

\( P(X < 2) = \frac{e^{-2} \cdot 2^0}{0!} \)

Normal Approximation
\[ P(X < 2) = P(Z < \frac{2 - 2}{\sqrt{2}}) \]

Test I
- \( P(Z < 0.63) \)
- \( P(Z < 0.63) = 0.7357 \)
- \( P(Z < 0.63) = 0.7357 \)
- Binomial from Table I
\[ P(X = 3) = 0.2870 \]

2) \( P(X = 1) \)

Normal Approximation
\[ P(0.5 < X < 1.5) = P\left( \frac{0.5 - 2}{\sqrt{2}} < Z < \frac{1.5 - 2}{\sqrt{2}} \right) \]

Test I
- \( P(Z < -0.56) \)
- Binomial from Table I
\[ P(X = 1) = P(0.5 < X < 1.5) \]
201. a) Show that the variance of a normal random variable with mean $\mu$ and variance $\sigma^2$ is $\sigma^2$.

Let $X$ be a normal random variable with mean $\mu$ and variance $\sigma^2$.

The variance of $X$ is given by $\text{Var}(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2$.

We know that $\mathbb{E}(X) = \mu$ and $\mathbb{E}(X^2)$ can be calculated using the moment generating function of the normal distribution.

By definition of the normal distribution, $\mathbb{E}(X) = \mu$.

Therefore, $\text{Var}(X) = \mathbb{E}(X^2) - (\mu)^2$.

For a normal distribution, $\mathbb{E}(X^2)$ is given by $\mu^2 + \sigma^2$.

Thus, $\text{Var}(X) = (\mu^2 + \sigma^2) - (\mu)^2 = \sigma^2$.

Hence, the variance of a normal random variable is $\sigma^2$.
Chapter 4 #50g

Chapter 6 #50g or 61g #30,59,41b

Note: To design a new, it must be a new product that will be sold...

The answer is that we are assuming a repeating pattern - which is not reasonable.

For the previous problem, we can integrate by substitution:

\[ \int e^{-x^2} \, dx \]

Let \( u = x^2 \), then \( du = 2x \, dx \)

\[ \int e^{-x^2} \, dx = \int e^{-u} (1/2) \, du = -1/2 \cdot e^{-x^2} + C \]

Following Page 5, we find the resulting function

\[ R(x) = 1 - \int e^{-x^2} \, dx \]

Now, we can find the second limit function, following theorem 4.2.13:
Diagram 2

1. Determine the number of gauge terms

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<th>2</th>
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<td>4</td>
<td>0</td>
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2. Using the formula (2.1) in Eq. 2.3,

\[ f(x) = \frac{\binom{N-1}{k}}{\binom{N}{k}} \]

where
- \( N \) = total number of objects
- \( k \) = required sample size
- \( f(x) \) = effective gauge

3. For the case where \( k = 2 \) and \( N = 4 \):

\[ f(2) = \frac{\binom{3}{2}}{\binom{4}{2}} = \frac{3}{6} = \frac{1}{2} \]

4. Final example continues (Refer page 9.2)

5. For the case where \( k = 2 \) and \( N = 3 \):

\[ f(2) = \frac{\binom{2}{2}}{\binom{3}{2}} = \frac{1}{3} \]
Are \( X \) and \( Y \) independent?

\( f(x,y) = \frac{1}{4} \) when \( 0 < x < 1 \)

\[ f(x,y) = f_X(x) f_Y(y) \Rightarrow \frac{1}{4} \neq 0 \] (since constant)

\[ f(x,y) = f(x)f(y) \]

\( X \) and \( Y \) are not independent.

8. Given joint density:

\[ f(x,y) = c(4x^2y+1), \quad 0 < x < \frac{1}{2}, \quad 0 < y < 1 \]

a) Find a value that makes this a density.

Since cumulative density \( F(x,y) = 1 \):

\[ F(x,y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f(t,u) \, dt \, du = 1 \]

\[ F(x,y) = \int_{0}^{x} \int_{0}^{y} c(4t^2u+1) \, dt \, du = 1 \]

\[ 1 = c \left[ \int_{0}^{x} \int_{0}^{y} 4t^2u+1 \, dt \, du \right] \]

\[ 1 = \left[ \frac{4}{3} x^3 y + \frac{y}{2} \right]_{0}^{1} \]

\[ 1 = \left( \frac{4}{3} x^3 + \frac{1}{2} \right) \]

\[ x = \frac{1}{2} \]

\[ c = 1 \]
2.2b) Find \( f(x, y) \) and \( g(x, y) \) \text{ (limit here is } d \) because we know the upper bound \( x > 0 \).\)

\[
\int_{x=0}^{\infty} \int_{y=0}^{\infty} \frac{4 \cdot 6}{(x+y+1)} \, dx \, dy
\]

\[
= \int_{x=0}^{\infty} \left[ \frac{4 \cdot 6}{(x+y+1)} \right]_{x}^{\infty} \, dx
\]

\[
= \int_{x=0}^{\infty} \frac{24}{6y+1} \, dx
\]

\[
= \frac{24}{6} \int_{x=0}^{\infty} \left( \frac{1}{2} \right) \, dx
\]

\[
= \frac{12}{6} \left[ x \right]_{0}^{\infty} = \infty
\]

Thus, the integral diverges.

2.2c) Find marginal densities for \( x \) and \( y \) \text{ (Definition 5.1.4 Eq.16)}

\[
f_x(x) = \int_{0}^{\infty} f(x, y) \, dy
\]

\[
f_x(x) = \int_{0}^{\infty} \frac{4 \cdot 6}{(x+y+1)} \, dy
\]

\[
= \frac{24}{6} \int_{0}^{\infty} \left( \frac{1}{2} \right) \, dy
\]

\[
= \frac{12}{6} \left[ y \right]_{0}^{\infty} = \infty
\]

Thus, the marginal density for \( x \) diverges.

\[
f_y(y) = \int_{0}^{\infty} f(x, y) \, dx
\]

\[
f_y(y) = \int_{0}^{\infty} \frac{4 \cdot 6}{(x+y+1)} \, dx
\]

\[
= \frac{24}{6} \int_{0}^{\infty} \left( \frac{1}{2} \right) \, dx
\]

\[
= \frac{12}{6} \left[ x \right]_{0}^{\infty} = \infty
\]

Thus, the marginal density for \( y \) diverges.
\[
\begin{array}{c}
\text{EX. 1)} \quad \text{Find:} \quad R(t) = \frac{1}{2} \int_{0}^{\infty} \left[ \frac{1}{t^2} \right] dt
\\
\text{Using}\:\:\int_{0}^{\infty} \frac{1}{x^2} \, dx = \frac{\pi}{2}
\\
\text{Is} \\ R(t) \text{a}\:\: \text{constant}\?:
\\
\text{Yes,}\: \: R(t) = \frac{\pi}{2}
\\
\end{array}
\]
\[ f(x,y) = \frac{1}{x+y} \]

...
32. \[ P(X > 20) = \frac{\alpha^{20}}{20!} e^{-\alpha} \]

33. \[ P(X = 10) \text{ and } X > 30 \]

34. \[ \int_{-\infty}^{\infty} \exp(-x^2) \, dx = \sqrt{\pi} \]

35. \[ \int_0^\infty x e^{-x} \, dx = 1 \]

36. \[ \int_0^\infty \frac{x^2}{e^{x^2}} \, dx = \frac{1}{2} \sqrt{\pi} \]

37. \[ \int_0^\infty \frac{xe^{-x}}{x+1} \, dx = \frac{\pi}{2} \]

Ex. Find the moment generating function of \( X \) such that:

\[ M_X(t) = \int_0^\infty \left( 1 - \frac{t}{x} \right)^{\alpha - 1} e^{-\frac{t}{x}} \, dx \]

\[ m_X(t) = \left[ e^{t \frac{t}{x} + \frac{t^2}{2x^2}} \right]_{x=0}^{x=\infty} = 1 + \frac{t^2}{2} + \cdots \]

\[ m_X(1) = e^{1 \cdot \frac{1}{x} + \frac{1^2}{2 \cdot x^2}} = e^{1 + \frac{1}{2} + \cdots} = e \]

\[ m_X(0) = 1 \]

\[ m_X(\infty) = \lim_{x \to \infty} \left[ 1 + \frac{t}{x} \right] = 1 \]
\[
\text{EOD} = \sum_{i=1}^{n} \psi_i(x_i, y_i) \quad \text{(shown in part 5.2.1)}
\]

\[
= \sum_{i=1}^{n} \left( x_i^2 + y_i^2 \right) \quad \text{(shown in part 5.2.1)}
\]

\[
= \left( \sum_{i=1}^{n} x_i^2 \right) + \left( \sum_{i=1}^{n} y_i^2 \right) \quad \text{(shown in part 5.2.1)}
\]

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\]

\[
\text{EOD} \approx 0.02
\]

\[
\text{EOD} = \sum_{i=1}^{n} \psi_i(x_i, y_i)
\]

\[
x(x, y) = EFD1, \ EFD2, \ EFD3(x, y) \quad \text{by theorem 5.2.1}
\]

\[
\text{EFD1} + (x, y) \approx (0.027)
\]

\[
\text{EFD1} + (x, y) \approx (0.027)
\]

The above shows that the expression for \( x(x, y) \) is well approximated by the given quadratic form in \( x, y \).
6) Derive (5.3.3) and (5.3.4) for general 

\[ f(t) = \int_{0}^{t} \exp(-st) \, dt \]

by substitution of \( s = 0 \) into \( f(t) \).