CHARACTERIZATION OF SEA CLUTTER BASED ON ESTIMATED SPACE-TIME COVARIANCE MATRIX FROM REAL DATASET

Y. Li, S. P. Sira, B. Moran†*, S. Suvorova*†*, D. Cochran, D. Morrell*‡ *and A. Papandreou-Suppappola**

SenSIP Center, Dept. of Electrical Engineering, Arizona State University, Tempe, AZ 85287, USA
†The University of Melbourne, Parkville, Vic 3010, Australia
‡ Dept. of Engineering, Arizona State University East, Mesa, AZ 85212, USA

ABSTRACT

We propose an estimation method for the space-time covariance matrix of sea clutter to support the application of waveform-agile sensing procedures that rely on accurate estimation of this matrix. The method exploits the special structure of the vectorized states of the scattering function for the dynamical system model governing the temporal evolution of the clutter matrix followed by a multiple particle filtering approach to estimate the covariance matrix and deal with the high dimensionality on the formulation. The effectiveness of the method is demonstrated by estimating the scattering function covariance matrix of both simulated sea clutter data and real sea clutter data from DSTO INGARA radar; and detecting a small moving target embedded in the clutter.

***Index Terms*—** Space-time covariance matrix, scattering function, sea clutter

1. INTRODUCTION

In radar signal processing applications, detection of a moving target in heavy sea clutter is a well-known and challenging problem. Accurate estimation of the space-time covariance matrix of sea clutter is a key ingredient in several possible mechanism for improving detection of small SCR targets. Although approaches to this estimation problem under slowly varying conditions have been proposed [1, 2], situations in which the radar scene varies quickly present significant additional difficulties.

In this work, we consider a rather realistic scenario where the clutter is fast changing instead of slow changing where the effect of Doppler is excluded. Our method introduces a formulation of the space-time representation of the clutter scene in the spreading function domain; this includes the effect of range as well as Doppler changes on the transmitted signal. The dimensionality of the vectorized dynamical system is very high in practical scenarios, so we use the multiple particle filter sequential Monte Carlo method [3] to sequentially estimate the dynamic scene. The special structure of the dynamical equations is used to reduce dimensionality. The effectiveness of our method is demonstrated by detecting a moving target in simulated sea clutter and in real sea data collected by DSTO INGARA radar.

The paper is organized as follows. In Section 2, we provide the system model for the radar scene representation. In Section 3, the multiple particle filtering method is described and used to develop our space-time covariance matrix estimation method. Simulation results illustrating performance of the approach are presented in Section 4.

2. RADAR SCENE SYSTEM REPRESENTATION

2.1. Scattering function

Consider a radar operating at a pulse repetition frequency (PRF) of f_s Hz that transmits a burst of K pulses in a dwell. The return from each burst is sampled at a rate f_b Hz to yield a sequence $y[m,k], m = m_0, m_1, \cdots, m_{M_n-1}, k = 0, 1, \cdots, K-1$, and the validation gate at dwell n consists of M_n range bins. Let the complex reflectivity of the aggregate scatterers on the sea surface at pulse k and range bin m be given by $x[m,k]$. Define B_n to be an $N_v \times K$ matrix consisting of the complex reflectivities $x[m,k]$ with range (*fast time or delay*) increasing down the columns and transmitted pulses (*slow time*) increasing across the rows, which is given by:

$$B_n = \begin{bmatrix} x[0,m_0] & \dots & x[K-1,m_0] \\ x[0,m_1] & \dots & x[K-1,m_1] \\ \vdots & \vdots & \vdots \\ x[0,m_{M_n+N-2}] & \dots & x[K-1,m_{M_n+N-2}] \end{bmatrix}.$$

Let $\mathbf{b}_n = \text{vec}(B_n)$ be the stacked columns of matrix B_n to form a vector of dimension KN_v. The covariance matrix of the reflectivity vector $\mathbf{b}_n$ provides both space and time covariance information. Our objective is to estimate $\mathbf{\Sigma}_{\mathbf{b},n} = \text{E}[\mathbf{b}_n\mathbf{b}_n^H]$, the space-time covariance matrix of the scatterers, where H denotes Hermitian transpose and $\text{E}[\cdot]$ is the expectation operator.

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Using continuous time representation, the spreading function $A(\tau, \nu)$ of a system underdoing time shift τ and frequency shift ν is the Fourier transform (FT) of the system time-varying impulse response. Specifically,

$$A(\tau, \nu) = \int h(t, \tau) e^{-j2\pi\nu t} dt \tag{1}$$

Thus, due to discretization, the elements of matrix A_n, which is called scattering function, are obtained by taking the short-time Fourier transform of (STFT) the elements of B_n in the slow-time direction (corresponding to t in (1)). Specifically,

$$A_n = B_n D \tag{2}$$

where D is the discrete Fourier transform matrix, given by

$$D = \frac{1}{\sqrt{K}} \begin{bmatrix} 1 & \dots & 1 & \dots & 1 \\ W^{-\frac{(K-1)}{2}} & \dots & 1 & \dots & W^{\frac{(K-1)}{2}} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ W^{-\frac{(K-1)(K-1)}{2}} & \dots & 1 & \dots & W^{\frac{(K-1)(K-1)}{2}} \end{bmatrix},$$

and $W = \exp(-j2\pi/K)$. Thus, A_n contains the range-Doppler description of the complex reflectivities in B_n with the zero Doppler in the middle of each row.

2.2. Observation model

Assume that the same signal $s[n], n = 0, 1, \cdots, N-1$ is transmitted repeatedly throughout each burst. Then, the radar return at the kth pulse is modeled by $y[m,k] = \sum_{n=0}^{N-1} x[m-n, k]s[n] + v[m,k]$, where $v[m,k]$ is white Gaussian noise. For notational convenience, we assume K is odd.

The observation can be written in a matrix manner as follows. Let the signal matrix P be an M_n by N_v matrix, where $N_v = M_n + N - 1$, with the signal sequence $s[n]$ beginning at the pth element in the pth row, $p = 1, 2, \cdots, M_n$. Then,

$$Y_n = PB_n + V_n \tag{3}$$

where Y_n is the $M_n \times K$ observation matrix with elements $y[m,k]$ and V_n is the $M_n \times K$ noise matrix at time n with elements $v[m,k]$.

2.3. Dynamic model

We assume a near-constant velocity model for each of the scatterers, including the target. With this assumption, the vectorized scattering matrix $\mathbf{a}_n = \text{vec}(A_n)$ evolves according to the dynamic equation

$$\mathbf{a}_n = F\mathbf{a}_{n-1} + \mathbf{w}_n \tag{4}$$

where $\mathbf{w}_n$ is zero-mean, complex Gaussian noise with covariance Q_n. The matrix F incorporates the movement of the scatterers between dwells and populates the range-Doppler cells that move into the validation gate.

To set up the matrix F, note that the first $(K-1)/2$ columns of matrix A_n represent negative Doppler shifts, the center column represents zero Doppler, and the last $(K-1)/2$ columns represent positive Doppler shifts. These correspond to scatterers moving away from the sensor, not moving, and moving toward the sensor, respectively. In each column of A_n, some scatterers will move out of the validation gate while others will move in. The latter effect requires us to populate the *empty* range-Doppler cells. This is accomplished by using an exponentially weighted sum of the complex reflectivities in the immediate neighborhood of these cells. As defined, F is a $KN_v \times KN_v$ block-diagonal matrix, with F_k ($N_v \times N_v$) presenting the kth block, $k = -(K-1)/2, \dots, -1, 0, 1, \dots, (K-1)/2$. When $k = -(K-1)/2, \dots, -1$, F_k is given by:

$$\begin{bmatrix} 2^{|k|-1}\mathrm{e}^{-|k|\alpha} & \dots & (2^{|k|-1} - |k| + 1)\mathrm{e}^{-(N_v+|k|-1)\alpha} \\ \vdots & \vdots & \vdots \\ \mathrm{e}^{-\alpha} & \dots & \mathrm{e}^{-N_v\alpha} \\ 1 & \dots & 0 \\ 0 & \vdots & \vdots \\ \vdots & \dots & 0 \end{bmatrix},$$

when $k = 1, \dots, (K-1)/2$, the corresponding block is given by:

$$\begin{bmatrix} 0 & \dots & \vdots \\ \vdots & \vdots & 0 \\ 0 & \dots & 1 \\ \mathrm{e}^{-N_v\alpha} & \dots & \mathrm{e}^{-\alpha} \\ \vdots & \vdots & \vdots \\ (2^{k-1} - k + 1)\mathrm{e}^{-(N_v+k-1)\alpha} & \dots & 2^{k-1}\mathrm{e}^{-k\alpha} \end{bmatrix},$$

and when $k = 0$, there is no movement of scatterers and the corresponding block is simply $\mathbb{I}_{N_v}$. The relationship between matrix A_n and B_n and the evolvement of A_n is shown in Figure 1.

2.4. Dynamic and observation models of the scattering function covariance matrix

According to (2) and (3), $Y_n = PA_nD^{-1} + V_n$. Using the property $\text{vec}(GJL) = (L^H \otimes G)\mathbf{j}$ where G, J, L are three arbitrary matrices and $\mathbf{j} = \text{vec}(J)$ [4], the observation can be vectorized as

$$\mathbf{y}_n = (D^{-H} \otimes P)\mathbf{a}_n + \mathbf{v}_n = \check{P}\mathbf{a}_n + \mathbf{v}_n \tag{5}$$

with $\mathbf{y}_n = \text{vec}(Y_n)$ and $\check{P} = D \otimes P$. Then, (4) and (5) give the original dynamic and observation models, respectively. By taking covariance, The dynamical system model can be further described as:

$$\mathbf{\Sigma}_{\mathbf{a},n} = \mathrm{E}[\mathbf{a}_n\mathbf{a}_n^H] = F\mathbf{\Sigma}_{\mathbf{a},n-1}F^H + Q_n, \tag{6}$$

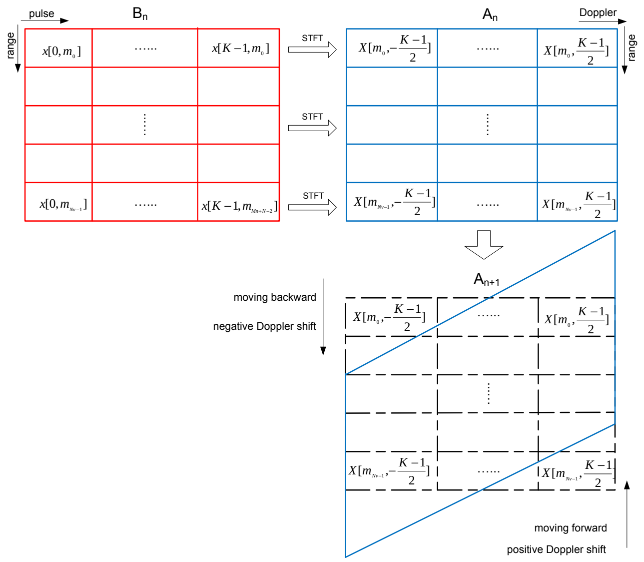

Fig. 1. Relationship between matrix A_n and B_n and the evolution of A_n.

where Q_n is the covariance matrix of $\mathbf{w}_n$. The observation model is then

$$\mathbf{\Sigma}_{\mathbf{y},n} = \check{P}\mathbf{\Sigma}_{\mathbf{a},n}\check{P}^H + R_n, \tag{7}$$

where R_n is the covariance matrix of $\mathbf{v}_n$. Using (7), we can obtain $p(\mathbf{y}_n|\mathbf{\Sigma}_{\mathbf{a},n})$ in order to update the filter. Note that Q_n in (6) and R_n in (7) are assumed to be Wishart distributed to describe the covariance matrix of multinormal samples $\mathbf{w}_n$ and $\mathbf{v}_n$.

Although the objective is to estimate the space-time covariance matrix $\mathbf{\Sigma}_{\mathbf{b},n}$ of the scatterers, it is obvious that, from (2), $B_n = A_n D^{-1} = \mathbb{I}_{N_v} \cdot A_n D^{-1}$, and by vecterization, $\mathbf{b}_n = (D \otimes \mathbb{I}_{N_v})\mathbf{a}_n$, then, $\mathbf{\Sigma}_{\mathbf{b},n} = (D \otimes \mathbb{I}_{N_v})\mathbf{\Sigma}_{\mathbf{a},n}(D^H \otimes \mathbb{I}_{N_v})$,which means $\mathbf{\Sigma}_{\mathbf{b},n}$ can be expressed in terms of $\mathbf{\Sigma}_{\mathbf{a},n}$.

3. PROPOSED ESTIMATION METHOD

3.1. Multiple particle filtering

Particle filtering is a sequential Monte Carlo method that is based on sampling to approximate probability density functions. Particle filtering is often an attractive alternative to Kalman filtering when the system equations are nonlinear or the noise is non-Gaussian [5, 6]. However, in many problems, the dimensionality of the state space may be large such as in our case, where the state space dimensionality is very large since the covariance matrix in (6) is the state. In this situation, a huge set of particles is needed to provide sufficient support and the computational complexity of the algorithm becomes prohibitive. As discussed in [3], multiple particle filtering can be used to overcome this dimensionality problem.

Suppose the dynamic and measurement models of a system can be expressed as: $\boldsymbol{\alpha}_n = f_n(\boldsymbol{\alpha}_{n-1}, \boldsymbol{\omega}_{n-1})$ and $\boldsymbol{\beta}_n = h_n(\boldsymbol{\alpha}_n, \boldsymbol{\gamma}_n)$, where $\boldsymbol{\alpha}_n$ is the $d_{\boldsymbol{\alpha}}$-dimensional system state vector at time step n, f_n and h_n are (possibly nonlinear) functions, and $\boldsymbol{\omega}_n$ and $\boldsymbol{\gamma}_n$ are noise vectors. Using the multiple particle filtering approach in [3], $\boldsymbol{\alpha}_n$ is divided into L subvectors, $\boldsymbol{\alpha}_n = [\boldsymbol{\alpha}_{1,n}^T \ \boldsymbol{\alpha}_{2,n}^T \ \cdots \ \boldsymbol{\alpha}_{L,n}^T]^T$. Each $\boldsymbol{\alpha}_{l,n}$, $l = 1, 2, \cdots, L$, is estimated using a different particle filter. The weights at time step n are updated by:

$$w_{l,n}^{(i)} = w_{l,n-1}^{(i)} \frac{p(\boldsymbol{\beta}_n|\boldsymbol{\alpha}_{l,n}^{(i)}, \tilde{\boldsymbol{\alpha}}_{-l,n})p(\boldsymbol{\alpha}_{l,n}^{(i)}|\boldsymbol{\alpha}_{l,n-1}^{(i)}, \hat{\boldsymbol{\alpha}}_{-l,n-1})}{\pi_l(\boldsymbol{\alpha}_{l,n}^{(i)}|\boldsymbol{\alpha}_{l,n-1}^{(i)}, \hat{\boldsymbol{\alpha}}_{-l,n-1}, \boldsymbol{\beta}_n)}$$

where l indexes the individual particle filters, $l = 1, 2, \cdots, L$, i is the index for the ith particle, $i = 1, 2, \cdots, I$ and $\tilde{\boldsymbol{\alpha}}_{-l,n}$ and $\hat{\boldsymbol{\alpha}}_{-l,n-1}$ are the predicted and estimated values of all the states at time step n except of $\boldsymbol{\alpha}_{l,n}$, respectively.

3.2. Multiple particle filtering for the estimation of scattering function covariance matrix

To take the advantage of Bayesian techniques to solve the system, we vectorize the dynamic and observation models in (6) and (7) as our proposed models:

$$\tilde{\mathbf{\Sigma}}_{\mathbf{a},n} = (F \otimes F)\tilde{\mathbf{\Sigma}}_{\mathbf{a},n-1} + \tilde{\mathbf{Q}}_n, \tag{8}$$

$$\tilde{\mathbf{\Sigma}}_{\mathbf{y},n} = (\check{P} \otimes \check{P})\tilde{\mathbf{\Sigma}}_{\mathbf{a},n} + \tilde{\mathbf{R}}_n, \tag{9}$$

where $\tilde{\mathbf{\Sigma}}_{\mathbf{a},n} = \text{vec}(\mathbf{\Sigma}_{\mathbf{a},n})$, $\tilde{\mathbf{\Sigma}}_{\mathbf{y},n} = \text{vec}(\mathbf{\Sigma}_{\mathbf{y},n})$, $\tilde{\mathbf{Q}}_n = \text{vec}(\mathbf{Q}_n)$ and $\tilde{\mathbf{R}}_n = \text{vec}(\mathbf{R}_n)$. After the vectorization, the dimensionality of $\tilde{\mathbf{\Sigma}}_{\mathbf{a},n}$ is given by $\Xi = (KN_v)^2$. The value of Ξ can be quite large, even if we consider a small number of pulses. For example, if we use $K = 9$ pulses and $M_n = 10$ range bins, then even if we reduce the signal length to $N = 6$, we obtain $N_v = M_n + N - 1 = 15$ and thus $\Xi = 18,225$. Then, the state dynamic formulation suffers from high dimensionality that prevents direct implementation of particle filtering and Kalman filter estimation techniques, even if the transformations are linear. We therefore apply the multiple particle filtering method [3] that we summarize in Section 3.1.

In (8), the evolution matrix $F \otimes F$ is block diagonal,

$$F \otimes F = \begin{bmatrix} F_1 \otimes F & 0 & \cdots & 0 \\ 0 & F_2 \otimes F & \cdots & 0 \\ & & \cdots & \\ 0 & 0 & \cdots & F_K \otimes F \end{bmatrix}$$

and F_k is defined in Section 2.3. The structure of $F \otimes F$ leads to a natural decomposition of the dynamics of the state vector into K independent subsystems:

$$\tilde{\mathbf{\Sigma}}_{\mathbf{a},n} = [\ \mathbf{\Lambda}_{1,n}^T, \ \mathbf{\Lambda}_{2,n}^T, \ \ldots \ , \mathbf{\Lambda}_{K,n}^T \ ]^T$$

where each vector $\mathbf{\Lambda}_{k,n}$, $k = 1, 2, \cdots, K$, has dimension KN_v^2. It is appropriate, then, to invoke the multiple particle filter method with $L = K$ particle filters applied simultaneously, one on each of these K subsystems. For the kth subsystem, the estimation of this segment of the current state is obtained using the dynamic and measurement models

$$\begin{aligned} \mathbf{\Lambda}_{k,n} &= (F_k \otimes F)\mathbf{\Lambda}_{k,n-1} + \mathbf{V}_{k,n} \\ \tilde{\mathbf{\Sigma}}_{\mathbf{y},n} &= (\check{P} \otimes \check{P})\tilde{\mathbf{\Sigma}}_{\mathbf{a},n} + \tilde{\mathbf{R}}_n \end{aligned} \quad (10)$$

The weight for the ith particle is updated according to

$$w_{k,n}^{(i)} \propto w_{k,n-1}^{(i)} \frac{p(\mathbf{\Sigma}_{\mathbf{y},n}|\mathbf{\Lambda}_{k,n}^{i}\tilde{\mathbf{\Lambda}}_{-k,n}^{(i)})p(\mathbf{\Lambda}_{k,n}^{(i)}|\mathbf{\Lambda}_{k,n-1}^{(i)}\hat{\mathbf{\Lambda}}_{-k,n-1})}{\pi_k(\mathbf{\Lambda}_{k,n}^{(i)}|\mathbf{\Lambda}_{k,n-1}^{(i)}, \hat{\mathbf{\Lambda}}_{-k,n-1}, \tilde{\mathbf{\Sigma}}_{\mathbf{y},n})}$$

where $\tilde{\mathbf{\Lambda}}_{-k,n} = [\hat{\mathbf{\Lambda}}_{1,n}^T, \cdots, \hat{\mathbf{\Lambda}}_{k-1,n}^T, \hat{\mathbf{\Lambda}}_{k+1,n}^T, \cdots, \hat{\mathbf{\Lambda}}_{k,n}^T]^T$ and $\hat{\mathbf{\Lambda}}_{j,n}^T = \sum_{i=1}^{Ms} w_{j,n-1}^{(i)} \mathbf{\Lambda}_{j,n}^{(i)}, \quad j \neq k$. The observation, that is used in the kth particle filter, has a complex Gaussian distribution with zero-mean and covariance matrix $\Sigma_{\mathbf{y},n}$.

4. DEMONSTRATION OF EFFECTIVENESS

To demonstrate the effectiveness of the estimation method of the space-time covariance matrix, we will investigate the performance of detecting a moving target in heavy sea clutter. The simulation will be executed on both simulated and real sea clutter data. In both scenarios, a total of $K = 9$ pulses are transmitted in each dwell. The validation gate size M_n is chosen to be 11. The waveform transmitted is chosen to be an LFM chirp with signal length $N = 6$. Accordingly, $L = 9$ particle filters are running simultaneously with 50 particles for each.

4.1. Using simulated sea clutter data

Our simulation model consists of a constant-velocity target that is observed by a single sensor in the presence of simulated sea clutter. Sea clutter is generated using the compound-Gaussian model described in [7]. 200 Monte Carlo simulations are used to obtain the space-time covariance matrix of reflectivities, which generated the simulated observations. Then, using our method, the space-time covariance matrix of sea clutter can be estimated.

Target returns at different pulses and range bins are added to the observations, and we assume that the target is moving with a constant velocity. The amplitudes of the target returns are sampled from a zero-mean, complex Gaussian process with variance σ^2, which is assumed known and determined by the specified SCR values and clutter energy. The GLRT detector [8] is then applied to the observations based on the estimates of the space-time covariance matrix.

The detection performance is shown in Figure 2.

4.2. Using real sea clutter data

We further apply our method to the real dataset which was collected by DSTO INGARA radar in Darwin, Australia in

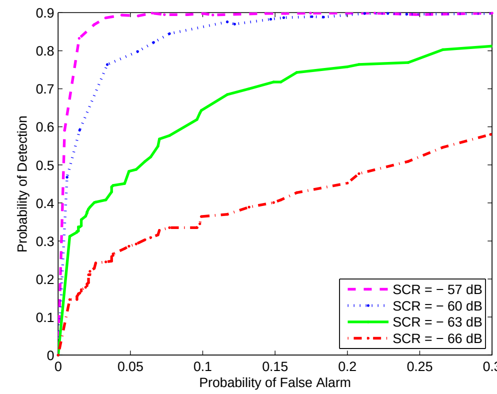

Fig. 2. Receiver operating characteristic curves for a GLRT detector operating at various SCR values (given in dB).

1999. The transmitted signal was an LFM chirp with bandwidth 96 MHz and A/D sampling rate 100 MHz. The carrier frequency is 9.375 GHz, a pulse width 8 us and PRF 500 Hz. The peak power is 5 kW. Wind speed is 13 m/s. The space-time covariance matrix is estimated using our proposed method based on the real data, which is then used in the detection procedure.

To demonstrate the effectiveness of our method, we add returns of a moving target with a constant velocity into the raw data. Based on the estimated covariance matrix, a GLRT detection is then applied. Note that, the clutter strength is obtained using the estimated covariance matrix.

The receiver operating characteristic curves are shown in Figure 3 with different SCR values. The performance is reasonably not as good as the results got from simulated sea clutter.

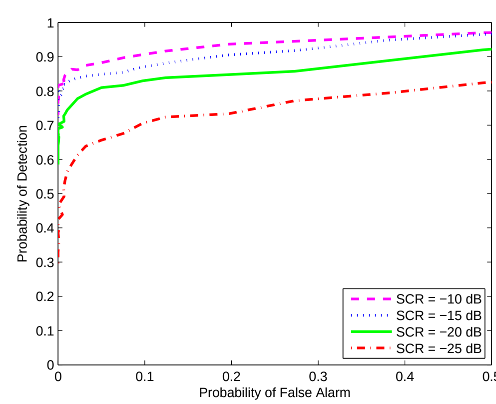

Fig. 3. Receiver operating characteristic curves for a GLRT detector operating at various SCR values (given in dB) using real data collected by DSTO INGARA radar.

5. CONCLUSION

We proposed a sea clutter space-time covariance matrix estimation method using the following two steps. We first vectorize the dynamic system equations to yield a structure in the state matrix. Then we use that structure in a multiple particle filtering approach to reduce the high dimensionality in the formulation. Simulations based on both simulated and real sea clutter data have been undertaken to demonstrate the effectiveness of our method. Note that the proposed dynamic estimation is particularly important in the context of waveform-agile radars where estimation of the clutter characteristics in one dwell can be used in the design of the transmitted waveform in the next dwell.

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