Abstract

We extend the class of phase coded waveforms to a new class of signals coded by finite Gabor systems leading to some ambiguity function performances, which cannot be reached by the class of phase coded waveforms alone.

1. Introduction

1.1 Signals:

\[ u(t) = \sum_{n=0}^{M-1} c[m]g(t-n) \]

\( g \) is a square integrable function and vanishing outside of \([0, 1)\) and \( c = \{c[m]\}_{m=0}^{M-1} \) is a code of length \( M \).

1.2 The ambiguity function:

\[ \Lambda_\gamma(t, \tau) = \int \left| \sum_{n=0}^{M-1} c[m] \exp(\frac{j 2\pi n t}{M}) \exp(j 2\pi \gamma m \tau) \right|^2 \, dt \]

1.3 Peak sidelobe levels

- The peak sidelobe level \( PSL_{\gamma} \) is the second greatest maximum of: \( \gamma = \int \left| \sum_{n=0}^{M-1} c[m] \exp(\frac{j 2\pi n t}{M}) \exp(j 2\pi \gamma m \tau) \right|^2 \, dt \)
- The peak sidelobe level at a nonzero \( \gamma \) \( PSL_{\gamma}(\gamma) \) is the maximum of: \( t \rightarrow \Lambda_{\gamma}(t, \gamma) \).

2. PCW

2.1 Phase coded waveforms:

\[ w(t) = \sum_{n=0}^{M-1} \left( \sum_{m=0}^{N-1} c[m] \text{rect}(t-n/M) \right) \exp(2\pi j n \tau) \exp(2\pi j m \gamma t) \]

2.2 Examples of codes:

- Chu codes: \( c[m] = e^{j m \beta} \)
- P4 codes: \( c[m] = e^{j m \beta} \exp(j \pi m^2/M) \)

2.3 The ambiguity function:

\[ \Lambda_\gamma(t, \tau) = \int \left| \sum_{m=0}^{N-1} \sum_{n=0}^{M-1} c[m] \exp(2\pi j n \tau) \exp(2\pi j m \gamma t) \right|^2 \, dt \]

3. SCFGS

3.1 Finite Gabor systems:

\[ G(t, m) = e^{j 2\pi m \gamma t} \sum_{n=0}^{M-1} c[m] \exp(2\pi j n \tau) \exp(2\pi j m \gamma t) \]

3.2 Signals coded by finite Gabor systems:

\[ v(t) = \sum_{n=0}^{M-1} \sum_{m=0}^{N-1} c[m] \exp(2\pi j n \tau) \exp(2\pi j m \gamma t) g(t-n) \]

3.3 The ambiguity function:

\[ \Lambda_\gamma(t, \tau) = \int \left| \sum_{m=0}^{N-1} \sum_{n=0}^{M-1} c[m] \exp(2\pi j n \tau) \exp(2\pi j m \gamma t) \right|^2 \, dt \]

4. Application

- Find a signal \( u(t, \gamma) \) such that \( |\Lambda_\gamma(t, \gamma)| < 0.24 \), for \( -1 < t < 1 \) and \( 0.01 < \gamma < 1.5 \).
- For every PCW \( w \), \( 0 < PSL_{\gamma}(\gamma) < 0.72 PSL_{\gamma}(1) \)
- PCW do not lead to an ambiguity function with a near thumbtack shape.
- Asymptotic behavior tending to \( 1/\pi \).

5. DSLR

- Domain of sidelobe level reductions for \( T_{\gamma}(\gamma) \)

6. Detection-tracking

- Probability of false alarm, and \( SNR \) the signal to noise ratio.

7. Resolution Cells

\[ SNR = 50, P_{Dq} = 10^{-3}, \text{ and } P_{Dq} > 0.8 \]

References


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