Waveform Libraries for Radar Tracking Applications: Maneuvering Targets

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***Abstract*— In this paper we extend the idea of adaptive waveform selection for radar target tracking to interacting multiple model (IMM) trackers to permit the modelling of maneuvering targets by allowing multiple possible dynamical models. We develop a one step ahead solution to the problem of waveform selection, which is designed to decrease dynamic model uncertainty for the target of interest. It is based on maximization of the expected information obtained about the dynamical model of the target from the next measurement. We also discuss the design of waveform libraries for target tracking applications.**

I. Introduction

Radars systems capable of waveform agility are becoming practical for operational use. The problems involved in fully utilizing such capability are manifold and we address just two of them here. Before discussing the first, we point out that there are two approaches to the design of waveforms for use in such a system. One way is to design each waveform by some optimal choice of parameters (such as chirp rate in an LFM waveform) based on the current scenario and operational requirements. The other is to have a finite library of waveforms from which a choice is made based on the same considerations. We adopt the second approach here, as it is the one more likely to be implemented because of computational feasibility. With this in mind, the first problem addressed in this paper is how to optimally choose a collection of waveforms with which to stock the waveform library. In particular, it is advantageous to be able to identify redundant waveforms or to identify waveforms which will significantly complement an existing library. The second problem is how to use such a waveform library to best achieve optimum performance of the radar system for particular tasks. The particular task considered here is the target tracking functions of the radar.

The idea of selecting waveforms adaptively based on tracking considerations was introduced in the papers of Kershaw and Evans [1], [2]. Further, work has been done in this area by Niu, Willet and Bar-Shalom [3]. In a recent paper [4], we have taken an information theoretic approach to the design of waveform libraries for tracking and have defined a measure of *utility* for waveform libraries, which measures the degree to which the waveforms in the library complement each other, and through which such libraries can be compared. This utility function was then used to develop a number of candidate libraries form tracking purposes.

Here we extend the idea of adaptive waveform selection for target tracking to the so-called interacting multiple model (IMM) trackers, which model maneuvering targets by attributing multiple possible dynamical models to them. For this type of tracker, the correct identification of the current dynamical model is important to the performance of the tracking system. We develop a one step ahead solution to the problem of waveform selection designed to decrease dynamic model uncertainty for the target of interest. In particular, we propose a criterion for choosing the library waveform for the next measurement of the tracker which is based on maximizing the expected information obtained about the dynamical model of the target from that measurement.

The paper is organised as follows. Sections II and III formulate the waveform adaptive IMM tracking problem and give an overview of the IMM tracker. In Section IV we give a brief description of our tracking waveform library utility function and go on to describe a number of waveform libraries which we subsequently use for the IMM tracking problem. In Section V we derive a mutual information criterion, for waveform selection for IMM trackers, and go on to show the effectiveness of this criterion, in conjunction with particular waveform libraries, in Section VI.

II. Problem Formulation and Modelling

The basic sensor model proposed in [2] is used. That is, the sensor is characterised by a measurement noise covariance matrix which is waveform dependent:

$$\mathbf{R}_\phi = \mathbf{T}\mathbf{J}_\phi^{-1}\mathbf{T}, \tag{1}$$

where $\mathbf{J}_\phi$ is the Fisher information matrix corresponding to the measurement using waveform $\phi \in \mathrm{L}^2(\mathbf{R})$ and $\mathbf{T}$ is the transformation matrix between the time delay and Doppler measured by the receiver and the target range and velocity. The Fisher information is given by an expression involving the normalised second order time and frequency moments of the waveform ϕ.

We assume that the dynamic models and the sensor measurement processes are linear and described by the following equations:

$$\mathbf{x}(k) = \mathbf{F}_j \mathbf{x}(k-1) + \boldsymbol{\nu}_j \quad (2)$$
$$\mathbf{z}(k) = \mathbf{H}\mathbf{x}(k) + \boldsymbol{\omega}_\phi \quad (3)$$

where $j = 1, \cdots, M$ is the dynamic model at time k, and $\phi = 1, \cdots, N$ is the waveform used to obtain the measurements at time k. A finite number of waveforms is considered. These waveforms form a library, to be described in the following sections. We write $\mathbf{x}(k)$ for the state of the track and $\mathbf{z}(k)$ for the measurement at time k. $\mathbf{F}_1, \cdots, \mathbf{F}_M$ are the state propagation matrices for the different maneuvers, $\mathbf{H}$ is the measurement matrix. Process noise is denoted by $\boldsymbol{\nu}_1, \cdots, \boldsymbol{\nu}_M$ and the measurement noise by $\boldsymbol{\omega}_1, \cdots, \boldsymbol{\omega}_N$. These both form zero mean, white, and uncorrelated Gaussian noise sequences with covariance matrices $\mathbf{Q}_1, \cdots, \mathbf{Q}_M$ and $\mathbf{R}_1, \cdots, \mathbf{R}_N$ respectively.

We assume that changes in target trajectory can be modelled as a Markov chain with given transition probabilities as follows:

$$\pi_{j,l} = P\{M(k) = l | M(k-1) = j\}, \quad j, l \in 1, \cdots, M. \quad (4)$$

The trajectory of the target can be described at any time by one of the M dynamic models. The tracker switches between the dynamic models using the data, and thereby facilitates tracking of maneuvering targets. Our problem is at each epoch to choose the waveform which will best reduce uncertainty of the current dynamic models. In other words, we wish to tune adaptively the waveform to the dynamic model.

III. The IMM Filter

Here we give a brief summary of the interacting multiple model (IMM) filter. When used with an Kalman filter, the IMM at time k has the following structure. State estimates at time $k-1$, given the model at time k are calculated. This step is referred to as "IMM mixing" [5]. Let $p_j(k|k) = P\{M(k) = j|\mathbf{Z}^k\}$, denote the probability of model j being correct at time k, where $\mathbf{Z}^k$ are the measurements collected up to time k. Further, let

$$\begin{aligned}\mu_{j,l}(k-1|k) &\equiv P\{M(k-1) = j | M(k) = l, Z^{k-1}\} \\ &= \frac{\pi_{j,l} p_l(k-1|k-1)}{p_j(k|k-1)},\end{aligned} \quad (5)$$

denote the backward transition probabilities, where the $p_j(k|k-1)$ are the predictive model probabilities

$$p_j(k|k-1) \equiv P\{M(k) = j|\mathbf{Z}^{k-1}\} = \sum_{l=1}^{M} \pi_{j,l} p_l(k-1|k-1). \quad (6)$$

Then the mixed input to the next processing stage is

$$\hat{\mathbf{x}}^j(k-1|k-1) = \sum_{l=1}^{M} \mu_{l,j}(k-1|k) \hat{\mathbf{x}}_l(k-1|k-1), \quad (7)$$

$$\mathbf{P}^j(k-1|k-1) = \sum_{l=1}^{M} \mu_{l,j}(k-1|k) \times \quad (8)$$

$$\left(\mathbf{P}_l(k-1|k-1) + \mathbf{X}_l^j(k-1|k-1)\mathbf{X}_l^j(k-1|k-1)^T\right), \quad (9)$$

where

$$\mathbf{X}_l^j(k-1|k-1) = \hat{\mathbf{x}}_l(k-1|k-1) - \hat{\mathbf{x}}^j(k-1|k-1). \quad (10)$$

The *a priori* state and error covariance for each model are calculated as follows:

$$\hat{\mathbf{x}}_j(k|k-1) = \mathbf{F}_j \hat{\mathbf{x}}^j(k-1|k-1) \quad (11)$$
$$\mathbf{P}_j(k|k-1) = \mathbf{F}_j \mathbf{P}^j(k-1|k-1)\mathbf{F}_j^T + \mathbf{Q}_j. \quad (12)$$

The update of the state estimate and error covariance are calculated for each IMM using the Kalman Filter update equations, i.e.,

$$\hat{\mathbf{x}}_j(k|k) = \hat{\mathbf{x}}_j(k|k-1) + \mathbf{K}_{j,\phi}(\,\mathbf{z}(k) - \mathbf{H}\hat{\mathbf{x}}_j(k|k-1)\,) \quad (13)$$
$$\mathbf{P}_j(k|k) = \mathbf{P}_j(k|k-1) - \mathbf{K}_{j,\phi}\mathbf{S}_{j,\phi}\mathbf{K}_{j,\phi}^T, \quad (14)$$

where

$$\mathbf{S}_{j,\phi} = \mathbf{H}\mathbf{P}_j(k|k-1)\mathbf{H}^T + \mathbf{R}_\phi \quad (15)$$

is an innovation covariance matrix and

$$\mathbf{K}_{j,\phi} = \mathbf{P}_j(k|k-1)\mathbf{H}^T\mathbf{S}_{j,\phi}^{-1} \quad (16)$$

is the Kalman gain. The predictive probability density for the result of the k^{th} measurement, given the model at k, is

$$\begin{aligned} f(\mathbf{z}|M(k) = j, \mathbf{Z}^{k-1}, \phi) = \frac{1}{\sqrt{2\pi|\mathbf{S}_{j,\phi}|}} \times \\ \exp(-\frac{1}{2}(\mathbf{z} - \mathbf{H}\hat{\mathbf{x}}_j(k|k-1))\mathbf{S}_{j,\phi}^{-1}(\mathbf{z} - \mathbf{H}\hat{\mathbf{x}}_j(k|k-1))^T). \end{aligned} \quad (17)$$

The probability of each dynamic model is calculated using Bayes rule:

$$p_j(k|k) = \frac{f(\mathbf{z}(k)|M(k) = j, \mathbf{Z}^{k-1}, \phi) p_j(k|k-1)}{\sum_{l=1}^{M} f(\mathbf{z}(k)|M(k) = l, \mathbf{Z}^{k-1}, \phi) p_l(k|k-1)} \quad (18)$$

where $p_j(k|k-1)$ is given by (6). We note that $p_j(k|k)$ depends on our choice of waveform to use for the k^{th} measurement.

The state of the tracker is estimated by combining the state estimates for each model into a single Gaussian. This process is referred to as "IMM output combination" [5].

$$\hat{\mathbf{x}}(k|k) = \sum_{j=1}^{M} p_j(k|k)\hat{\mathbf{x}}_j(k|k) \quad (19)$$

$$\begin{aligned}\mathbf{P}(k|k) = \sum_{j=1}^{M} p_j(k|k)\left(\mathbf{P}_j(k|k) + \hat{\mathbf{x}}_j(k|k)\hat{\mathbf{x}}_j(k|k)^T\right) \\ - \hat{\mathbf{x}}(k|k)\hat{\mathbf{x}}(k|k)^T.\end{aligned} \quad (20)$$

IV. Waveform Libraries

In this section we give a summary of an information theoretic approach to the design of radar waveform libraries for tracking purposes that we have developed recently [4]. The motivation for our approach is to define measures for the utility of a waveform library. With such utility measure we can determine how much the addition of a particular set of waveforms to the library will improve the library or on the other hand, how much will removing some waveforms reduce the utility of the library.

In developing a waveform library utility function for tracking applications, we again use the basic sensor model [2] described in Section II. Thus, representing the measurement obtained using the waveform ϕ as a Gaussian measurement with covariance $\mathbf{R}_\phi$, the expected information obtained from a measurement with such a waveform is, given that our current state of knowledge about the range and Doppler of the target is represented by the state covariance matrix $\mathbf{P}$,

$$I(X;Z) = \log|\mathbf{I} + \mathbf{R}_\phi^{-1}\mathbf{P}|. \tag{21}$$

This is the mutual information between target state (range and Doppler) X and the radar return, Z, arising from the use of waveform ϕ. Here $\mathbf{I}$ is the identity matrix.

We assume some knowledge of the possible state covariances P able to be generated by our tracking system, represented by means of a probability distribution $F(\mathbf{P})$ over the space of positive definite matrices. The *utility* of a waveform library $\mathcal{L} \subset \mathrm{L}^2(\mathbf{R})$, with respect to a distribution F, is defined to be

$$G_F(\mathcal{L}) = \int_{\mathbf{P}>0} \max_{\phi\in\mathcal{L}} \log|\mathbf{I} + \mathbf{R}_\phi^{-1}\mathbf{P}|\, dF(\mathbf{P}). \tag{22}$$

We consider two libraries $\mathcal{L}$ and $\mathcal{L}'$ to be *weakly equivalent*, with respect to the distribution F, if $G_F(\mathcal{L}) = G_F(\mathcal{L}')$ and *strongly equivalent* if $G_F(\mathcal{L}) = G_F(\mathcal{L}')$, for all F.

In what follows we work in receiver coordinates, i.e., treat $\mathbf{T}$ above as $\mathbf{I}$, this amounts to a change in parameterisation of the positive definite matrices in the integral in (22).

Having defined the utility of a waveform library, we investigate the utilities of a number of libraries. In particular, libraries generated from a fixed waveform ϕ_0, usually an unmodulated pulse of some fixed duration, by so-called symplectic transformations are considered. These form a group of unitary transformations on $\mathrm{L}^2(\mathbf{R})$ and include linear chirping (to be explained later), as well as the Fractional Fourier transform (FrFT). Under such transformations $\phi = \mathbf{U}\phi_0$, the ambiguity function of the waveform ϕ_0, transforms, such that

$$|A_\phi(\mathbf{x})| = |A_{\phi_0}(\mathbf{S}^{-1}\mathbf{x})| \tag{23}$$

where $\mathbf{x} = (t,\omega)^T$ and $|\mathbf{S}| = 1$. Furthermore, we are able to show that under symplectic transformations, the measurement covariance matrix transforms as,

$$\mathbf{R}_\phi = \mathbf{S}^{-1}\mathbf{R}_{\phi_0}\mathbf{S}. \tag{24}$$

A. LFM Waveform Library

As an example of this approach to library construction, we describe the linear frequency modulation (LFM) or "chirp" waveform library. In this case the library consists of

$$\mathcal{L}_{\mathrm{LFM}} = \{\exp(i\lambda\mathbf{t}^2/2)\phi_0\ |\lambda_{\min} \le \lambda \le \lambda_{\max}\} \tag{25}$$

where ϕ_0 is an unmodulated pulse, $\lambda_{\min}$ and $\lambda_{\max}$ are the minimum and maximum chirp rates supported by the radar, and $\mathbf{t}$ is the (unbounded) operator on $\mathrm{L}^2(\mathbf{R})$ defined by

$$\mathbf{t}\phi(t) = t\phi(t). \tag{26}$$

It follows that

$$(\exp(i\lambda\mathbf{t}^2/2)\phi)(t) = \exp(i\lambda t^2/2)\phi(t). \tag{27}$$

For this library the corresponding measurement covariance matrices are given by (24) with

$$\mathbf{S}(\lambda) = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix}. \tag{28}$$

In range-Doppler coordinates, the error covariance matrix for each LFM can be represented by

$$\mathbf{R}(\lambda) = \mathbf{S}(\lambda)\mathbf{R}_0\mathbf{S}(\lambda)^T, \tag{29}$$

where $\mathbf{R}_0$ is a diagonal matrix with ρ_1, ρ_2 on the diagonal; that is, a covariance matrix for the rectangular pulse [6], [2], and

$$\mathbf{S}(\lambda) = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix} \tag{30}$$

is the transformation to the constant sweep rate λ for every ϕ. In [4] we have shown that the sub-library

$$\mathcal{L}'_{\mathrm{LFM}} = \{\exp(i\lambda_{\min}\mathbf{t}^2/2)\phi_0, \exp(i\lambda_{\max}\mathbf{t}^2/2)\phi_0\} \tag{31}$$

is strongly equivalent to $\mathcal{L}_{\mathrm{LFM}}$. That is, we do just as well if we keep only the chirps with the maximum and minimum rates.

B. LFM-Rotation Library

An alternative approach is to rotate the ambiguity of the unmodulated pulse. This is considered here. The rotation is performed by the fractional Fourier transform. Thus this waveform library consists of arbitrary fractional Fourier transforms of a single waveform, i.e

$$\mathcal{L}_{\mathrm{FrFT}} = \{\exp(i\theta(\mathbf{t}^2 + \boldsymbol{\omega}^2)/2)\phi_0 \mid \theta \in \Theta\}, \tag{32}$$

where the set $\Theta \subset [0, 2\pi]$ can be chosen so as not to violate the bandwidth constraints of the radar, and $\boldsymbol{\omega}$ is the operator on $\mathrm{L}^2(\mathbf{R})$ defined by

$$\boldsymbol{\omega}\phi(t) = i\phi'(t). \tag{33}$$

For this library the corresponding transformation in range-Doppler space is given by the rotation

$$\mathbf{S}(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}. \tag{34}$$

For the library under consideration here, we start with an unmodulated waveform ϕ_0 and allow both the "chirping" transformations (27) and the fractional Fourier transformations (32); that is, we consider all waveforms of the following form:

$$\mathcal{L}_{\mathrm{LFMR}} = \{\exp(i\theta(\mathbf{t}^2 + \boldsymbol{\omega}^2)/2)\exp(i\lambda\mathbf{t}^2/2)\phi_0 \mid \lambda_{\min} \le \lambda \le \lambda_{\max}, \theta \in \Theta\} \tag{35}$$

where the set Θ is chosen so as not to violate the bandwidth constraints of the radar, and $\boldsymbol{\omega}$ is the operator on $\mathrm{L}^2(\mathbf{R})$ defined by

$$\boldsymbol{\omega}\phi(t) = i\phi'(t), \tag{36}$$

where $\cdot'$ denotes differentiation in time.

For this library the corresponding measurement covariance matrices are given by (24) with

$$\mathbf{S}(\theta,\lambda) = \begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}1 & 0\\ \lambda & 1\end{pmatrix}. \tag{37}$$

As in the pure chirp library case, we have shown in [4], that the sub-library

$$\mathcal{L}'_{\text{LFMR}} = \{\exp(i\theta(\mathbf{t}^2+\boldsymbol{\omega}^2)/2)\exp(i\lambda\mathbf{t}^2/2)\phi_0 \mid \lambda\in\{\lambda_{\min},\lambda_{\max}\}, \theta\in\Theta\} \tag{38}$$

is strongly equivalent to $\mathcal{L}_{\text{LFMR}}$.

C. A Six Waveform Library

Based on the above considerations we now define the waveform library containing six waveforms that we will be used in our subsequent analysis of the maneuvering target tracking problem. The library, which is a sub-library of (38), contains

$$\begin{aligned}\mathcal{L}_6 = \{ & \exp(i\lambda_{\min}\mathbf{t}^2/2)\phi_0,\\ & \exp(-0.2\pi i(\mathbf{t}^2+\boldsymbol{\omega}^2)/2)\exp(i\lambda_{\min}\mathbf{t}^2/2)\phi_0,\\ & \exp(-0.4\pi i(\mathbf{t}^2+\boldsymbol{\omega}^2)/2)\exp(i\lambda_{\min}\mathbf{t}^2/2)\phi_0,\\ & \exp(i\lambda_{\max}\mathbf{t}^2/2)\phi_0,\\ & \exp(0.2\pi i(\mathbf{t}^2+\boldsymbol{\omega}^2)/2)\exp(i\lambda_{\max}\mathbf{t}^2/2)\phi_0,\\ & \exp(0.4\pi i(\mathbf{t}^2+\boldsymbol{\omega}^2)/2)\exp(i\lambda_{\max}\mathbf{t}^2/2)\phi_0\}\end{aligned} \tag{39}$$

V. Application to Tracking Maneuvering Targets

In this section we consider criteria for the adaptive selection of a waveform from a waveform library for an IMM tracker. At each epoch we would like to select a waveform (i.e the error covariance matrix) so the the measurement will minimize the uncertainty of the dynamic model of the target. We take a single step ahead (greedy) approach to the problem. That is, we choose the waveform which minimises the expected entropy of the probability distribution of the model at the next epoch. This implies, that the mutual information between the model and the measurement needs to be maximized.

$$\begin{aligned}I_\phi(M;Z) = & -\sum_{j=1}^{M} p_j(k|k{-}1)\log p_j(k|k{-}1)\\ & + \int f(\mathbf{z}(k)|\mathbf{Z}^{k-1},\phi)\sum_{j=1}^{M} p_j(k|k)\log p_j(k|k)\; d\mathbf{z}(k),\end{aligned} \tag{40}$$

where

$$f(\mathbf{z}|\mathbf{Z}^{k-1},\phi) = \sum_{j=1}^{M} p_j(k|k{-}1) f(\mathbf{z}|M(k)=j,\mathbf{Z}^{k-1},\phi) \tag{41}$$

with $f(\mathbf{z}|M(k)=j,\mathbf{Z}^{k-1},\phi)$ given by (17). We recall that $p_j(k|k)$ explicitly depends on the waveform ϕ. (40) can be rewritten as

$$\begin{aligned}I_\phi(M;Z) = & -\sum_{j=1}^{M} p_j(k|k{-}1)\log|\mathbf{S}_{j,\phi}|\\ & - \int f(\mathbf{z}(k)|\mathbf{Z}^{k-1},\phi)\log f(\mathbf{z}(k)|\mathbf{Z}^{k-1},\phi)\; d\mathbf{z}(k).\end{aligned} \tag{42}$$

We now proceed by approximating $f(\mathbf{z}(k)|\mathbf{Z}^{k-1},\phi)$ by a Gaussian density with covariance equal to the covariance of (41), i.e.,

$$\sum_{j=1}^{M} p_j(k|k{-}1)\mathbf{S}_{j,\phi}$$

The approximate mutual information may then be written as

$$I_\phi(M;Z) = -\sum_{j=1}^{M} p_j(k|k{-}1)\log\frac{|\mathbf{S}_{j,\phi}|}{|\sum_{l=1}^{M} p_l(k|k{-}1)\mathbf{S}_{l,\phi}|}. \tag{43}$$

Thus, by the criterion of maximum mutual information between the current state of knowledge of the model and the measurement, we select the waveform from the library $\mathcal{L}$ satisfying

$$\phi^\star = \arg\min_{\phi\in\mathcal{L}}\left(-\sum_{j=1}^{M} p_j(k|k{-}1)\log\frac{|\mathbf{S}_{j,\phi}|}{|\sum_{l=1}^{M} p_l(k|k{-}1)\mathbf{S}_{l,\phi}|}\right). \tag{44}$$

VI. Simulation Results

The simulations were performed for the cost function identified in equation (44). Target trajectories in range and Doppler were randomly created. The maneuvers for the trajectories were generated using given transition probability matrix. For simplicity, we specified four maneuvers: 0 m/s^2 acceleration; 10m/s^2 acceleration; 50 m/s^2 acceleration; -10 m/s^2 acceleration.

In the experiments three rotation-LFM waveform libraries were tested:

- A library containing a single waveform – the maximum up-sweep chirp.
- A library containing two waveforms – the maximum up-sweep and maximum down-sweep chirps.
- The six waveform library $\mathcal{L}_6$ waveforms described in Section IV.

The simulations were performed 1000 times and the results are presented in Figures 1, 2. Clearly, waveform scheduling with the six waveform library out performs waveform scheduling using the two waveform library, which in turn out performs no waveform scheduling (one waveform), in both estimation accuracy (Figure 1) and correct identification of target maneuver (Figure 2).

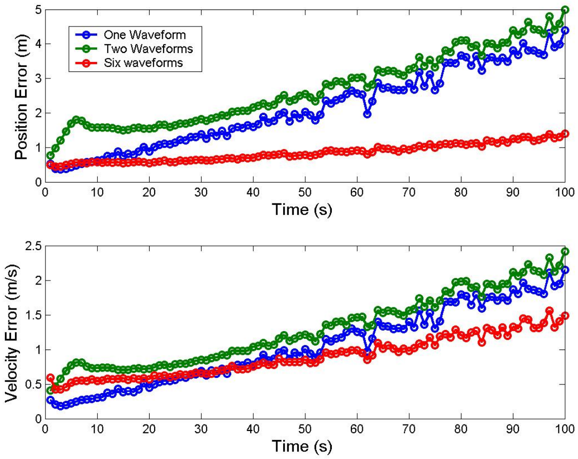

Fig. 1. Root mean square error in range and velocity for waveform selection using cost (44)

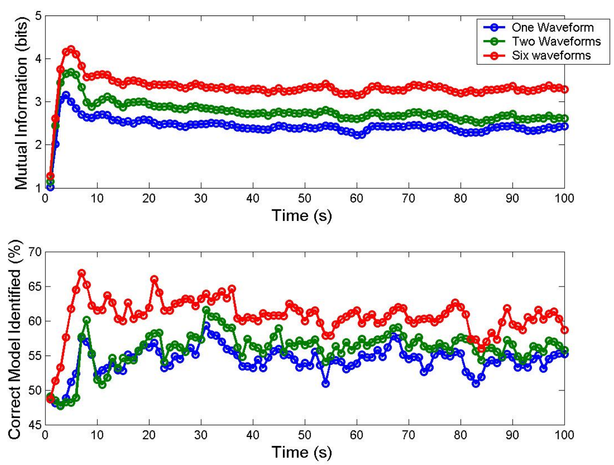

Fig. 2. Cost function value and correct maneuver identification for for waveform selection using cost (44)

VII. Conclusion

We have developed a waveform selective version of the IMM tracker. The waveform selection is based on maximising the mutual information between our state of knowledge of the dynamical model for the target at the next epoch and the measurement defined by the waveform. We have shown the effectiveness of such an approach, when used with appropriate waveform libraries, through simulation. However, considerable work remains to be done in the areas of waveform library design and waveform selection criteria for target tracking.

Finally, we note that the approach adopted here is based on looking a single step ahead. Future will also focus on waveform adaptive tracking algorithms which base decisions on looking multiple steps into future.

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