ADAPTIVE PATH DESIGN OF A MOVING RADAR

Martin Hurtado and Arye Nehorai

Department of Electrical and Systems Engineering
Washington University in St. Louis
One Brookings Drive, St. Louis, MO 63130, USA

ABSTRACT

We consider the problem of designing the trajectory of a radar mounted on a moving platform. We develop an adaptive algorithm that, at each time step, optimally selects the radar path in response to the estimated and predicted target parameters to improve the tracking accuracy. We derive our approach under a framework of sequential Bayesian filtering. We apply a sequential Monte Carlo method (particle filter) to implement the filter for the case of nonlinear measurement models. We design the criterion for the path optimization based on the posterior Cramér-Rao bound.

***Index Terms*—** Adaptive path design, target tracking, sequential Bayesian filtering, posterior Cramér-Rao bound

1. INTRODUCTION

Recent advances in technology have enabled the mounting of radar systems on robots such as unmanned ground or aerial vehicles (UGAs or UAVs) for remote sensing, surveillance, and tracking applications. Developing algorithms for path planning is a significant part of designing these autonomous systems. In planning the radar trajectory, the goal is to determine the best possible path in order to accomplish a specific task optimally, e.g., intercepting the target. Finding the optimal path presents several challenges. First, the target parameters are measured through nonlinear processes corrupted by noise and interference. Second, the target parameters are not steady. Third, the algorithms have to be efficient for implementing in real-time.

Recent work in this area applied the trace of the Cramér-Rao bound (CRB) on the target estimates as the criterion for selecting the optimal path [1]. In [2], the same problem was solved by maximizing the mutual information between the target trajectory and system measurements. In [3], the design of the optimal observer trajectory was addressed using the framework of Markov decision process. However, these procedures considered the problem of estimating the entire target trajectory and designing the entire radar path after processing a complete set of measurements. In previous work we move the sensors in opposite direction to the gradient of the CRB for localizing stationary chemical sources [4].

In this paper, we develop a sequential algorithm for designing the optimal path of a moving radar. At each time step, our proposed method adaptively selects the radar trajectory based on the estimated and predicted target parameters to improve the tracking accuracy for the next time step. We use a sequential Bayesian framework to solve jointly the problem of target tracking and radar path design. We implement the Bayesian filter using a sequential Monte Carlo method (particle filter) that is suitable for nonlinear and non-Gaussian models. We also propose a new criterion based on posterior Cramér-Rao bound to optimally select the radar trajectory. In addition, we develop an efficient method to evaluate this cost function. We note that this research work extends our work on adaptive radar waveform design [5].

2. STATE AND MEASUREMENT MODELS

In this section we present the dynamic state model that describes the target position and velocity. We also introduce the nonlinear measurement model and define the statistical assumptions of the process and measurement noise.

2.1. Dynamic State Model

For the target tracking problem, we define the target state relative to the radar state as

$$\boldsymbol{x}_k = \boldsymbol{x}_{\mathrm{t}k} - \boldsymbol{x}_{\mathrm{r}k} = [x\ y\ z\ \dot{x}\ \dot{y}\ \dot{z}]^{\mathrm{T}}, \tag{1}$$

where $\boldsymbol{x}_{\mathrm{t}k}$ and $\boldsymbol{x}_{\mathrm{r}k}$ are the target and radar state at time step k. Then, x, y, z and $\dot{x}$, $\dot{y}$, $\dot{z}$ represent the target position and velocity relative to the radar in a Cartesian coordinate system.

We assume a target moving at a nearly constant velocity. Then, the dynamic state model is given by

$$\boldsymbol{x}_k = F\boldsymbol{x}_{k-1} + \boldsymbol{v}_{k-1}, \tag{2}$$

where F is the transition matrix defined as

$$F = \begin{bmatrix} I_3 & T_{\mathrm{PRI}} I_3 \\ \mathbf{0} & I_3 \end{bmatrix}, \tag{3}$$

This work was supported by the DARPA Grant No. HR0011-07-1-0036 and the Department of Defense under the Air Force Office of Scientific Research MURI Grant FA9550-05-0443.

where I_n is the identity matrix of size n and T_{PRI} is the pulse repetition interval (PRI). The process noise $\boldsymbol{v}_k$ represents the uncertainty about the state model and is assumed to be zero mean Gaussian distributed with known covariance Σ_v.

2.2. Measurement Model

We consider a mono-static radar transmitting narrow band signals that illuminates a point target located in the far field region. The target is characterized by azimuth ϕ, elevation ψ, range r, and Doppler shift ω_{D}. Then, the output of the array of M sensors receiving the echoes from the target can be expressed as

$$\boldsymbol{y}(t) = \boldsymbol{p}(\phi, \psi)s(t-\tau)e^{j\omega_{\mathrm{D}}t} + \boldsymbol{e}(t), \quad t = t_1, \ldots, t_N, \quad (4)$$

where $\boldsymbol{p}(\phi,\psi) = [e^{j2\pi \boldsymbol{u}^{\mathrm{T}}\boldsymbol{r}_1/\lambda}, \ldots, e^{j2\pi \boldsymbol{u}^{\mathrm{T}}\boldsymbol{r}_M/\lambda}]^{\mathrm{T}}$ is the array response vector, $\boldsymbol{u} = [\cos\phi\cos\psi,\ \sin\phi\cos\psi,\ \sin\psi]^{\mathrm{T}}$ is the planewave direction of arrival, $\boldsymbol{r}_m$ is the position of the m-th sensor ($m = 1, \ldots, M$), λ is the wavelength of the carrier signal, $s(t)$ is the transmitted waveform, $\tau = 2r/c$ is the signal delay, and c is the propagation velocity of the signal. The vector $\boldsymbol{e}(t)$ is the additive noise corrupting the measurements; and it represents the thermal noise at the sensors and the interference from the environment. We assume that $\boldsymbol{e}(t)$ is a zero mean white Gaussian process with known covariance $\sigma^2 I_M$. N denotes the number of samples during the observation time T_{PRI}.

It can be verified that the relationship between the target parameters $[\phi,\ \psi,\ r,\ \omega_{\mathrm{D}}]$ and the state vector $\boldsymbol{x}$ is given by

$$\phi = \arctan(y/x) \qquad \psi = \arctan(z/\sqrt{x^2+y^2})$$
$$r = \sqrt{x^2+y^2+z^2} \qquad \omega_{\mathrm{D}} = \frac{2\pi}{\lambda r}(\dot{x}x + \dot{y}y + \dot{z}z)$$

Therefore, the measurement model is a nonlinear function of the state parameters:

$$\boldsymbol{y}_k(t) = \boldsymbol{h}(t;\boldsymbol{x}_k) + \boldsymbol{e}_k(t), \quad t = t_1, \ldots, t_N, \quad (5)$$

where $\boldsymbol{h}(t;\boldsymbol{x}) = \boldsymbol{p}(\phi,\psi)s(t-\tau)e^{j\omega_{\mathrm{D}}t}$. When we lump $\{\boldsymbol{y}_k(t), t = t_1, \ldots, t_N\}$ together into a vector, we obtain the measurement model as

$$\begin{aligned}\boldsymbol{y}_k &= \begin{bmatrix}\boldsymbol{y}_k(t_1)\\ \vdots \\ \boldsymbol{y}_k(t_N)\end{bmatrix} = \begin{bmatrix}\boldsymbol{h}(t_1;\boldsymbol{x}_k)\\ \vdots \\ \boldsymbol{h}(t_N;\boldsymbol{x}_k)\end{bmatrix} + \begin{bmatrix}\boldsymbol{e}_k(t_1)\\ \vdots \\ \boldsymbol{e}_k(t_N)\end{bmatrix} \\ &= \boldsymbol{h}(\boldsymbol{x}_k) + \boldsymbol{e}_k. \end{aligned} \quad (6)$$

3. SEQUENTIAL BAYESIAN TRACKING

The problem of target tracking consists of recursively estimating the state $\boldsymbol{x}_k$ based on the measurements $\boldsymbol{y}_{1:k}$ up to time k. Then, it is required to compute the posterior probability density function (pdf) $p(\boldsymbol{x}_k \mid \boldsymbol{y}_{1:k})$. Under the Bayesian inference framework, we can obtain recursive formulas to calculate this pdf when the new measurements $\boldsymbol{y}_k$ are available [6]. However, this recursive procedure cannot be determined analytically when the models are nonlinear or non-Gaussian. Under these conditions, numerical methods are used to find approximate solutions to the optimal Bayesian filter.

3.1. Particle Filter

We apply particle filter to implement the former Bayesian algorithm for our tracking problem characterized by a nonlinear measurement model. The particle filter algorithm is a sequential Monte Carlo method in which the key idea is to represent the required posterior density function by a set of random samples with associated weights. Then, the parameters of interest are estimated based on these samples and weights.

Let $\{\boldsymbol{x}_k^{(i)}\}$ be a set of random points with associated weights $\{w_k^{(i)}\}$ for $i = 1, \ldots, N_{\mathrm{s}}$, where N_{s} is the number of particles. Then, the posterior density $p(\boldsymbol{x}_k \mid \boldsymbol{y}_{1:k})$ can be approximated as [6]

$$p(\boldsymbol{x}_k \mid \boldsymbol{y}_{1:k}) \approx \sum_{i=1}^{N_{\mathrm{s}}} w_k^{(i)} \delta(\boldsymbol{x}_k - \boldsymbol{x}_k^{(i)}), \quad (7)$$

where the weights are normalized such that $\sum_i w_k^{(i)} = 1$. The samples $\boldsymbol{x}_k^{(i)}$ are easily generated from a proposal distribution $q(\cdot)$ called importance density function. The weights are computed using the principle of importance sampling and depend on the selected importance density. For a sequential filtering case, we can choose an importance density $q(\cdot)$ such that we obtain a recursive weight equation as [6]

$$w_k^{(i)} \propto w_{k-1}^{(i)} \frac{p(\boldsymbol{y}_k \mid \boldsymbol{x}_k^{(i)}) p(\boldsymbol{x}_k^{(i)} \mid \boldsymbol{x}_{k-1}^{(i)})}{q(\boldsymbol{x}_k^{(i)} \mid \boldsymbol{x}_{k-1}^{(i)}, \boldsymbol{y}_k)}. \quad (8)$$

In our particle filter, we choose the importance density to be the transitional prior: $q(\boldsymbol{x}_k \mid \boldsymbol{x}_{k-1}, \boldsymbol{y}_k) = p(\boldsymbol{x}_k \mid \boldsymbol{x}_{k-1})$.

4. ADAPTIVE RADAR PATH DESIGN

In this section we present an algorithm for the optimal design of the radar trajectory. It is combined with the above target tracking algorithm and forms an adaptive path planning approach. In order to solve the optimization at time step k, we create a criterion that evaluates the system performance for the predicted target at time step $k+1$ when employing specific radar state parameters. Then, we select those radar parameters that optimize this performance criterion.

We use the posterior Cramér-Rao bound (PCRB) as a performance criterion for selecting the optimal radar path. The PCRB provides a lower bound on the mean square error (MSE) matrix for the random state vector.

4.1. Path Selection Based on PCRB

Considering our target tracking problem at time step k, we want to estimate a state trajectory $\boldsymbol{x}_{0:k}$ using the measurements $\boldsymbol{y}_{1:k}$. Then, the Bayesian information matrix (BIM) of the trajectory, whose inverse is the PCRB, is defined as

$$\bar{J}_k \triangleq \mathrm{E}_{\boldsymbol{y}_{1:k},\, \boldsymbol{x}_{0:k}} \left[-\triangle_{\boldsymbol{x}_{0:k}}^{\boldsymbol{x}_{0:k}} \log p(\boldsymbol{y}_{1:k},\ \boldsymbol{x}_{0:k}) \right], \qquad (9)$$

where $\triangle_{\alpha}^{\beta}$ denotes the second order derivative with respect to α and β. The lower right $n_x \times n_x$ block ($n_x = \dim(\boldsymbol{x})$) of $\bar{J}_k^{\,-1}$ is the PCRB on the estimation of $\boldsymbol{x}_k$; and its inverse is the BIM on the estimation of $\boldsymbol{x}_k$, denoted as J_k.

To derive the optimal path selection criterion, we adopt the recursive equation in [7] to update BIM J_{k+1}. For the particular case of a linear state model with additive Gaussian noise, this recursive BIM can be written as (see [7])

$$J_{k+1} = \left[\Sigma_v + F J_k(\boldsymbol{\theta}_k)^{-1} F^{\mathrm{T}} \right]^{-1} + \Gamma_{k+1}(\boldsymbol{\theta}_{k+1}), \qquad (10)$$

where $\boldsymbol{\theta}_k$ and $\boldsymbol{\theta}_{k+1}$ are the radar parameters at the time step k and $k+1$, respectively, and

$$\Gamma_{k+1} = \mathrm{E}_{\boldsymbol{y}_{k+1},\, \boldsymbol{x}_{k+1}} \left[-\triangle_{\boldsymbol{x}_{k+1}}^{\boldsymbol{x}_{k+1}} \log p(\boldsymbol{y}_{k+1} \mid \boldsymbol{x}_{k+1}) \right]. \qquad (11)$$

Note that Γ_{k+1} is calculated by averaging over all possible values of $\boldsymbol{y}_{k+1}$. That means we do not need to know the specific values of the next measurements to calculate J_{k+1}.

In our sequential path design algorithm, we attempt to minimize the error on the target state estimation. Then, we propose the weighted trace of the predicted PCRB as a criterion for selecting the optimal radar trajectory:

$$\boldsymbol{\theta}_{k+1}^{*} = \arg \min_{\boldsymbol{\theta}_{k+1} \in \boldsymbol{\Theta}} \mathrm{Tr}\{\Pi J_{k+1}^{-1}(\boldsymbol{\theta}_{k+1})\} \qquad (12)$$

where $\boldsymbol{\Theta}$ denotes a set of allowed values for $\boldsymbol{\theta}_{k+1}$ or a library of all possible radar trajectories, and Π is a weighting matrix used to equalize the magnitude of different parameters in the state vector.

4.2. Computation of the Criterion Function

The proposed criterion function depends not only on the information provided by the state model but also by the measurement model, through the term Γ_{k+1}. To compute this matrix, in general, the expectation in (11) has no closed-form analytical solution and must be solved numerically; for example using Monte Carlo integration. However, this method is computationally intensive and time demanding because the integrand of Γ_{k+1} must be evaluated for every particle. Therefore, we propose a suboptimal method.

It can be verified that Γ_{k+1} can be calculated as

$$\Gamma_{k+1} = \mathrm{E}_{\boldsymbol{x}_{k+1}}[\Xi_{k+1}] \qquad (13)$$

$$\Xi_{k+1} = \mathrm{E}_{\boldsymbol{y}_{k+1}|\boldsymbol{x}_{k+1}} \left[-\triangle_{\boldsymbol{x}_{k+1}}^{\boldsymbol{x}_{k+1}} \log p(\boldsymbol{y}_{k+1} \mid \boldsymbol{x}_{k+1}) \right] \qquad (14)$$

where Ξ_{k+1} is the standard Fisher information matrix (FIM) for estimating $\boldsymbol{x}_{k+1}$ based on $\boldsymbol{y}_{k+1}$. We note that Ξ_{k+1} can be computed analytically for Gaussian measurement models. Then, we propose to replace the matrix Γ_{k+1} by Ξ_{k+1} evaluated at the predicted state. Therefore, the suboptimal criterion can be computed by as follows:

- For $i = 1, \ldots, N_{\mathrm{s}}$, draw samples $\boldsymbol{x}_{k+1}^{(i)} \sim p(\boldsymbol{x}_{k+1} \mid \boldsymbol{x}_{k}^{(i)})$.
- The expectation of the predicted state is approximated as

$$\hat{\boldsymbol{x}}_{k+1} \approx \sum_{i=1}^{N_{\mathrm{s}}} w_k^{(i)} \boldsymbol{x}_{k+1}^{(i)}.$$

- Replace Γ_{k+1} by $\Xi_{k+1}(\hat{\boldsymbol{x}}_{k+1})$ in (10).

This suboptimal criterion significatively reduces computation time at the expense of accuracy in computing the integral in (11). This procedure has the advantage that it can be merged into the sequential Monte Carlo method for target tracking.

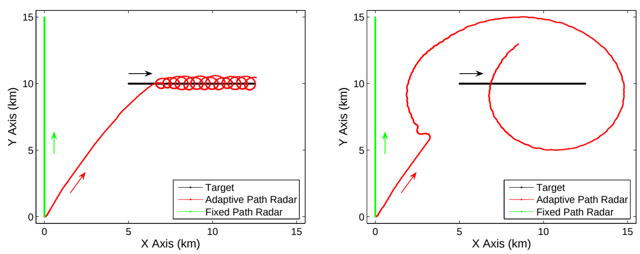
5. NUMERICAL EXAMPLES

In this section, we use numerical examples to illustrate the behavior of the developed algorithm. We consider the problem of tracking a moving target in a 2D environment (XY plane). The target moves East (to the right) at a speed of 25m/s on a straight trajectory. The radar transmits linear frequency modulated (LFM) pulses with Gaussian envelop of 1MHz bandwidth. The carrier frequency is 10GHz and pulses are transmitted with a PRI of 0.5s. The receiver array consists of a uniform circular array of six sensor separated by 0.5λ. For our simulations, we assume that the signal-to-noise ratio (SNR) is 10dB at the receiver, independent of the target range (neglecting the attenuation of the signal due to free-space propagation).

Through the examples we demonstrate the advantages of the adaptive path scheme compared with a radar system that follows a fixed path. This radar travels North (upward) at a speed of 50m/s on a straight trajectory. The results reported in this section correspond to an average over 100 Monte Carlo simulations.

Example 1. In this example, we assume that the radar follows some simple kinematic constraints. We assume that the radar can maneuver by selecting its velocity vector at each time step k. However, its speed must remains in between 50m/s and 200m/s, and the speed can be increased or reduced in 10% at each time step. The radar can modify its bearing angles in steps of 5°, in between -10° and 10°, with respect to its current direction. Fig. 1a shows an example of the trajectory of the radar for fixed and adaptive schemes, as well as the trajectory of the target. We note that the adaptive radar moves toward the target, gradually decreasing the range. Then, the radar circles around the target remaining at a short distance, because it is not allowed to stop or make sharp turns. Fig. 2 shows that the adaptive radar improves the tracking performance because it can resolve better the x and y positions of the target for a shorter range.

Example 2. In this new example, we add some tactical constraints to the problem of selecting the optimal radar trajectory. We assume that the radar cannot be closer to the target than 5km. To implement this constraint, we penalize those radar actions which do not satisfy the range condition. Fig. 1b shows an example of the target and radar trajectories. The adaptive radar behaves similarly to the previous example, but it circles the target from a further distance. Fig. 3 shows the tracking performance of the adaptive path radar in comparison to the fixed path system.



(a) (b)

Fig. 1. Examples of target and radar trajectories. Adaptive radar path: (a) with kinematic constraints and (b) with kinematic and tactical constraints.

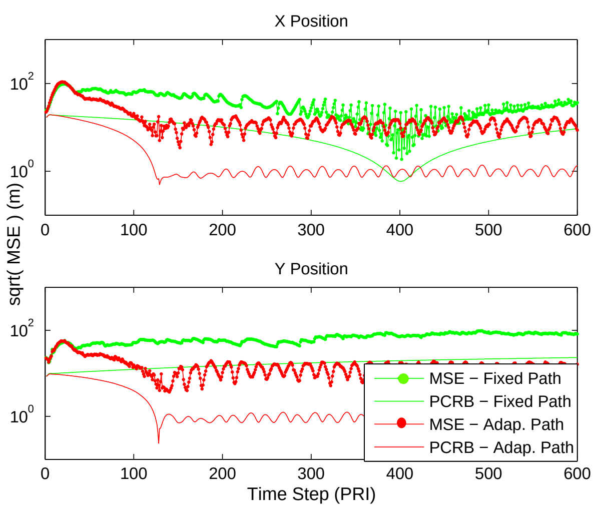


Fig. 2. Location errors and bounds on the error estimates as a function of time for the case of adaptive radar path with kinematic constraints.

6. CONCLUSIONS

We developed an adaptive algorithm for designing the optimal trajectory of a moving radar. Our algorithm selects the optimal path based on the estimated and predicted target in order to improve the system tracking performance. We proposed a sequential Bayesian framework for solving the target tracking

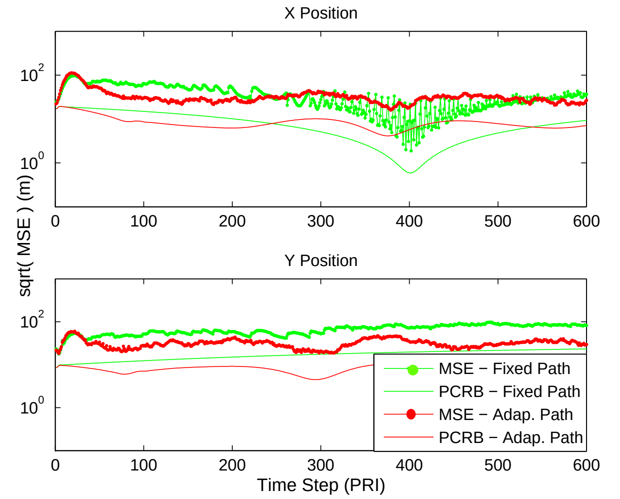


Fig. 3. Location errors and bounds on the error estimates as a function of time for the case of adaptive radar path with kinematic and tactical constraints.

problem. For the path optimization, we proposed a cost function based on the posterior Cramér-Rao bound and applied a Monte Carlo method to compute it. Numerical examples demonstrated the advantages of the adaptive path scheme.

7. REFERENCES

[1] J. P. Helferty and D. R. Mudgett, “Optimal observer trajectories for bearings only tracking by minimizing the trace of the Cramer-Rao lower bound,” in *Proc. IEEE Conf. Decision Control*, Dec. 1993, vol. 1, pp. 936-939.

[2] A. Logothetis, A. Isaksson, and R. J. Evans, “An information theoretic approach to observer path design for bearings-only tracking,” in *Proc. IEEE Conf. Decision Control*, Dec. 1997, vol. 4, pp. 3132-3137.

[3] O. Tremois and J. P. Le Cadre, “Optimal observer trajectory in bearings-only tracking for manoeuvring sources,” *IEE Proc. Radar, Sonar and Navig.*, vol. 146, pp. 31-39, Feb. 1999.

[4] B. Porat and A. Nehorai, “Localizing vapor-emitting sources by moving sensors,” *IEEE Trans. Signal Process.*, vol. 44, pp. 1018-1021, Apr. 1996.

[5] M. Hurtado, T. Zhao, and A. Nehorai, “Adaptive polarized waveform design for target tracking based on sequential Bayesian inference,” *IEEE Trans. Signal Process.*, vol. 56, pp. 1120-1133, Mar. 2008.

[6] M. S. Arulampalam, S. Maskell, N. Gordon, and T. Clapp, “A tutorial on particle filters for online nonlinear/non-Gaussian Bayesian tracking,” *IEEE Trans. Signal Process.*, vol. 50, pp. 174-188, Feb. 2002.

[7] P. Tichavský, C.H. Muravchik, and A. Nehorai, “Posterior Cramér-Rao bounds for discrete-time nonlinear filtering,” *IEEE Trans. Signal Process.*, vol. 46, pp. 1386-1396, May 1998.