ADAPTIVE SENSING OF DYNAMIC TARGET STATE IN HEAVY SEA CLUTTER

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ABSTRACT

We propose an adaptive estimation method for the spatio-temporal covariance matrix of sea clutter. The motivation is to enable adaptive detection approaches that rely on accurate estimation of this matrix. The method involves vectorization of the equations for the dynamical system model governing the temporal evolution of the clutter matrix followed by a multiple particle filtering approach to deal with the high dimensionality of the formulation. The estimated sea clutter covariance matrix is applied to the problem of detection of a small target in heavy clutter; effectiveness is demonstrated via simulations.

Index Terms— Sea clutter estimation, waveform scheduling, waveform diversity, space-time covariance matrix, radar scattering function

1. INTRODUCTION

The next generation of active sensing systems will offer profound increases in agility — particularly at the transmitter, where opportunities to exert real-time control over a proliferation of degrees of freedom are rapidly developing [1]. In radar, waveform agility presents new possibilities for sensing rapidly changing scenes. The well-studied problem of detecting a moving target in heavy sea clutter presents a particularly challenging situation where waveform agility, coupled with adaptive algorithms encompassing waveform design, scheduling and receiver processing, may hold potential for substantial performance gains. Exploitation of waveform agility for target detection in the presence of sea clutter has been demonstrated through the use of short-time "snapshot" estimation of the spatio-temporal covariance matrix of the clutter [2, 3]. In this paper, we seek to extend this circle of ideas into situations in which a dynamical model of rapidly changing parameters of a scene, including both target and clutter states, is available to allow leveraging of prior parameter estimates. The goal with regard to clutter is to use prior information in conjunction with the dynamical model to obtain a more accurate estimate of the space-time covariance matrix in each sensing epoch, thereby enabling increased fidelity in waveform design and receiver processing.

In this paper, we propose a method to adaptively estimate the space-time covariance matrix for rapidly varying radar scenes. The method introduces a formulation of the space-time representation of the clutter scene in the scattering function domain. This formulation is vectorized to obtain a convenient dynamical system description for the temporally evolving clutter model. The dimensionality of the resulting dynamical system is very high in practical scenarios, so we use the multiple particle filter sequential Monte Carlo method [4] to sequentially estimate the dynamic scene. In this step, structure of the dynamical equations is capitalized upon to reduce dimensionality.

The paper is organized as follows. In Section 2, we provide the system model for the radar scene representation. In Section 3, the multiple particle filtering method is described and used to develop our space-time covariance matrix estimation method. Simulation results illustrating performance of the approach are presented in Section 4.

2. RADAR SCENE SYSTEM REPRESENTATION

2.1. Space-time covariance matrix characterization

Consider a radar operating at a pulse repetition frequency (PRF) of f_s Hz that transmits a burst of K pulses in a dwell. The return from each burst is sampled at a rate f_b Hz to yield a sequence $y[m,k] = y(m\,\Delta T_b, k), m = m_0, m_1, \cdots, m_{M_n-1}$, $k = 0, 1, \cdots, K-1$, where $\Delta T_b = 1/f_b$ is the *fast* sampling interval, m_0 is the lowest range bin in the validation gate at time step n, and the validation gate at dwell n consists of M_n range bins. Let the complex reflectivity of the aggregate scatterers on the sea surface at pulse k and range bin m be given by $x[m,k]$, which could represent sea clutter or targets. Then, the radar return at the kth pulse is modeled by

$$y[m,k] = \sum_{i=0}^{N-1} x[m-i,k]s[i] + v[m,k] \qquad (1)$$

where $v[m,k]$ is white Gaussian noise. Note that we have assumed that the same signal $s[n], n = 0, 1, \cdots, N-1$ is

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transmitted repeatedly throughout each burst.

We let the signal matrix P be an M_n by N_v matrix, where $N_v = M_n + N - 1$, with the signal sequence $s[n]$ beginning at the pth element in the pth row, $p = 1, 2, \cdots, M_n$:

$$P = \begin{bmatrix} s[0] & s[1] & \cdots & s[N-1] & \cdots & 0 \\ 0 & s[0] & s[1] & \cdots & s[N-1] & \cdots \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & \cdots & s[N-2] & s[N-1] \end{bmatrix}$$

We also define B_n to be an $N_v \times K$ matrix consisting of the complex reflectivities $x[m,k], m = m_0, m_1, \cdots, m_{M_n-1+N-1}, k = 0, 1, \cdots, K-1$, with range (fast time or delay) increasing down the columns and transmitted pulses (slow time) increasing across the rows. For notational convenience in the following development, we will assume K is odd.

We can then write the observation matrix as

$$Y_n = PB_n + V_n \tag{2}$$

where Y_n is the observation matrix with elements defined in (1) and V_n is the noise matrix at time n.

We denote by $\tilde{B} = \text{vec}(B)$ the stacked columns of matrix B to form a vector of dimension KN_v. The covariance matrix of the reflectivity vector $\tilde{B}_n$ provides both space and time covariance information. Our objective is to estimate $\mathbf{\Sigma}_{\tilde{B},n} = \text{E}[\tilde{B}_n \tilde{B}_n^H]$, the space-time covariance matrix of the scatterers given the sequence of observations $\tilde{Y}_1, \tilde{Y}_2, \cdots, \tilde{Y}_n$. Here, H denotes Hermitian transpose and $\text{E}[\cdot]$ is the expectation operator.

2.2. Scattering function characterization

To estimate the space-time covariance matrix $\mathbf{\Sigma}_{\tilde{B},n}$, we first define a scattering matrix A_n whose elements are obtained by taking the short-time Fourier transform of the elements of B_n in the slow-time direction; specifically,

$$A_n[m,l] = \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} B_n[m,k] W^{kl}$$

where $W = \exp(-j2\pi/K)$ and $l \in [-\frac{K-1}{2}, \cdots, \frac{K-1}{2}]$. Then,

$$A_n = B_n D \tag{3}$$

where D is the discrete Fourier transform matrix. Thus, A_n contains the range-Doppler description of the complex reflectivities in B_n.

We assume a near-constant velocity model for each of the scatterers, including the target. With this assumption, the vectorized scattering matrix $\tilde{A}_n$ evolves according to the dynamic equation

$$\tilde{A}_n = F\tilde{A}_{n-1} + \tilde{\mathbf{W}}_n \tag{4}$$

where $\tilde{\mathbf{W}}_n$ is zero-mean complex Gaussian noise with covariance Q_n. The matrix F incorporates the movement of the scatterers between dwells and populates the range-Doppler cells that move into the validation gate.

The structure of the matrix F follows from underlying physical considerations. Note that the first $(K-1)/2$ columns of matrix A_n represent negative Doppler shifts, the center column represents zero Doppler, and the last $(K-1)/2$ columns represent positive Doppler shifts. These correspond to scatterers moving away from the sensor, not moving, and moving toward the sensor, respectively. We will assume that the range bin size and Doppler resolution are such that the scatterer in the lth column of A_n, $l = -\frac{K-1}{2}, \cdots, 0, \cdots, \frac{K-1}{2}$, moves a total of l bins between dwells. In each column of A_n, some scatterers will move out of the validation gate while others will move in. The latter effect requires us to populate the *empty* range-Doppler cells. This is accomplished by using an exponentially weighted sum of the complex reflectivities in the immediate neighborhood of these cells. The $KN_v \times KN_v$ matrix F is block-diagonal, with its kth $N_v \times N_v$ block, F_k, given by

$$F_k = \begin{bmatrix} e^{-|k|\alpha} & e^{-(|k|+1)\alpha} & \cdots & \cdots & e^{-(N_v+|k|-1)\alpha} \\ e^{-(|k|-1)\alpha} & \cdots & \cdots & \cdots & e^{-(N_v+|k|-2)\alpha} \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ e^{-\alpha} & e^{-2\alpha} & \cdots & \cdots & e^{-N_v\alpha} \\ 1 & 0 & \cdots & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & \cdots & 1 & \cdots & 0 \end{bmatrix}$$

when $k = -(K-1)/2, \cdots, -1$, and

$$F_k = \begin{bmatrix} 0 & \cdots & 0 & 1 & \cdots & 0 \\ \vdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ 0 & \cdots & \cdots & \cdots & 0 & 1 \\ e^{-N_v\alpha} & e^{-(N_v-1)\alpha} & \cdots & \cdots & \cdots & e^{-\alpha} \\ e^{-(N_v+1)\alpha} & e^{-N_v\alpha} & \cdots & \cdots & \cdots & e^{-2\alpha} \\ \vdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ e^{-(N_v+k-1)\alpha} & e^{-(N_v+k-2)\alpha} & \cdots & \cdots & \cdots & e^{-k\alpha} \end{bmatrix}$$

when $k = 1, \cdots, (K-1)/2$. At $k = 0$, there is no movement of scatterers and the corresponding block is simply $\mathbb{I}_{N_v}$, the identity matrix of dimension N_v.

The update of the filtering formulation is given by the observation equation. Specifically, from (2),

$$\tilde{Y}_n = \bar{P}\tilde{B}_n + \tilde{V}_n$$

where $\bar{P} = \mathbb{I}_K \otimes P$ is the Kronecker product of $\mathbb{I}_K$ and P. According to (2) and (3),

$$Y_n = PA_nD^{-1} + V_n \tag{5}$$

Using the property $\text{vec}(GJL) = (L^H \otimes G)\tilde{J}$ where G, J, L are three arbitrary matrices [5], (5) can be vectorized as

$$\begin{aligned}\tilde{Y}_n &= (D^{-H} \otimes P)\tilde{A}_n + \tilde{V}_n \\ &= \check{P}\tilde{A}_n + \tilde{V}_n\end{aligned}$$

with $\check{P} = D \otimes P$.

The dynamical system model can be derived from (4):

$$\mathbf{\Sigma}_{\tilde{A},n} = \mathrm{E}[\tilde{A}_n \tilde{A}_n^H] = F\mathbf{\Sigma}_{\tilde{A},n-1}F^H + Q_n \tag{6}$$

The observation model is then

$$\mathbf{\Sigma}_{\tilde{Y},n} = \check{P}\mathbf{\Sigma}_{\tilde{A},n}\check{P}^H + R_n \tag{7}$$

Using (7), we can obtain $p(\tilde{Y}_n|\mathbf{\Sigma}_{\tilde{A},n})$ in order to update the filter. Note that Q_n in (6) and R_n in (7) are assumed to be Wishart distributed.

Our goal is to estimate the space-time covariance matrix $\mathbf{\Sigma}_{\tilde{B},n}$ of the scatterers. By vectorizing (3),

$$\tilde{B}_n = (D \otimes \mathbb{I}_{N_v})\tilde{A}_n$$

Then,

$$\mathbf{\Sigma}_{\tilde{B},n} = (D \otimes \mathbb{I}_{N_v})\mathbf{\Sigma}_{\tilde{A},n}(D^H \otimes \mathbb{I}_{N_v})$$

Thus, $\mathbf{\Sigma}_{\tilde{B},n}$ can be obtained by the estimation of $\mathbf{\Sigma}_{\tilde{A},n}$.

3. PROPOSED ESTIMATION METHOD

3.1. State dimensionality

After vectorization, the dimensionality of $\tilde{\mathbf{\Sigma}}_{\tilde{A},n}$ is given by $\Xi = (KN_v)^2$. The value of Ξ can be quite large, even if we consider a small number of pulses. For example, if we use $K = 9$ pulses and $M_n = 10$ range bins, then even if we reduce the signal length to $N = 6$, we obtain $N_v = M_n + N - 1 = 15$ and thus $\Xi = 18,225$. Thus, the state dynamic formulation suffers from high dimensionality that prevents direct implementation of particle filtering estimation techniques. We therefore apply the multiple particle filtering method [4] that we summarize next.

3.2. Multiple particle filtering

Particle filtering is a sequential Monte Carlo method that is based on sampling to approximate probability density functions. Particle filtering is often an attractive alternative to Kalman filtering when the system equations are nonlinear or the noise is non-Gaussian [6, 7]. However, in many problems, the dimensionality of the state space may be very large, as occurs when using the covariance matrix as the state in this paper. In this situation, a huge set of particles is needed to provide sufficient support and the computational complexity of the algorithm becomes prohibitive. As discussed in [4], multiple particle filtering can be used to overcome this dimensionality problem.

Suppose the dynamic and measurement models of a system can be expressed as:

$$\begin{aligned}\alpha_n &= f_n(\alpha_{n-1}, \omega_{n-1}) \\ \beta_n &= h_n(\alpha_n, \gamma_n)\end{aligned}$$

where α_n is the d_α-dimensional system state vector at time step n, f_n and h_n are (possibly nonlinear) functions, and ω_n and γ_n are noise vectors. Using the multiple particle filtering approach [4], α_n is divided into L subvectors:

$$\alpha_n = \begin{bmatrix} \alpha_{1,n} \\ \alpha_{2,n} \\ \cdots \\ \alpha_{L,n} \end{bmatrix}$$

Each $\alpha_{l,n},\ l = 1, 2, \cdots, L$, is estimated using a different particle filter. The weights at time step n are updated by:

$$w_{l,n}^{(i)} = w_{l,n-1}^{(i)} \frac{p(\beta_n|\alpha_{l,n}^{(i)}, \tilde{\alpha}_{-l,n})p(\alpha_{l,n}^{(i)}|\alpha_{l,n-1}^{(i)}, \hat{\alpha}_{-l,n-1})}{\pi_l(\alpha_{l,n}^{(i)}|\alpha_{l,n-1}^{(i)}, \hat{\alpha}_{-l,n-1}, \beta_n)}$$

where l indexes the individual particle filters, $l = 1, 2, \cdots, L$, i is the index for the ith particle, $i = 1, 2, \cdots, I$ and $\tilde{\alpha}_{-l,n}$ and $\hat{\alpha}_{-l,n-1}$ are the predicted and estimated values of all the states at time step n except of $\alpha_{l,n}$, respectively.

3.3. Multiple particle filtering of scattering function covariance matrix

After vectorization, (6) can be expressed as

$$\tilde{\mathbf{\Sigma}}_{\tilde{A},n} = (F \otimes F)\tilde{\mathbf{\Sigma}}_{\tilde{A},n-1} + \tilde{Q}_n$$

where the matrix $F \otimes F$ is block diagonal, $F \otimes F = \text{diag}[F_1 \otimes F\ \cdots\ F_K \otimes F]$, and F_k is defined in Section 2.2. The structure of $F \otimes F$ leads to a natural decomposition of the dynamics of the state vector into K independent subsystems:

$$\tilde{\mathbf{\Sigma}}_{\tilde{A},n} = \left[\ \Lambda_{1,n}^T,\ \Lambda_{2,n}^T,\ \ldots\ , \Lambda_{K,n}^T\ \right]^T$$

where each vector $\Lambda_{k,n},\ k = 1, 2, \cdots, K$, has dimension KN_v^2. It is appropriate, then, to invoke the multiple particle filter method with $L = K$ particle filters applied simultaneously, one on each of these K subsystems. For the kth subsystem, the estimation of this segment of the current state is obtained using the dynamic and measurement models

$$\begin{aligned}\Lambda_{k,n} &= (F_k \otimes F)\Lambda_{k,n-1} + \mathbf{V}_{k,n} \\ \tilde{\mathbf{\Sigma}}_{\tilde{Y},n} &= (\check{P} \otimes \check{P})\tilde{\mathbf{\Sigma}}_{\tilde{A},n} + \tilde{\mathbf{R}}_n\end{aligned} \tag{8}$$

The weight for the ith particle is updated according to

$$\frac{p(\mathbf{\Sigma}_{\tilde{Y},n}|\Lambda_{k,n}^i \tilde{\Lambda}_{-k,n}^i)p(\Lambda_{k,n}^{(i)}|\Lambda_{k,n-1}^{(i)}\hat{\Lambda}_{-k,n-1})}{\pi_k(\Lambda_{k,n}^{(i)}|\Lambda_{k,n-1}^{(i)}, \hat{\Lambda}_{-k,n-1}, \tilde{\mathbf{\Sigma}}_{\tilde{Y},n})}$$

where

$$\tilde{\Lambda}_{-k,n} = [\hat{\Lambda}_{1,n}^T, \cdots, \hat{\Lambda}_{k-1,n}^T, \hat{\Lambda}_{k+1,n}^T, \cdots, \hat{\Lambda}_{k,n}^T]^T$$

$$\hat{\Lambda}_{j,n}^T = \sum_{i=1}^{Ms} w_{j,n-1}^{(i)} \Lambda_{j,n}^{(i)}, \quad j \neq k$$

The observation then used has the complex Gaussian distribution with zero-mean and covariance matrix $\Sigma_{\tilde{Y},n}^{(i)}$.

4. SIMULATION

To demonstrate the effectiveness of the estimation method of the space-time covariance matrix, we will investigate the performance of detecting a moving target in heavy sea clutter.

Our simulation model consists of a constant-velocity target that is observed by a single sensor in the presence of simulated sea clutter. Sea clutter is generated using the compound-Gaussian model described in [8]. The range bin size and Doppler resolution are chosen such that from one dwell to another, the scatterer in the lth column, $l = -\frac{K-1}{2}, \cdots, 0, \cdots, \frac{K-1}{2}$, of the scattering function moves l bins. Monte Carlo simulations are used to obtain the space-time covariance matrix of reflectivities matrix, which generated the simulated observations. A total of $K = 9$ pulses are transmitted in each dwell. The validation gate size M_n is chosen to be 11. The waveform transmitted is chosen to be a linear frequency-modulated (LFM) chirp with signal length $N = 6$. Using our method, the space-time covariance matrix of sea clutter can be estimated.

Target returns at different pulses and range bins are added to the observations, and we assume that the target is moving with a constant velocity. The amplitudes of the target returns are sampled from a zero-mean, complex Gaussian process with variance σ^2, which is assumed known and determined by the specified SCR values. The general likelihood ratio test (GLRT) detector [9] is then applied to the observations based on the estimates of the space-time covariance matrix.

The detection performance is shown in Figure 1: as the SCR increases, the detection performance is improved. The performance of the target detection suggests the effectiveness and correctness of the estimation of the space-time covariance matrix of heavy sea clutter, though further comparative evaluation is needed.

5. CONCLUSION

We have proposed an adaptive estimation method for the space-time covariance matrix of sea clutter to support the application of adaptive detection approaches that rely on accurate estimation of this matrix. The method involves vectorization of the equations for the dynamical system model governing the temporal evolution of the clutter matrix followed by a multiple particle filtering approach to deal with the high dimensionality on the formulation. The estimated sea clutter covariance matrix is then applied to the problem of detection of a small target in heavy clutter. The effectiveness of this approach is demonstrated via simulations.

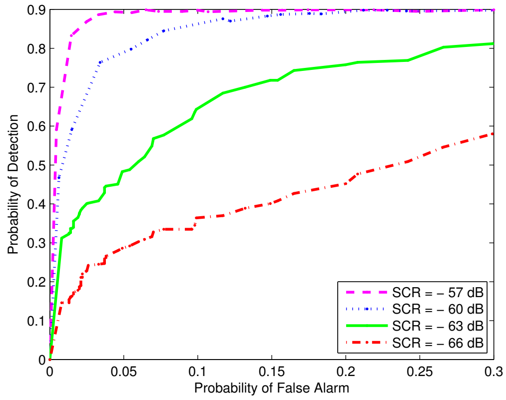

Fig. 1. Receiver operating characteristic curves for a GLRT detector operating at various SCR values (given in dB).

Ultimately, dynamic estimation approaches of this kind are expected to play important roles in the context of waveform-agile radars where the ability to propagate information about clutter characteristics from dwell to dwell supports optimal design/scheduling of transmitted waveforms.

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