Improved Space-Time Coding for Multiple Antenna Multicasting

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***Abstract*— Multicasting is the general method of conveying the same information to multiple users. For multicasting applications over multiple-input multiple-output (MIMO) channels, it is of great importance to ensure that all users experience an adequate level of performance while utilizing the transmit diversity available in MIMO channels. One technique, called max-min beamforming, provides a way to increase the received signal-to-noise ratio (SNR) of the worst-case user. However, due to its limited degrees of freedom, this method suffers severe performance degradation when the number of users increases. Alternatively, space-time codes can be employed to transmit data simultaneously to all users. By introducing additional degrees of freedom in the time domain, open-loop orthogonal space-time block coding (OSTBC) schemes achieve better performance than max-min beamforming when the number of users is large. In this paper, a precoded OSTBC scheme is proposed that optimizes the performance of the worst-case user. By pre-multiplying the OSTBC codeword by a precoding matrix, the transmitter can adapt to current channel conditions which results in a higher worst-case receive SNR. Simulation results demonstrate that the proposed scheme performs universally better than open-loop OSTBC schemes and outperforms the max-min beamforming scheme for a large number of users.**

I. Introduction

With the widespread deployment of wireless networks and the fast-improving capabilities of mobile devices, there is an increasing demand for wireless applications employing group communication among mobile users. These applications include group-oriented mobile commerce, military command and control, distance education, and intelligent transportation systems [1]. Compared to unicasting and broadcasting, multicasting consumes less network resources to transmit information to multiple destinations. In internet environment, it is usually implemented in data-link layer and/or the network layer for fixed users. However, due to the different channel characteristics of wired links and wireless links, several issues should be addressed before one can implement multicasting in wireless environments [1]. Among these, the requirement for reliable transmission are usually considered the most important. To achieve this, one has to deal with the performance degradation caused by multi-path fading which is common to all wireless channels.

This work was supported in part by Spectral Systems, Inc., the Department of Defense (administered by the United States Air Force Office of Scientific Research) under Grant No. AFOSR F49620-03-1-0149, the SBC Foundation, and the National Science Foundation under Grant CCF-0513916.

In point-to-point communication systems, multiple-input multiple-output (MIMO) techniques are widely used to combat channel fading. It is well established that MIMO systems provide numerous improvements over single antenna systems [2]. This improvement can be two-fold, namely, diversity and spatial multiplexing [3]. In most communication systems, the receiver is able to obtain the channel state information (CSI). This can be achieved by embedding pilot symbols into the transmitted signal. In some cases, the transmitter can also obtain CSI through the use of feedback from the receiver in frequency division duplexing (FDD) systems or direct channel estimation in time division duplexing (TDD) systems. Most of the point-to-point MIMO work presumes that the receiver knows the exact channel state information (CSI). However, a variety of different scenarios have been considered for the extent of CSI knownledge at the transmitter side. At one extreme is the case of no CSI at the transmitter. In this case, open-loop orthogonal space-time block coding (OSTBC) [4], [5] is a typical method to achieve optimal diversity gain, in which the transmitted symbol is encoded into a matrix form such that the transmitted signals are orthogonal in both space and time domain. This in turn results in a simple decoding algorithm. At the other extreme is the case of full CSI at the transmitter. One typical approach is to maximize the instantaneous signal to noise (SNR) value at the receiver through beamforming (BF) which is based on the singular value decomposition (SVD) of the channel matrix.

For MIMO multicasting, the OSTBC method is a natural candidate as it does not require any CSI and can transmit signals to each user as if other users did not exist. The benefits of channel adaptation in point-to-point MIMO communication systems motivate the problem of finding ways to efficiently incorporate transmit-side CSI into system design. In [9], a beamforming method is presented to optimize the SNR averaged over all receivers. However, the drawback of this scheme is the unfair performance among the receivers, i.e. users with poor channel condition may be allocated with unacceptably low SNR. Practical systems, e.g. future digital video/audio/data applications, often require that each user is able to meet a minimum quality of service. Hence, it is natural to optimize the SNR of the worst receiver. In [10], a max-min beamforming scheme that aims to maximize the worst-case SNR is proposed. When the number of users is small, this method provides some benefit over using OSTBCs. However, as the number of users grows, the performance of this scheme degrades very fast and at some point, it performs considerably worse than the open-loop OSTBC method. In [11], two multicast beamforming problems with different design criteria are considered. It is proved that both of the optimization problems are NP-hard. It is further proposed to use semidefinite relaxation (SDR) techniques to obtain the suboptimal solution.

In this paper, a MIMO multicasting scheme based on OSTBC precoding is presented. By pre-multiplying the open-loop OSTBC code by a precoding matrix, the transmitter can adapt itself with current CSI to improve the receive SNR. Similar to [10], [11], we aim to maximize the receive SNR of the worst-case user. The proposed approach can achieve performance that is universally better than the OSTBC method. The scheme outperforms the max-min beamforming when the number of users exceeds a moderate value. In this sense, the proposed scheme can be considered as a trade-off between max-min beamforming and OSTBC. Another benefit of this scheme is that it is released from the burden of switching between beamforming and OSTBC as suggested in [10].

The organization of this paper is as follows. Section II gives the system model that will be used throughout the paper. A brief review and some insight into the max-min beamforming method is presented in Section III. In Section IV, a precoded OSTBC scheme is discussed. Simulation results is given in Section V. Section VI concludes the paper.

In this paper, we use lowercase boldface letters to denote vectors and uppercase bold letters to denote matrices. $\|\cdot\|$ denotes the norm of a vector, $(\cdot)^T$ denotes matrix transposition, $(\cdot)^H$ denotes the matrix Hermitian transpose, $E\cdot$ denotes expectation, and $|\cdot|$ denotes the absolute value.

II. System model

In this paper, we consider a narrowband multicast system with one transmitter and N users. The transmitter is equipped with M antennas and each user only has one antenna. However, the results in this paper is readily extended to the case where each user has multiple antennas.

The received signal is represented as

$$y_i(t) = \sum_{j=1}^{M} h_{i,j} x_j(t) + n_i(t) \tag{1}$$

where $x_j(t)$ and $y_i(t)$ denote the complex baseband signal transmitted from the jth transmit antenna, and the received signal at the ith user, at time instant t, respectively, for $j = 1, 2, \cdots, M$ and $i = 1, 2, \cdots, N$. $h_{i,j}$ denotes the complex gain of the Rayleigh fading channel from the jth transmit antenna to user i. We assume that the transmit antennas and users are sufficiently apart such that $h_{i,j}$ can be modelled as i.i.d. zero-mean circularly symmetric complex Gaussian random variables. $n_i(t)$ denotes the noise at ith receiver at time instant t, which is modelled as i.i.d circularly symmetric complex Gaussian noise with zero mean and variance σ_N^2 for $i = 1, 2, \cdots, N$.

Two signalling schemes are considered in this study, namely, beamforming and space-time blocking coding (STBC). When beamforming is employed, we can drop the time index, and reformulate (1) in matrix form given by

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n} \tag{2}$$

where $\boldsymbol{y} = [y_1, \cdots, y_N]^T$ and $\boldsymbol{n} = [n_1, \cdots, n_N]^T$. $\boldsymbol{H} = [\boldsymbol{h}_1^T, \boldsymbol{h}_2^T, \cdots, \boldsymbol{h}_N^T]^T$ is the channel matrix, where $\boldsymbol{h}_i = [\boldsymbol{h}_{i,1}, \boldsymbol{h}_{i,2}, \cdots, \boldsymbol{h}_{i,j}]$ is the channel matrix for user i. Since we only consider the case where each user has only one antenna, $\boldsymbol{h}_i$ is a vector. Denoting the beamforming vector as $\boldsymbol{v}$, $\boldsymbol{x}$ can be written as

$$\boldsymbol{x} = \sqrt{P}\boldsymbol{v}s$$

where s is the transmitted signal, P is the total average transmit power, and $\boldsymbol{v}^H\boldsymbol{v} = 1$. Throughout the paper, we assume $\mathrm{E}|s|^2 = 1$. On the other hand, when STBC is employed, the input-output relation in matrix form is given by

$$\boldsymbol{Y} = \boldsymbol{H}\boldsymbol{X} + \boldsymbol{N} \tag{3}$$

where $\boldsymbol{H}$ is same as in (2), $\boldsymbol{X}$ is the $M \times T$ STBC code and $\mathrm{E}\{\mathrm{tr}(\boldsymbol{X}^H\boldsymbol{X})\} = TP$, and $\boldsymbol{Y}$ is an $N \times T$ matrix, containing the received signals for N users over T time slots. Similarly, $\boldsymbol{N}$ contains the noise at N users over T time slots and is modelled as circularly symmetric complex Gaussian random matrix with i.i.d. entries.

We assume that the ith user only has access to $\mathbf{h}_i$ and $\boldsymbol{y}_i$ (where $\boldsymbol{y}_i$ is a complex number for beamforming and a $1 \times T$ vector for OSTBC). In addition, the users do not collaborate at all. This creates issues in performance analysis and signal design because the point-to-point analysis assumes joint detection.

III. Performance Analysis of Max-Min Beamforming

In [10], the max-min beamforming method is proposed to optimize the worse-case SNR. The problem is posed as a max-min optimization problem. Specifically, when beamforming is used, the receive SNR at the receiver i, denoted by γ_i^{BF}, is given by

$$\gamma_i^{\mathrm{BF}} = \frac{P}{\sigma_N^2} \boldsymbol{v}^H \boldsymbol{h}_i^H \boldsymbol{h}_i \boldsymbol{v}. \tag{4}$$

The beamforming vector $\boldsymbol{v}$ is obtained by solving the max-min problem

$$\max_{\boldsymbol{v}\in\mathbb{C}^M} \min_{i\in\{1,2,\cdots,N\}} \gamma_i^{\text{BF}}$$
$$\text{subject to: } \boldsymbol{v}^H\boldsymbol{v} = 1 \quad (5)$$

. Generally, there is no closed-form solution to this problem. In [10], this problem is solved numerically by the sequential quadratic programming (SQP) method presented in [12]

However, it is observed in [10] that as N, the number of users, increases, the bit error rate (BER) of the max-min beamforming scheme quickly degrades. When N exceeds a certain threshold, the performance of max-min beamforming is even worse than that of the open-loop OSTBC scheme where the transmitter exploits no CSI. In fact, it can be proved that with the max-min beamforming the SNR of the worst users approaches zero as the number of users increases. On the other hand, the SNR of the OSTBC scheme for the worst-case user is given by

$$\gamma^{\text{OSTBC}} = \frac{P}{M\sigma_N^2}\min_{1\le i\le N}\|\boldsymbol{h}_i\|^2.$$

According to order statistics, γ_{OSTBC} also goes to zero as N grows to infinity. However, the convergence rate of γ_{OSTBC} is much smaller. Consider the case where there are two transmit antennas. In this case, the optimal beamforming vector can be written as $\frac{1}{\sqrt{2}}[1\ \exp(j\theta)]$. The angle θ of the beamforming vector is uniformly distributed. When the number of users is large, intuitively, for an arbitrary vector $\boldsymbol{x}$ in the two-dimensional space, there will always be one channel vector $\boldsymbol{h}_i$ that is almost perpendicular to it, which makes $|\boldsymbol{h}_i^H\boldsymbol{x}|^2$ smaller than γ_{OSTBC}. As a result, the SNR obtained from max-min beamforming is smaller than that of OSTBC when the number of users is large.

IV. Max-Min Criterion for OSTBC Precoding

The max-min beamforming method only utilizes spatial degrees of freedom. As a result, the number of parameters that the technique can manipulate is small. When the number of users is large, max-min beamforming has limited capability to favor every channel. On the other hand, with space-time block coding, temporal degrees of freedom are exploited. As a result, a large number of users can be handled with OSTBC, which is demonstrated in [10]. However, OSTBC schemes are open-loop schemes that do note exploit any form of CSI at the transmitter. As demonstrated in [6], [7], [8], combining beamforming with OSTBC can substantially enhance the performance. This motivates us to employ precoding schemes for the OSTBC method with application to the MIMO multicast problem.

Let $\boldsymbol{C}$ be the OSTBC code matrix. With precoding, we pre-multiply $\boldsymbol{C}$ with a precoding matrix $\boldsymbol{Q}$. Thus the input signal in (3) becomes $\boldsymbol{X} = \sqrt{P}\boldsymbol{QC}$. Since $\mathrm{E}\{\boldsymbol{CC}^H\} = \boldsymbol{I}$, the power constraints dictate that

$$\mathrm{tr}(\boldsymbol{QQ}^H) = T.$$

The precoding matrix $\boldsymbol{Q}$ is determined according the current CSI of all users. From another perspective, pre-multiplying the channel with $\boldsymbol{Q}$ can also be regarded as a way to re-condition the channel matrix to become one that is more friendly to the OSTBC code matrix $\boldsymbol{C}$.

The receive SNR for user i is given by

$$\gamma_i = \frac{P}{T\sigma_N^2}\boldsymbol{h}_i\boldsymbol{QQ}^H\boldsymbol{h}_i^H \quad (6)$$

. As discussed in previous sections, it is more important to ensure fair performance among all the users. In this regard, we adopt the max-min criterion for the receive SNR's. The OSTBC precoding matrix $\boldsymbol{Q}$ is obtained by maximizing the SNR of the worst-case user, which is given by

$$\max_{\boldsymbol{Q}\in\mathbb{C}^{M\times M}} \min_{i\in\{1,2,\cdots,M\}} \gamma_i$$
$$\text{subject to: } \mathrm{tr}(\boldsymbol{QQ}^H) = T. \quad (7)$$

To simplify implementation, we only consider the case where $\boldsymbol{Q}$ is a diagonal matrix. This means that $\boldsymbol{Q}$ can be efficiently implemented by varying the transmit antenna powers. Let $\boldsymbol{Q} = \sqrt{T}\,\mathrm{diag}\{\lambda_1, \lambda_2, \cdots, \lambda_M\}$, where λ_i, $i = 1, 2, \ldots, M$, are real numbers. Equation (7) becomes

$$\max_{j\in\{1,2,\cdots,M\}} \min_{i\in\{1,2,\cdots,N\}} \frac{P}{\sigma_N^2}\sum_{j=1}^{M}\lambda_j^2|h_{i,j}|^2$$
$$\text{subject to: } \sum_{j=1}^{M}\lambda_j^2 = 1 \quad (8)$$

. This problem can be solved by the SQP method [12].

Note that the matrix $\sqrt{T/M}\boldsymbol{I}$ satisfies the power constraint, so it is a feasible choice. Plugging it into the SNR expression in equation (6), yields

$$\gamma_i = \frac{P}{M\sigma_N^2}\|\boldsymbol{h}_i\|^2.$$

The worst-case SNR is exactly the one that the open-loop OSTBC scheme without precoding can achieve. Hence the resultant worse-case SNR of this optimization problem in (8) is always greater than that in the open-loop OSTBC scheme.

V. Simulation Results

In this section, the performance of the proposed max-min OSTBC precoding scheme is compared with the max-min beamforming scheme and the open-loop OSTBC scheme. We assume that the variance of the channel coefficients is set to one. We also assume the noise variance $\sigma_N^2 = 1$ thus the transmit SNR is totally controlled by the transmit power P.

Figure 1 plots the average worst-case instantaneous SNR of the three schemes as the number of users increases. In this simulation, the transmit SNR is fixed to be 0dB. The transmitter is equipped with 4 antennas. One can see that when the number of users is small, the max-min beamforming scheme outperforms both max-min OSTBC and open-loop OSTBC scheme. Although all the average worst-case SNRs of the three schemes degrade as the number of users increases, the one of the max-min beamforming scheme drops much faster and the curve of max-min beamforming scheme goes below that of the open-loop OSTBC when the number of users is greater than 30. This observation agrees with the analysis in Section III. On the other hand, the average worst-case SNR of the OSTBC with max-min precoding is always above that of the OSTBC, demonstrating the effectiveness of the proposed precoding scheme.

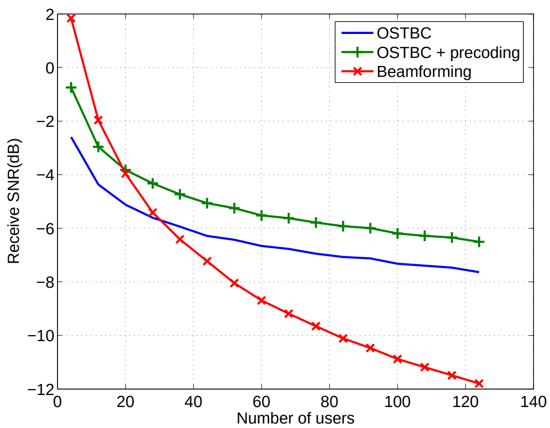

Fig. 1. Comparison of worst-case SNR for OSTBC, max-min beamforming and OSTBC with precoding. The input SNR is 0dB, and there are 4 antennas at the transmitter

A similar trend can be seen in Figure 2, where the worst-case symbol rate error (SER) of the three schemes is plotted versus different number of users. The channel is assumed to be quasi-static in the sense that the channel remains constant for one frame. In this simulation, the frame size is 10 time slots. Since rate one OSTBC codes do not exist when the number of transmitter is greater than two, for a fair comparison in this simulation the transmitter is equipped with two transmit antennas and an Alamouti code [4] is used. In addition, QPSK is used as the modulation scheme. From this figure, one can see that the max-min beamforming scheme works better when the number of users is small but degrades fast as the number of users increases, while the max-min precoding OSTBC method performance drops mildly as the number of users increases but keeps above the OSTBC scheme.

In Figure 3, the mean symbol error rates (SERs) for each of the three schemes are plotted against the number of users. One can see that different from Figure 2, the open-loop OSTBC performs better than the other two schemes when the number of users is in realistically large. Moreover, compared to the max-min beamforming scheme, the performance of the proposed scheme is more close to that of the open-loop

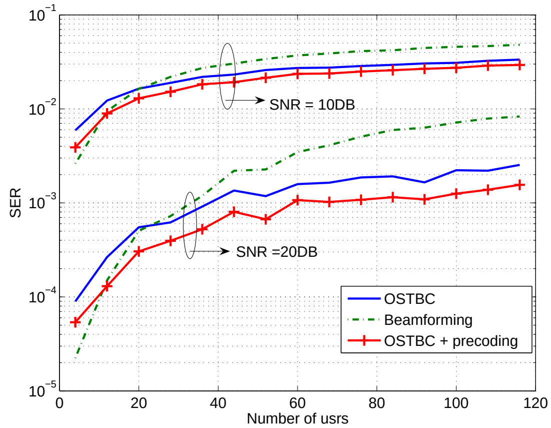

Fig. 2. Comparison of worst-case SER for OSTBC, max-min beamforming and OSTBC with precoding. QPSK is used. The transmitter has 2 antennas.

OSTBC scheme. When the number of users is small, the max-min beamforming technique performs better than the other two schemes. This figure is another example that demonstrates that the proposed diagonal precoding scheme is a good trade-off between open-loop OSTBC and max-min beamforming.

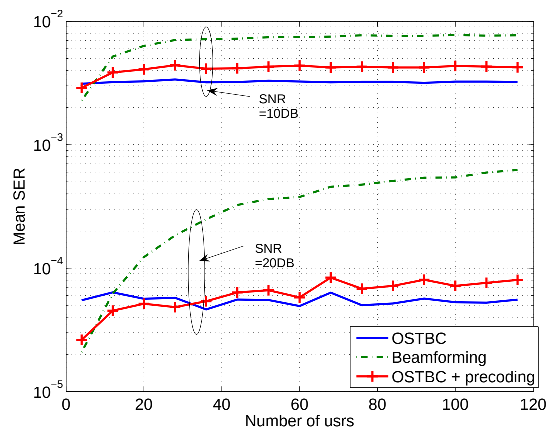

Fig. 3. Comparison of mean SER for OSTBC, max-min beamforming and OSTBC with precoding. QPSK is used. The transmitter has 2 antennas.

Figure 4 demonstrates the benefit of employing more antennas at the transmitter. When the number of transmit antennas increases, the average worst-case SNRs for all the three schemes gain considerate enhancement. The performance gain of the precoding scheme over the open-loop OSTBC scheme also increases as the number of transmit antennas becomes large.

VI. Conclusion

In this paper, an OSTBC with diagonally precoding scheme is proposed to ensure reliable transmission in multicasting application over MIMO channels. Compared to open-loop OSTBC schemes, this scheme achieves universally better

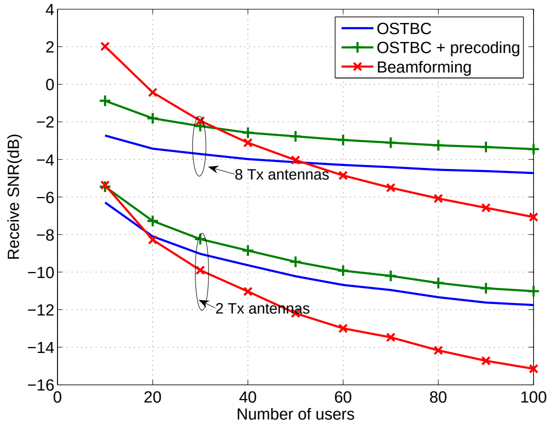

Fig. 4. Effect of different number of transmit antennas for OSTBC, max-min beamforming and OSTBC with precoding.

performance. The proposed scheme also outperforms the max-min beamforming method when the number of users is large.

Future work should be on determining the more general precoding matrix. With more free elements in the precoding matrix, one can gain more freedom to adapt the transmitter to current channel state hence better performance is expected.

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