tau)\) are qualitatively of the same shape as those in the ideal case when \(\tau \neq 0\). In the vicinity of \(\tau = 0\), the \(g_{\text{LL}}^{(2)}(\tau)\) flattens out [green line in Fig. 3(e) inset] and \(g_{\text{LL}}^{(2)}(0)\) is roughly 46\% smaller, which indicates a higher quality of antibunching. The backscattering-assisted mechanism provides new possibilities to engineer quantum correlations.

To illustrate the physics, hereafter, we have assumed the non-dissipative case. We now investigate the effects of intrinsic dissipations \(\Gamma/g = 0.6\), \(\hbar = 0\). Specifically, we choose \(\gamma_a = \gamma_r = 0.1g\), which are typical values in current experiments. The single-photon spectrum can be straightforwardly obtained as in the ideal case (not plotted here). Numerically, we find there are only two reflection peaks. When the two-photon wave packet operates at the left reflection peak, \(\omega_k \approx \omega_L - 1.179g\), antibunching and bunching still emerge after scattering. Nonetheless, the quality degrades, and \(g_{\text{LL}}^{(2)}(0)\) is twice as large as that in the ideal case [blue line in Fig. 3(d) inset].

In addition, a new optical phenomenon emerges in the dissipative case. Compared to the ideal case [Fig. 3(b)], the wave functions in the RL and LR branches spread out in the off-diagonal regions (Fig. 4). The spread indicates that dissipations lead to a probability leakage in the RL and LR branches, which gives rise to the possibility of observing two photons propagating counter-directionally with an even wider range of time lag. We call this optical phenomenon of probability spreading a “quantum photonic halo” phenomenon (similar to [27]). As can be seen from Fig. 3(d) (blue line), the peak value of \(g_{\text{RL}}^{(2)}(\tau)\) drops significantly to only 1/4 of the ideal case.

The correlation function \(g^{(2)}(\tau)\) is an important measure for photon correlations. Many discussions center on its local behavior around \(\tau = 0\). Here, we discuss its global quantitative behavior. In our numerical investigations, we notice that a visible overshoot sometimes appears on both sides of the center dip of \(g_{\text{LL}}^{(2)}(\tau)\) [Fig. 3(e)]. We call such an overshooting a “shoulder effect,” which has been observed in some cQED experiments [28]. Here, we provide a quantitative explanation of the shoulder effect. The numerator of \(g_{\text{LL}}^{(2)}(\tau)\) is the projection of \(\phi_L\) onto \(x_1 = -x_2\) (Ref. [14]). Consequently, the width of \(g_{\text{LL}}^{(2)}(\tau)\) is characterized by the spatial width of the wave packet, \(\sigma\). On the other hand, the photon-atom interaction gives rise to a dip at the center of \(g_{\text{LL}}^{(2)}(\tau)\), of which the width is determined by the interaction strength, \(g\). In the following, we investigate the dependence of the shoulder effect on \(g\) and \(\sigma\).

Fig. 3. (a) and (b) plot probability density \(|\phi(x_1, x_2)|^2\) for the outgoing state in the ideal case. The two-photon wave packet operates at \(\omega_k\) in (a) and \(\omega_L\) in (b). (c)–(e) plot the second-order correlation functions \(g_{\text{RR}}^{(2)}(\tau)\), \(g_{\text{RL}}^{(2)}(\tau)\), and \(g_{\text{LL}}^{(2)}(\tau)\), respectively. The red, green, and blue lines refer to the ideal, backscattering, and dissipative cases, respectively.

Fig. 4. Probability density plot for the dissipative case, illustrating the quantum halo phenomenon. The contrast of the figure is adjusted to aid the visualization.
Fig. 5. (a) and (b) illustrate the interplay between the width of the wave packet [in (a)] and the width of the dip [in (b)] in the shoulder effect. (c)–(f) are numerical conformations of the shoulder effect. (c) shows $g = g_0$ for varying $\sigma$. (d) shows $g = 1.5g_0$ for varying $\sigma$. (e) shows $g = 2g_0$ for varying $\sigma$. (f) shows $\sigma = \sigma_0$ for varying $g$. $g_0$ and $\sigma_0$ are the same as the parameters used in Fig. 3(b).

As illustrated in Figs. 5(a) and 5(b), the shoulder effect corresponds to the interplay between the width of the wave packet (characterized by $\sigma$) and the width of the dip (characterized by $v_\gamma/g$). Apparently, a larger value for either $\sigma$ or $g$ yields a weaker shoulder effect. In Figs. 5(c)–5(e), we plot $g^{(2)}(\tau)$ for progressively increasing $g$ with varying $\sigma$. These results demonstrate that the largest $\sigma$ gives the weakest shoulder effect for fixed $g$. In Fig. 5(f), we plot $g^{(2)}(\tau)$ for fixed $\sigma$ with varying $g$. Again, the numerical results confirm that the largest $g$ gives the weakest shoulder effect. In typical experiments, the atom-photon interaction is difficult to alter. The shoulder effect can be probed by alternatively using wider wave packets.

In conclusion, we have presented numerical investigations of the two-photon dynamics in a ring-resonator waveguide QED system. Our numerical results indicate that the WARC is versatile in manipulating the two-photon dynamics. Furthermore, we discuss a new mechanism to enhance the correlation quality by engineering the intermode-coupling-induced backscattering. Finally, we also identify the quantum photonic halo phenomenon and the shoulder effect. Our numerical framework can be straightforwardly extended to investigate more complicated cQED systems.

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