

# ADAPTIVE POLARIMETRY DESIGN FOR A TARGET IN COMPOUND-GAUSSIAN CLUTTER

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## Abstract

We develop optimal adaptive design of radar waveform polarizations for a target in compound-Gaussian clutter. We present the maximum likelihood (ML) estimates of the target's scattering matrix and clutter parameters using a parameter-expanded expectation-maximization (PX-EM) algorithm. We compute the Cramér-Rao bound (CRB) on the target's scattering matrix and use it as the optimization cost function. We illustrate the performance of our algorithm and compare it with traditional ones through numerical examples.

## 1. INTRODUCTION

The problem of optimally selecting the polarization state of radar transmitted waveforms has received much attention. An optimal polarization state selection approach was first considered in [1], which enables treating symmetric, asymmetric, monostatic, and bistatic cases in an identical way, see also [2]. The optimization is in the sense of maximizing the voltage at the receiving antenna under the assumption that the scattering matrix  $S$  is known, see [2, 3] and Section 2. However, in practice, the scattering matrix of the target of interest is usually unknown because the target is either completely unknown or its posture is unknown. Additionally, the backscattered clutter from the unsteady environment, such as sea surface and foliage covered terrain, is typically non-Gaussian, see e.g. [4], which makes it more challenging to estimate the scattering matrix accurately.

In this paper, we consider adaptive optimal waveform design for polarized signals under compound-Gaussian clutter models with inverse gamma distributed texture, which is often used to model clutter in high-resolution and low-grazing-angle radar, see [5]-[7]. In this model, the complex clutter can be written as

$$e(t) = \sqrt{u(t)}\chi(t), \quad (1)$$

where  $\chi(t)$  is the fast-changing component, accounting for lo-

cal scattering, and is referred to as *speckle*. It is assumed to be a stationary complex Gaussian process with zero mean, whose covariance matrix  $\Sigma$  is unknown. The slow-changing component  $u(t)$  describes the variation of the local power due to the tilting of the illuminated area, and is called *texture*. The texture  $u(t)$  is modeled as a stationary non-negative real random process with unknown parameter  $\nu$ . We assume  $u(t)$  follows the inverse gamma texture distribution because of the simplicity in its computation, see [5], [6], and [8]. It has been shown in [8] to fit well real data.

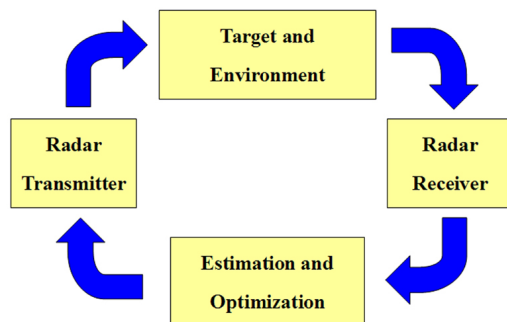


Fig. 1. Polarized radar waveform optimization block diagram.

We illustrate the concept of adaptive polarimetry design in Fig. 1. The transmitted polarized signal is scattered by the target and environment. At the receiver end, measurements of the sum of the scattered signal from the target and the interference from the environment (clutter) are obtained. Using these measurements, we estimate the parameters of both the target and the environment, and optimally choose the signal variables for the transmission to achieve the best system performance. We develop optimal adaptive design of polarized signals fitting the estimated target to get the best estimation performance of the scattering matrix. First we compute the maximum likelihood (ML) estimate of the scattering matrix  $S$  using a parameter-expanded expectation-estimation (PX-EM) algorithm, see [9]. We compute also the Cramér-Rao bound (CRB) on  $S$  and use it as the performance measure for the optimal design. Then we design algorithms to optimally or suboptimally select the

polarization state parameters for the next transmitted signals under the criterion of minimizing the CRB.

The paper is organized as follows. In Section 2, we introduce the polarized radar signal measurement model. In Section 3, we present the ML estimates of the scattering matrix  $S$ , speckle covariance matrix  $\Sigma$  and texture distribution parameter  $\nu$ . In Section 4, we first compute the CRB on  $S$ , then present the algorithms for designing the polarization state of the transmitted waveform. We evaluate the performance of the algorithm with numerical examples in Section 5.

## 2. MEASUREMENT MODEL

In this section, we present our parametric measurement model of a polarized signal with additive compound-Gaussian distributed clutter. To focus the discussion on the adaptive design process, we consider the simple coherent one-transceiver-antenna case (monostatic). We assume that the target is an ideal point scatterer in far field, which is stationary during the observation time and that the target's range and direction are known.

Let  $\xi(t) = [\xi_h(t), \xi_v(t)]^T$  be the transmitted polarized electric field impinging on the target, and  $\mathbf{y}(t) = [y_h(t), y_v(t)]^T$  be the complex envelope of the received (scattered) electric field, where the subscripts "h" and "v" denote the horizontal and vertical polarization components of a fully polarized signal. Under the far-field assumption, the scattered field is related to the incident field by [2, 10, 11]:

$$\mathbf{y}(t) = g(r) S \xi(t) + \mathbf{e}(t), \quad t = 1, 2, \dots, N, \quad (2)$$

where  $r$  denotes the range from the transmitting antenna to the target,  $S$  is the  $2 \times 2$  scattering matrix, and  $\mathbf{e}(t)$  denotes additive clutter. For simplicity, we ignore the measurement noise here. Assume  $\mathbf{e}(t) = [e_h(t), e_v(t)]^T$  to be independent and identically distributed (i.i.d.), under compound-Gaussian with parameters  $\Sigma$  and  $\nu$ , where the superscript " $T$ " denotes the transpose. The variable  $N$  is the number of temporal observations. Denoting the wave number of the carrier waveform by  $k$ , we have  $g(r) = e^{-2jkr}/2kr$  corresponding to the propagation magnitude attenuation and phase shift, which is a (known) complex scalar. We will estimate the scattering matrix of the target

$$S = \begin{bmatrix} s_{hh} & s_{hv} \\ s_{vh} & s_{vv} \end{bmatrix} \quad (3)$$

which characterizes the polarization features of the target [3, 10, 11]. The first and second subscripts of each element in  $S$  denote the polarization components in the receiving and transmitting signals, respectively. For the reciprocal monostatic case, we can assume  $s_{hv} = s_{vh}$ .

We parameterize the transmitted signal using the notations of [12] and [13] (omitting time index for notational simplicity):

$$\xi = \|\xi\| e^{j\varphi} Q \mathbf{w}, \quad (4)$$

where

$$Q = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} \cos \beta \\ j \sin \beta \end{bmatrix}, \quad (5)$$

$\|\xi\| e^{j\varphi}$  is the complex envelope of the source signal,  $\alpha$  denotes the rotation angle between the coordinates and the electric ellipse axes, and  $\beta$  determines the ellipse's eccentricity, see [12]. The definition space of these parameters are:  $\|\xi\|, \varphi \in (-\pi, \pi]$ ,  $\alpha \in (-\pi/2, \pi/2]$ ,  $\beta \in [-\pi/4, \pi/4]$ . Considering that the transmitted signal is power-limited, we exclude  $\|\xi\|$  out of the design by setting  $\|\xi\| = 1$ , i.e.,

$$\xi = e^{j\varphi} \begin{bmatrix} \cos \alpha \cos \beta + j \sin \alpha \sin \beta \\ -\sin \alpha \cos \beta + j \cos \alpha \sin \beta \end{bmatrix}. \quad (6)$$

## 3. SCATTERING MATRIX ESTIMATION

The traditional way of measuring the scattering matrix of a given target is to transmit intermittently vertically and horizontally polarized signals and recording both components of the received wave. From the measurement, the scattering matrix is calculated by its definition. Previous work on measuring the scattering matrix assumes no interference in the measurement [14]. In the following, we estimate the scattering matrix under compound-Gaussian noise assumption.

We now propose the ML estimate for the scattering matrix  $S$  and clutter parameters  $\Sigma, \nu$  in the measurement model (2). By definition, the reciprocal of  $u(t)$  follows a gamma distribution with mean one and unknown shape parameter  $\nu$ , i.e. the probability density function (pdf) of  $u(t)$  is

$$p_u(u(t); \nu) = \frac{1}{\Gamma(\nu)} \nu^\nu u(t)^{-\nu-1} e^{-\nu/u(t)} \sim \text{iGamma}(\nu, 1/\nu), \quad (7)$$

where  $\Gamma(\cdot)$  is the gamma function. The conditional distribution of  $\mathbf{y}(t)$  given  $u(t)$  is

$$p_{\mathbf{y}|u}(\mathbf{y}(t) | u(t); S, \Sigma) = \exp \left\{ -[\mathbf{y}(t) - g(r)S\xi(t)]^H \cdot [u(t)\Sigma]^{-1} \cdot [\mathbf{y}(t) - g(r)S\xi(t)] \right\} / |\pi u(t)\Sigma|. \quad (8a)$$

Since it is impossible to find a closed-form solution for the ML estimates of  $S$ ,  $\Sigma$  and  $\nu$  from the observed data likelihood, we use expectation-maximization (EM) algorithm to find the ML estimates from the *complete data log-likelihood*, which is easier to manipulate mathematically. Define the *unobserved data*  $w(t) = u(t)^{-1}$ . Our goal is to find the estimates that maximize the *complete data log-likelihood*.

The model (2) is a special GMANOVA model (see [15]) with spatial-temporal matrix  $A = g(r) \cdot I$ , where  $I$  is the  $2 \times 2$  identical matrix. Thus, we simplify the PX-EM algorithm we proposed in [16] as below for our problem.

The estimation algorithm is composed of two loops. In the inner loop, the ML estimates of  $S$  and  $\Sigma$  are computed for a fixed  $\nu$  value using PX-EM algorithm. In the outer loop,

we estimate  $\nu$  with the estimation results in inner step using alternate projection until  $\nu$  converges.

**Estimating  $S$  and  $\Sigma$ :** With a fixed  $\nu^{(j)}, j = 1, 2, \dots$ , the resulting PX-EM algorithm consists of iterating between the following PX-E and PX-M steps:

**PX-E Step:** Here we compute the conditional expectations  $\mathcal{T}_k(\hat{\nu}^{(j)}) = \mathbb{E}\{T_k(w; \hat{\nu}^{(j)}) | \mathbf{y}\}$  of the sufficient statistics  $T_k(\hat{\nu}^{(j)})$ ,  $k = 1, 2, 3$  for  $\{S, \Sigma, \nu\}$ .

$$\hat{w}^{(i)}(t; \nu^{(j)}) = (\nu^{(j)} + 2) \cdot \left\{ \nu^{(j)} + [\mathbf{y}(t) - g(r)S^{(i)}\boldsymbol{\xi}(t)]^H \cdot [\Sigma^{(i)}]^{-1} [\mathbf{y}(t) - g(r)S^{(i)}\boldsymbol{\xi}(t)] \right\}^{-1} \quad (9a)$$

for  $t = 1, 2, \dots, N$  and

$$\mathcal{T}_1^{(i)}(\hat{\nu}^{(j)}) = \frac{1}{N} \cdot \sum_{t=1}^N \mathbf{y}(t)\boldsymbol{\xi}(t)^H \cdot \hat{w}^{(i)}(t; \nu^{(j)}), \quad (9b)$$

$$\mathcal{T}_2^{(i)}(\hat{\nu}^{(j)}) = \frac{1}{N} \cdot \sum_{t=1}^N \mathbf{y}(t)\mathbf{y}(t)^H \cdot \hat{w}^{(i)}(t; \hat{\nu}^{(j)}), \quad (9c)$$

$$\mathcal{T}_3^{(i)}(\hat{\nu}^{(j)}) = \frac{1}{N} \cdot \sum_{t=1}^N \boldsymbol{\xi}(t)\boldsymbol{\xi}(t)^H \cdot \hat{w}^{(i)}(t; \hat{\nu}^{(j)}). \quad (9d)$$

**PX-M Step:** Compute the ML estimates using the conditional expectations of the sufficient statistics from the E step:

$$\hat{S}^{(i+1)}(\hat{\nu}^{(j)}) = \frac{1}{g(r)} \mathcal{T}_1^{(i)}(\hat{\nu}^{(j)}) (\mathcal{T}_3^{(i)}(\hat{\nu}^{(j)}))^{-1}, \quad (10a)$$

$$\hat{\Sigma}^{(i+1)}(\hat{\nu}^{(j)}) = \frac{N}{\sum_{t=1}^N \hat{w}^{(i)}(t; \hat{\nu}^{(j)})} \cdot$$

$$\left[ \mathcal{T}_2^{(i)}(\hat{\nu}^{(j)}) - \mathcal{T}_1^{(i)}(\hat{\nu}^{(j)}) (\mathcal{T}_3^{(i)}(\hat{\nu}^{(j)}))^{-1} (\mathcal{T}_1^{(i)}(\hat{\nu}^{(j)}))^H \right], \quad (10b)$$

$i = 1, 2, \dots$

The above iteration is performed until  $\hat{S}^{(i)}(\hat{\nu}^{(j)})$  and  $\hat{\Sigma}^{(i)}(\hat{\nu}^{(j)})$  converge. Denote by  $\hat{S}^{(\infty)}(\hat{\nu}^{(j)})$  and  $\hat{\Sigma}^{(\infty)}(\hat{\nu}^{(j)})$  the estimates of  $\hat{S}$  and  $\hat{\Sigma}$  obtained upon convergence.

**Estimating  $\nu$ :** We compute the ML estimate of  $\nu$  by maximizing the observed-data concentrated log-likelihood function with respect to  $\hat{S}^{(\infty)}(\hat{\nu}^{(j)})$  and  $\hat{\Sigma}^{(\infty)}(\hat{\nu}^{(j)})$ :

$$\hat{\nu}^{(j+1)} = \arg \max_{\nu} \sum_{t=1}^N \ln p_{\nu}(\mathbf{y}(t); \hat{S}^{(\infty)}(\hat{\nu}^{(j)}), \hat{\Sigma}^{(\infty)}(\hat{\nu}^{(j)}), \nu). \quad (11)$$

Next we estimate  $S$  and  $\Sigma$  using this new estimate of  $\hat{\nu}^{(j+1)}$ , etc.

#### 4. OPTIMAL WAVEFORM PARAMETER SELECTION

Since the estimation algorithm we use is maximum-likelihood, its asymptotical accuracy is expected to attain the CRB. So by

minimizing the CRB cost-function with respect to the transmitted signal, we will minimize the corresponding asymptotic estimation error. We will then choose the polarization state that minimizes the CRB to be transmitted in the next pulse. Thus, in this section we will consider the optimal polarization design subject to minimizing the CRB function.

#### 4.1. Cramér-Rao Bound

Define the vector of parameters of interest

$$\boldsymbol{\rho} = [\text{Re}\{\text{vech}(S)\}^T, \text{Im}\{\text{vech}(S)\}^T]^T, \quad (12)$$

where the ‘‘vech’’ matrix operator creates a single column vector by stacking elements below (including) the main diagonal columnwise. Considering that  $S_{\text{hv}} = S_{\text{vh}}$ , then  $\boldsymbol{\rho}$  is a  $6 \times 1$  vector and the CRB with respect to  $\boldsymbol{\rho}$  is a  $6 \times 6$  matrix. We minimize the determinant of the CRB matrix, which is also known as the D-optimality criterion [17]. This is equivalent to maximizing the determinant of the Fisher information matrix (FIM), whose elements are (see [16]):

$$[I(\boldsymbol{\rho}; \Sigma)]_{pq} = \frac{2(\nu + 2)\|G(r)\|^2}{\nu + 3} \cdot \text{Re} \left[ \sum_{t=1}^N \boldsymbol{\xi}(t)^H \frac{\partial S^H}{\partial \rho_p} \cdot \Sigma^{-1} \cdot \frac{\partial S}{\partial \rho_q} \boldsymbol{\xi}(t) \right]. \quad (13a)$$

Hence, the FIM can be expressed as

$$I(\boldsymbol{\rho}; \Sigma) = \frac{2(\nu + 2)\|g(r)\|^2}{\nu + 3} \text{Re} \left( \sum_{t=1}^N F(\alpha_t, \beta_t, \Sigma, \boldsymbol{\rho}) \right), \quad (14)$$

where  $F(\alpha_t, \beta_t, \Sigma, \boldsymbol{\rho})$  is the contribution of the  $t$ th signal to the FIM. The  $(p, q)$ th entry of  $F(\alpha_t, \beta_t, \Sigma, \boldsymbol{\rho})$  has the form:

$$[F(\alpha_t, \beta_t, \Sigma, \boldsymbol{\rho})]_{pq} = \text{Re} \left( \boldsymbol{\xi}(t)^H \cdot \frac{\partial S^H}{\partial \rho_p} \Sigma^{-1} \frac{\partial S}{\partial \rho_q} \cdot \boldsymbol{\xi}(t) \right). \quad (15)$$

We now discuss the properties of FIM with respect to the transmitted signal.

**Property 1.** *The initial signal phase  $\varphi$  does not affect the FIM.*

Substituting the right side of (6) into (13a), we find that the FIM is independent of the initial phase  $\varphi$ , namely  $\varphi_{\text{opt}}$  can be arbitrarily selected in  $(-\pi, \pi]$  without affecting the estimation performance.

**Property 2.** *For  $N = 1$ , the determinant of FIM equals to zero.*

Considering that the covariance matrix of the speckle is Hermitian, and so is its inverse, we denote the inverse of the current estimate of the speckle covariance matrix as

$$\Sigma^{-1} = \begin{bmatrix} a_1 & a_2 - jb_2 \\ a_2 + jb_2 & a_3 \end{bmatrix} \quad (16)$$

where  $a_1, a_2, a_3, b_2 \in \mathbb{R}$ .

Substitute (4) and (16) into (15). Then, with the help of a symbolic function software, e.g., MATLAB, it is not hard to find that the determinant of  $F(\alpha_t, \beta_t, \Sigma, \rho)$  equals zero despite the polarization state  $\{\alpha_t, \beta_t, \varphi\}$  of the transmitted signal. Hence, the determinant of the FIM equals zero in this case. Note that the dimension of the scattering matrix  $S$  is  $2 \times 2$  and the transmitted signal  $\xi(t)$  is a  $2 \times 1$  vector. The returned single signal will enable identifying the scattering matrix.

Interestingly, if fixed polarized waveforms are transmitted in all  $N$  steps, the determinant of FIM will still be zero. That is, increasing the number of observations without changing the transmitted waveform will not improve the estimation quality – the non-identifiability will still exist [13].

#### 4.2. Adaptive Optimal Polarization State Design

We now present the proposed algorithm for optimal radar waveform polarization state design and a suboptimal version, under the criterion of minimizing the CRB cost function.

Denote

$$J_N(\Sigma) = \sum_{t=1}^N F(\alpha_t, \beta_t, \Sigma, \rho), \quad (17)$$

where  $F(\alpha_t, \beta_t, \Sigma, \rho)$  denote the contribution of the transmitted signal  $\xi(\alpha_t, \beta_t)$  to the FIM.

The optimal polarization state design method includes the following three steps:

Step 1: Update  $\Sigma$  with the estimation from  $N$  observations (see Section 3),  $\Sigma = \widehat{\Sigma}_N$ .

Step 2: Calculate  $J_N(\widehat{\Sigma}_N)$ .

Step 3: Find the optimal value  $\{\alpha_{N+1}^*, \beta_{N+1}^*\}$  within their definition space:

$$\{\alpha_{N+1}^*, \beta_{N+1}^*\} = \arg \max_{\{\alpha, \beta\} \in [-\frac{\pi}{2}, \frac{\pi}{2}] \times [-\frac{\pi}{4}, \frac{\pi}{4}]} \det [J_N(\widehat{\Sigma}_N) + F(\alpha, \beta, \widehat{\Sigma}_N, \rho)]. \quad (18a)$$

### 5. NUMERICAL EXAMPLES

We demonstrate the effectiveness of our algorithm through numerical examples. We set the true value of the scattering matrix to

$$S = \begin{bmatrix} 2j & 0.5 \\ 0.5 & -j \end{bmatrix}, \quad (19)$$

which is used also in [1] and [2]. The speckle covariance matrix  $\Sigma$  was generated using a model similar to that in [18, Sec. 2.6] with 1000 patches. The  $(p, q)$ th element of the covariance matrix of the speckle component was chosen as

$$\Sigma_{p,q} = \sigma^2 \cdot 0.9^{|p-q|} \cdot \exp[j(\pi/2)(p-q)], \quad p, q = 1, 2. \quad (20)$$

In the examples presented here, we selected  $\sigma^2 = 2.05$ . In the following numerical examples, without specification, we

set the speckle covariance matrix to the above value and the texture distribution parameter  $\nu = 2$ . Since we assume the target's range  $r$  and direction are known, the value of factor  $g(r)$  will not affect the results.

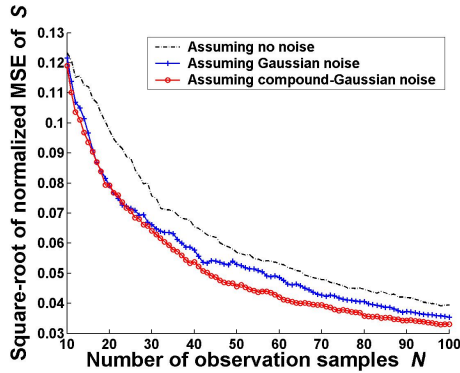
Note that the PX-EM algorithm can only converge after a certain number of measurements. Hence we send 10 pilot signals to start the algorithm. The pilot signal parameters are randomly selected in each realization.

In Fig. 2, we show the square-root of normalized MSEs of  $S$ , which is calculated by averaging the ratio of the square-root of MSE of each estimate to its true value, under three noise related assumptions. The actual scenario is set to that the scattered field from the target is contaminated by compound-Gaussian distributed noise. We first use the traditional method, i.e., we send intermittently changing vertical and horizontal polarized signals and assume no clutter noise in the measurement (e.g., see [10] and references therein). Then we assume the measurement is contaminated by Gaussian noise and estimate  $S$  based on this assumption. We also show the square-root of normalized MSE of the ML estimate for  $S$  under the compound-Gaussian noise assumption with our optimal design algorithm. The results show that the optimal polarization design algorithm under compound-Gaussian noise assumption has a smaller MSE value than the other two at every evaluation point.

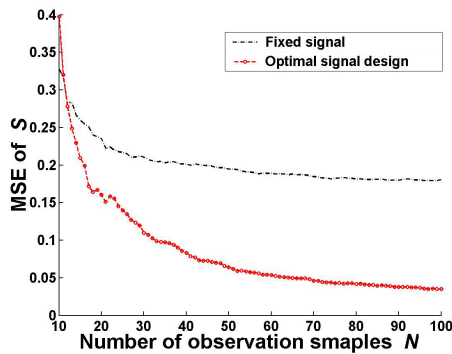
In Fig. 3, we show the average MSEs for the ML estimates of the scattering matrix parameters over 50 independent trials as functions of  $N$ . Two different signal transmitting schemes are compared here: (i) fixed signal with  $\alpha = \frac{\pi}{3}, \beta = \frac{\pi}{6}$ , and (ii) optimally designed signal. We vary the observation number from 10 to 100 in each scheme. Note: we just show the average of elements in scattering matrix here. Since at each step, the most efficient waveform (in terms of minimizing CRB) is selected, the result of optimal adaptive algorithm converges much faster than the other one when the number of observations increases. The average MSE of  $\widehat{S}$  decreases about 6dB, comparing with the fixed signal. Note that the average MSE of  $\widehat{S}$  is computed by averaging the MSE of each entries of  $S$ .

### 6. SUMMARY

We developed optimal adaptive design methods of the radar waveform polarization state under compound-Gaussian clutter. We first presented the maximum-likelihood estimates for the scattering matrix and the clutter distribution parameters based on a parameter-expanded expectation-maximization algorithm. Then we computed the Cramér-Rao bound on the scattering matrix  $S$  and used it as the performance measure. By minimizing the CRB cost function, we optimally selected the polarization state for the waveform to be transmitted in the next pulse. In the numerical examples, we compared the performance of these algorithms and showed a significant performance improvement in the estimation accuracy compared with conventional methods.



**Fig. 2.** Square-root of normalized mean-square errors of estimating the elements of the scattering matrix  $S$  for three different noise assumptions: (i) no noise, (ii) Gaussian noise, and (iii) compound-Gaussian noise, where  $\nu = 2$ ,  $\Sigma_1 = \Sigma$ .



**Fig. 3.** Averaged mean-square errors of estimating the elements of the scattering matrix  $S$  for three different transmit signal schemes: (i) fixed signal, and (ii) optimal design.

## 7. ACKNOWLEDGEMENT

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