Radar Performance of Temporal and Frequency Diverse Phase-Coded Waveforms

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***Abstract*—**

We explore the use of binary phase-coded waveforms encoded in frequency and temporally diverse modes for radar detection of targets in clutter. Specifically, in this paper we study the use of complementary binary phase-coded waveforms and compare their performance to other, more conventional, suites of waveforms. We also give an ambiguity calculation showing the effects of time and frequency separation.

I. Introduction

Complementary waveforms are transmitted in pairs in radar to produce, after match-filtering, effectively perfect range sidelobe performance, at least at zero Doppler. However, this disguises the fact that to achieve this level of performance the complementary waveforms need to be separated in some way in order that their returns are separable at the transmitter. It is possible, in theory at least, to use larger collections of complementary codes and to apply Doppler processing to them. Our aim is to address some of the implementational issues associated with the use of such waveforms.

Larger complementary sets comprised of many different waveforms, such as the Prometheus Orthonormal Set (PONS) and variants on it can be employed in a radar to the extent that separation mechanisms permit. Theoretical and simulation results show that such waveform suites are able to produce remarkably good ambiguity performance.

The limiting factor in achieving such performance is then in the implementation, and in particular in the extent to which separation of the returns can be achieved. The possible ways in which separation can be accomplished include the separation of the signals in time or in frequency or in some combination of these methods. For example, it is possible to take a collection of 8 waveforms and code pairs of them on two different frequency channels (in particular on the I and Q channels), followed by transmission of the four resulting signals on separate pulse repetition intervals. All of these methods lead to only imperfect separation, and result in less than perfect sidelobe performance. Moreover, bandwidth limitation required to implement these discrete waveforms places an extra constraint on performance.

We describe the various separation schemes in detail, and compare their performance when combined with Doppler processing in terms of sidelobes of the ambiguity. The trade-off in ambiguity performance for changes in the separation methods and the amount of separation in time and frequency will be explored, along with the effects of bandwidth limitation.

We undertake a theoretical analysis of the issues and performance results that can be expected from complementary waveforms separated in time, including a temporal diversity mode in which many different pulses are separated in time and combined using conventional Doppler processing, and in frequency. The degree of separation in each case has a profound effect on the ambiguity properties of the combined waveform. To a limited extent it is possible to obtain analytic performance measures from this analysis.

We will do this by means both of an ambiguity analysis, notably comparison of sidelobe performance in regions of the range-Doppler plane of interest in an operational scenario, and by means of simulations to compare complementary waveforms used in temporal diversity mode, with other discrete phase-coded waveforms such as PN codes, Frank codes, and the P4 codes of Kretschmer and Lewis.

II. Ambiguity of Multi-Waveforms

The radar ambiguity function (or rather its absolute value) describes the sensitivity of a transmitted waveform to targets of particular ranges and velocities. It is the response of the radar system to a return from a point target at varying range and velocity (Doppler).

Suppose that we are able to transmit several waveforms

$$\mathbf{s}(t) = (s_1(t),\ s_2(t),\ \ldots,\ s_M(t)) \tag{1}$$

and receive them separately with no interference between them. Then the processing of the signals will involve matched filtering of each individual waveform and summing the resulting signals.

The resulting ambiguity function for the multiple waveforms is then

$$A_{\mathbf{s}}(\tau, \phi) = \sum_{m=1}^{M} A_{s_m}(\tau, \phi), \tag{2}$$

where τ denoted time and ϕ denotes frequency.

It has been shown by several authors, and is a simple consequence of Moyal's identity (see [2]) that, provided the signal components $s_m(t)$ are orthogonal, the ambiguity of a signal of total energy 1 (that is, with $\sum_m ||s_m||^2_{L^2(\mathbf{R})} = 1$) satisfies

$$\|A_{\mathbf{s}}\|^2_{L^2(\mathbf{R}^2)} \geq \frac{1}{M}, \tag{3}$$

with equality if the components have equal energy. Thus the more orthogonal and separated signals we can use, the smaller will be the ambiguity. Since orthogonal (and we note here that orthogonality means just that, not that the signals are orthogonal, or near orthogonal under translation) signals occur in abundance, the key issue is one of separation.

The zero frequency section of the ambiguity function ($A_s(\tau, 0)$) is effectively the auto-correlation of the waveform with itself, and this applies equally well to the ambiguity of multiple waveforms. It follows that waveforms with good correlation properties will have good range ambiguity (that is, at least in the range direction, will approximate a 'thumb-tack'). While no single waveforms have perfect correlation properties of this kind, it is possible to find multiple waveforms of this kind. In fact, the Golay property given in (8) is exactly the perfect correlation property we need.

III. Separation Schemes for Multi-Waveforms

Multi-waveform schemes involve the separate transmission of several different waveforms in ways that allow the corresponding returns to be separated at the receiver before being match-filtered and added. The key to the implementation of multi-waveforms is in the separation of the returns resulting from them. Two methods are normally studied: time separation and frequency separation. Here we briefly review them both and give an analysis that shows the limitations on their performance. For the purposes of this analysis we shall consider only pairs of waveforms, which we shall assume to be *complementary* (see equation (8)).

Recall that the *cross-correlation* of two waveforms $\mathbf{w}_1$ and $\mathbf{w}_2$ is given by

$$\mathrm{corr}_{\mathbf{w}_1,\mathbf{w}_2}(\tau) = \int_{-\infty}^{\infty} \mathbf{w}_1(t)\mathbf{w}_2^*(t-\tau)\,dt, \tag{4}$$

where * denotes complex conjugation. The auto-correlation of $\mathbf{w}$ is its cross-correlation with itself

$$\mathrm{corr}_{\mathbf{w}}(\tau) = \mathrm{corr}_{\mathbf{w},\mathbf{w}}(\tau). \tag{5}$$

Such an expression typifies the working of a matched filter. The *cross-ambiguity* of a pair of waveforms $\mathbf{w}_1$ and $\mathbf{w}_2$ is

$$A_{\mathbf{w}_1,\mathbf{w}_2}(\tau, \phi) = \int_{-\infty}^{\infty} e^{i\phi t}\mathbf{w}_1(t)\mathbf{w}_2^*(t-\tau)\,dt. \tag{6}$$

Thus

$$\mathrm{corr}_{\mathbf{w}_1,\mathbf{w}_2}(\tau) = A_{\mathbf{w}_1,\mathbf{w}_2}(\tau, 0). \tag{7}$$

An appropriate definition of complementarity might be that a pair $\mathbf{w}_1$ and $\mathbf{w}_2$ of waveforms is *complementary* if they satisfy

$$\mathrm{corr}_{\mathbf{w}_1}(\tau) + \mathrm{corr}_{\mathbf{w}_2}(\tau) = C\delta_0(\tau). \tag{8}$$

In fact, such an equation is impossible since the right side must have finite (i.e., non-zero) time duration, so that the best we can hope for is

$$\mathrm{corr}_{\mathbf{w}_1}(\tau) + \mathrm{corr}_{\mathbf{w}_2}(\tau) = C\Delta_0(\tau), \tag{9}$$

where $\Delta_0(\tau)$ is the triangle function with height 1 and width equal to twice the chip length L_c of the (digital) waveforms used. C is twice the sum of the lengths of the two codes. This is the meaning of complementarity we shall use here. Throughout our discussions the codes will have the same length. Codes satisfying this equation will be discussed in detail in section IV.

Whether the waveforms are separated in time or frequency (or in some other novel way), ultimately only one waveform is transmitted and so the basic physical limitation (see Eq.(3) the case for $M = 1$) cannot be overcome. The difference is dramatic and requires some explanation. Of course the simple explanation is that the two waveforms cannot be truly separated. Separation is just a convenient fiction. Nonetheless, the ambiguity of multi-waveforms has significantly lower sidelobes in range for operationally realistic Doppler values, giving rise to the belief that there is some truth in equation (3).

A. Time Separation

Time separation of complementary waveforms $\mathbf{w}_1$ and $\mathbf{w}_2$ means that they are emitted sufficiently separated in time so that the returns cannot be mistaken for each other. In particular, the two waveforms do not overlap in time. Of course, separation in time means much more than this. It means that the returns from the waveform that is emitted first will have become of insignificant size before the returns from the second waveform arrive.

The emitted waveform is thus

$$\mathbf{w}(t) = \mathbf{w}_1(t) + \mathbf{w}_2(t - T), \tag{10}$$

where it is assumed that each waveform has finite temporal support and is defined to be zero outside of that time interval. The time delay T of the second waveform is sufficiently large that $\mathbf{w}_1(t) \times \mathbf{w}_2(t + T) = 0$ for all t and indeed so that

$$\mathbf{w}_1(t - \tau_f) \times \mathbf{w}_2(t - \tau_i - T) = 0, \tag{11}$$

where τ_i and τ_f are the initial and final delays of the returning waveforms. Thus

$$T \geq \tau_f - \tau_i + L \tag{12}$$

where L is the time extent of the waveforms.

The ambiguity of $\mathbf{w}$ is then

$$\begin{aligned} A_{\mathbf{w}}(\tau, \phi) &= \int_{-\infty}^{\infty} e^{i\phi t}\mathbf{w}(t)\mathbf{w}^*(t - \tau)\, dt \\ &= \int_{-\infty}^{\infty} e^{i\phi t}\big(\mathbf{w}_1(t) + \mathbf{w}_2(t - T)\big) \\ &\qquad \big(\mathbf{w}_1^*(t - \tau) + \mathbf{w}_2^*(t - \tau - T)\big)\, dt \\ &= \int_{-\infty}^{\infty} e^{i\phi t}\mathbf{w}_1(t)\mathbf{w}_1^*(t - \tau)\, dt \\ &\qquad + \int_{-\infty}^{\infty} e^{i\phi t}\mathbf{w}_1(t)\mathbf{w}_2^*(t - \tau - T)\, dt \\ &\qquad + \int_{-\infty}^{\infty} e^{i\phi t}\mathbf{w}_1^*(t - \tau)\mathbf{w}_2(t - T)\, dt \\ &\qquad + \int_{-\infty}^{\infty} e^{i\phi t}\mathbf{w}_2(t - T)\mathbf{w}_2^*(t - \tau - T)\, dt \\ &= A_{\mathbf{w}_1}(\tau, \phi) + A_{\mathbf{w}_1,\mathbf{w}_2}(\tau + T, \phi) \\ &\qquad + e^{i\phi T}A_{\mathbf{w}_2,\mathbf{w}_1}(\tau - T, \phi) + e^{i\phi T}A_{\mathbf{w}_2}(\tau, \phi). \end{aligned} \tag{13}$$

The first point to note from this calculation is that the ambiguity of $\mathbf{w}$ is not just the sum of the ambiguities of the two waveforms. If T is large in the sense we have described, then the two cross terms $A_{\mathbf{w}_1,\mathbf{w}_2}(\tau + T, \phi) + e^{i\phi T}A_{\mathbf{w}_2,\mathbf{w}_1}(\tau - T, \phi)$ are vanishingly small for τ corresponding to actual delays seen. Indeed this is what is used in the processing of the returned signals. The remaining terms are

$$A_{\mathbf{w}_1}(\tau, \phi) + e^{i\phi T}A_{\mathbf{w}_2}(\tau, \phi). \tag{14}$$

We remark that *true* separation would replace the term $e^{i\phi T}$ by 1. The extent to which we fail to achieve that separation is then manifest by this difference. If Doppler processing is being used then the Doppler will be known to within the width of a Doppler bin.

Note that the cross terms only disappear because delays of a size to cause problems do not happen in practice without considerable attenuation of the signal due to the $1/R^4$ fall-off in signal strength with distance R. The missing energy under the ambiguity surface is really in these cross terms but does not have any practical effect.

To illustrate the sort of numbers involved, we assume a PRI of 100 microseconds ($T = 100$ms) with the two waveforms being transmitted on alternate pulses. Assume too that the waveform length is small (around 1ms). If the targets are moving ground targets (of velocity 5 meters per second), and the radar is operating at X-band (10^{10} Hz), the value of the Doppler shift ϕ will be close to 300Hz and the difference

$$|e^{i\phi T} - 1| \tag{15}$$

will be around 0.03. This remains significant, but it must be borne in mind that this will be reduced by Doppler processing.

B. Frequency Separation

Here we consider the scheme of modulating the two waveforms onto slightly different carriers. There could be issues here concerned with the target's reflectivity at different frequencies, but for the purposes of this discussion we assume that for the kind of frequency shifts contemplated there is insignificant change in reflectivity. The waveform $\mathbf{w}$ to be modulated onto the carrier is, in this case, of the form

$$\mathbf{w}(t) = \mathbf{w}_1(t) + e^{if_s t}\mathbf{w}_2(t), \tag{16}$$

where f_s is the frequency separation used to permit separation of returns. Note that in order that this separation can be achieved on receipt of the signal, the frequency f_s should be larger than the bandwidth of the waveforms in question. Phase coded digital waveforms, in principal, have infinite bandwidth since they require instantaneous switching of phase. However, this is not achievable and the waveforms will inevitably suffer some low-pass filtering by the processing.

Now we perform the ambiguity calculation for the waveform $\mathbf{w}$ of equation (16).

$$\begin{aligned} A_{\mathbf{w}}(\tau,\phi) &= \int_{-\infty}^{\infty} e^{i\phi t}\mathbf{w}(t)\mathbf{w}^*(t-\tau)\,dt \\ &= \int_{-\infty}^{\infty} e^{i\phi t}\big(\mathbf{w}_1(t) + e^{if_s t}\mathbf{w}_2(t)\big) \\ &\qquad \big(\mathbf{w}_1^*(t-\tau) + e^{-if_s(t-\tau)}\mathbf{w}_2^*(t-\tau)\big)\,dt \\ &= A_{\mathbf{w}_1}(\tau,\phi) + e^{if_s\tau}A_{\mathbf{w}_1,\mathbf{w}_2}(\tau,\phi-f_s) \\ &\qquad + A_{\mathbf{w}_2,\mathbf{w}_1}(\tau,\phi-f_s) + e^{if_s\tau}A_{\mathbf{w}_2}(\tau,\phi). \end{aligned} \tag{17}$$

Frequency separation again kills the cross terms modulo the bandwidth issues just mentioned, and we are left with

$$A_{\mathbf{w}}(\tau,\phi) = A_{\mathbf{w}_1}(\tau,\phi) + e^{if_s\tau}A_{\mathbf{w}_2}(\tau,\phi). \tag{18}$$

The situation here is precisely dual to the time-separation case. Here the delay τ rather than the Doppler is the unknown in the exponential term that prevents the achievement of the perfect "separation ambiguity" $A_{\mathbf{w}}(\tau,\phi) = A_{\mathbf{w}_1}(\tau,\phi) + A_{\mathbf{w}_2}(\tau,\phi)$. If the separation is 100MHz, a delay of around 100 nanoseconds can be significant. We note here that there are implementation issues that need further work to align the phases. We will return to this topic in a a future paper.

Note that here, as in the time-separation case, the "missing energy" in the ambiguity is again in the cross terms but at Doppler frequencies close to the separation frequency f_d. Targets with velocities capable of producing a Doppler shift of 100MHz at X-band would have to be traveling in excess of 10^6 meters per second, and so are not operationally realistic.

IV. Complementary Waveforms

We will briefly review the mathematical formulation of complementary temporally and frequency diverse waveforms in this section. For temporally diverse waveforms, at the k-th PRI in the i-th frequency channel the waveform $p_k^{(i)}(t)$ is transmitted. Suppose we have a single, point target at a distance (time delay) of r with a Doppler shift of f_d. Each return pulse is correlated against a copy of itself (standard pulse compression/matched filtering). We briefly point out that in order to understand the imaging effects of the use of these waveforms in this way, it is enough to consider the effect on a point target of this type. The range-Doppler image of a scene resulting from such a waveform collection is the two-dimensional convolution of the actual scene with the result for a point target. The goal of this work is to construct sets of waveforms $p_k(t)$ such that

$$\sum_{i=1}^{M} q^{(i)}(u,t) = c_1\delta_0(u,t), \tag{19}$$

where $q(u,t)$ is a discreet Fourier transform of $p_k(t)$ in k-th variable, δ_0 is the delta function and c_1 is some constant. Waveforms that satisfy (19) would provide perfect sidelobes in range and Doppler, up to the resolution imposed by the system. An additional desirable feature of such waveforms is that

$$|p_k^{(i)}(t)| \approx c_2 \qquad \forall \quad i,k,t \tag{20}$$

where c_2 is some constant.

A. Two Dimensional Constant Amplitude Waveforms

In this section we describe a method for constructing waveforms that, in theory, provide a perfect "thumbtack" in range-Doppler space (subject to resolution limitations) and which have constant amplitude. These waveforms are based on PONS waveforms [1]. This construction technique generates sets of complementary waveforms where each row is the complement of the adjacent row. The temporally diverse waveforms are just obtained from the full PONS matrix or certain sub-matrices of it.

We recall the symmetric PONS construction [1]. It starts from any pair of complementary sequences $\mathbf{w}_1$ and $\mathbf{w}_2$, of length N, and obtains four vectors of length $2N$

by means of the recursion

$$\begin{bmatrix} \mathbf{w}_1 & \mathbf{w}_2 \\ \mathbf{w}_1 & -\mathbf{w}_2 \\ \mathbf{w}_2 & \mathbf{w}_1 \\ -\mathbf{w}_2 & \mathbf{w}_1 \end{bmatrix} \quad (21)$$

This construction technique generates sets of complementary waveforms where each row is the complement of the adjacent row.

The temporally diverse waveforms are just obtained from the full PONS matrix or certain submatrices of it. We begin the PONS recursion with the vectors

$$W_1 = \begin{bmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad (22)$$

and recursively apply the PONS construction (21) to each pair $\mathbf{w}_{2m-1}$ and $\mathbf{w}_{2m}$ of adjacent rows of the W_{s-1} PONS matrix to form the $2^s \times 2^s$ PONS matrix W_s.

Define the $N \times N$ matrix

$$G = [g_{vw}] \quad (23)$$

where

$$g_{vw} = \begin{cases} 1 & v = 2(s-1)+1, w = v+1, s = 1, \ldots, \frac{N}{2} \\ 1 & v = 2s, w = v-1, s = 1, \ldots, \frac{N}{2} \\ 0 & \text{otherwise.} \end{cases} \quad (24)$$

Then let $p_k^{(1)}(t)$ and $p_k^{(2)}(t)$ be the $2^s \times 2^s$ matrices

$$P^{(1)} = W_s \quad (25)$$

$$P^{(2)} = GW_s \quad (26)$$

In other words, $P^{(2)}$ is the PONS matrix with the odd and even rows swapped. Thus, when these waveforms are transmitted over $M = 2$ CPIs each waveform is complementary with the waveform in the adjacent PRI and also the equivalent PRI in the next CPI.

As these waveforms are binary coded waveforms, (20) is obviously satisfied. To show that these waveforms satisfy (19), recall that showing this holds is equivalent to showing that

$$\sum_{i=1}^{2} \left| FQ^{(i)}F' \right|^2 \quad (27)$$

is constant, where

$$Q^{(i)} = FP^{(i)}. \quad (28)$$

Therefore

$$\sum_{i=1}^{2} \left| FQ^{(i)}F' \right|^2 = \sum_{i} \left| FFP^{(i)}F' \right|^2 \quad (29)$$

$$= N^2 \sum_{i} \left| P_{\text{perm}}^{(i)}F' \right|^2 \quad (30)$$

where $P_{\text{perm}}^{(i)}$ is the matrix $P^{(i)}$ with the rows permuted. Equation (30) is equivalent to taking the Fourier transform along the rows of the PONS matrix, i.e. in the t index. By construction, the sum of the Fourier transform of a PONS waveform and its complement is a constant, therefore (19) holds.

The default method of construction of these waveforms generates waveforms that have the same number of PRIs as chips in each pulse. However, this is not a requirement. Provided the number of PRIs is a power of two, the number of PRIs can be reduced. In other words, the sets of waveforms for each frequency channel can be given by $p_k^{(i)}(t)$ for $k = 0, \ldots, (2^r - 1)$ and $t = 0, \ldots, (2^s - 1)$ where $r < s$.

B. 4-Complementary PONS

The “4 complementary” waveforms are binary coded waveforms constructed in such way that they are complementary in quartets. Their transmissions requires four separate time or/and frequency channels. Assembled over one two or four coherent (or rather Doppler) processing intervals, they provide essentially “perfect” range and doppler resolution. In this section we use 4-complementary PONS in a temporal diversity mode. The resulting matrix is complementary in fours. That is, the first four rows form a complementary quartet, as do the second four, as do any four rows numbered $4k+1,\ 4k+2,\ 4k+3, 4k+4$ for any k. The construction of these waveforms is described in [4]. There is considerable flexibility in their construction as described there.

We can construct these waveforms to form, for any power 4^M of 4, 4 matrices $D_M^1,\ D_M^2,\ D_M^3,\ D_M^4$ of size $4^M \times 4^M$, where the remaining three matrices are obtained from the first by cyclically permuting the adjacent rows forming complementary quartets.

We use these matrices in a temporal and frequency diversity context by spreading them across 4 frequency channels, so that the kth PRI of the mth channel is the kth row of D_M^j. These are individually match-filtered and then Doppler processed.

V. SIMULATIONS

In this section results of computer experiments are given for the waveforms described in Section IV-A and IV-B.

The simulations assume an S-band radar(3 GHz carrier frequency). The PRI is assumed to be equal to 0.1 milliseconds. The chip length is 10 nanoseconds for the 256-chip pulse, in order to keep time-scales relatively realistic.

Two figures are associated with each pulse length:

1) The first figure shows the ambiguity response to a point target of a given velocity. The target velocities are chosen to be 0, 20, 40, 60, 80 and 100 meters per second. These images have a color map in dB to the max absolute value of the ambiguity.
2) The second figure gives range sections across the ambiguities shown in the previous figure.

In all these figures the number of PRIs in each CPI was equal to the number of chips in each pulse (256). For comparison these experiments were performed using temporarily diverse pseudo-random (PN) waveforms, Frank code and P4 code, all of length 256 chips. The results of these experiments are represented in Figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

VI. Conclusion

As can be seen from the figures that constant amplitude and “4-complementary” outperform the other waveforms in this instance over most of the range of Dopplers under consideration. While this is not always the case, there are many circumstances where the complementary codes used appropriately outperform the others because of their low (essentially null) range sidelobes.

While we do not propose that complementary waveforms used in this way are optimal in all circumstances, we believe they have a role in a library of waveforms to be used in a waveform scheduled radar system. Such libraries have been tested in [3] and elsewhere.

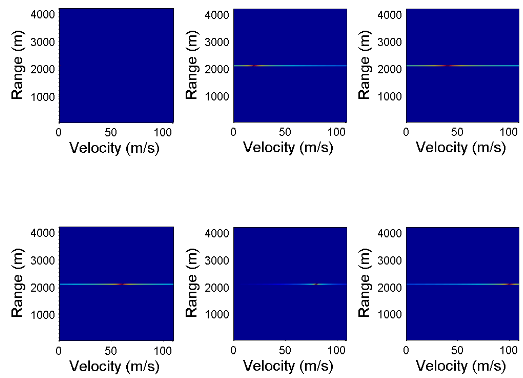

Fig. 1. Ambiguity Function (dB) for temporally diverse constant amplitude waveforms for various velocity targets

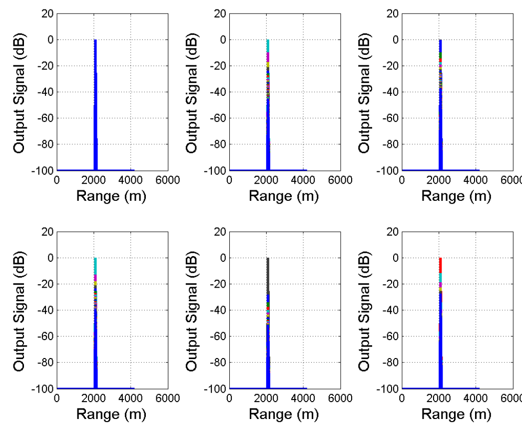

Fig. 2. Range plots for temporally diverse constant amplitude waveforms for various velocity targets

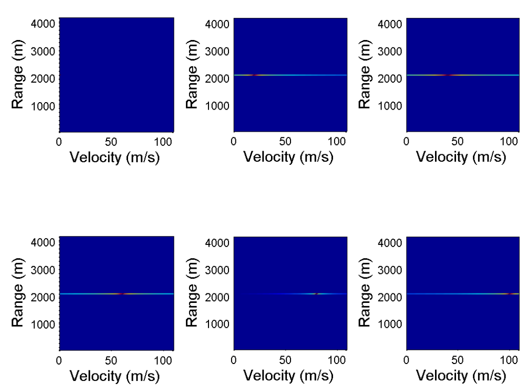

Fig. 3. Ambiguity Function (dB) for temporally diverse 4-complementary waveforms for various velocity targets

References

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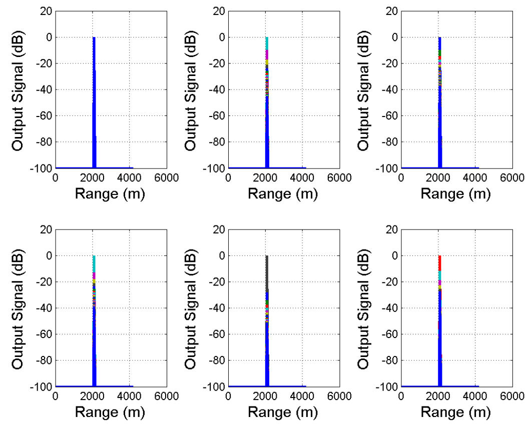

Fig. 4. Range plots for temporally diverse 4-complementary waveforms for various velocity targets

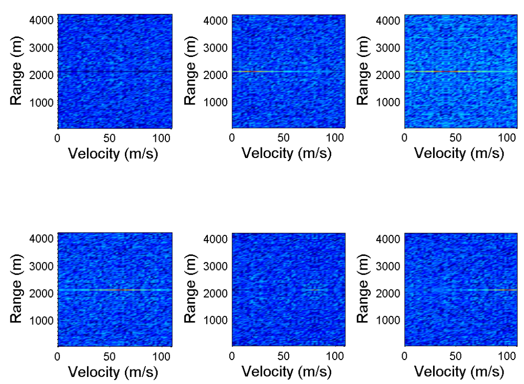

Fig. 5. Ambiguity Function (dB) for pseudo-random waveforms for various velocity targets

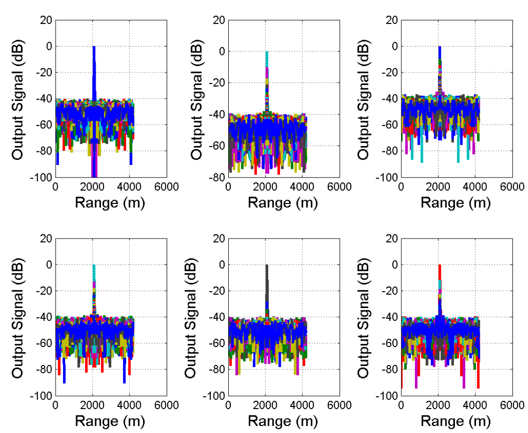

Fig. 6. Range plots for pseudo-random waveforms for various velocity targets

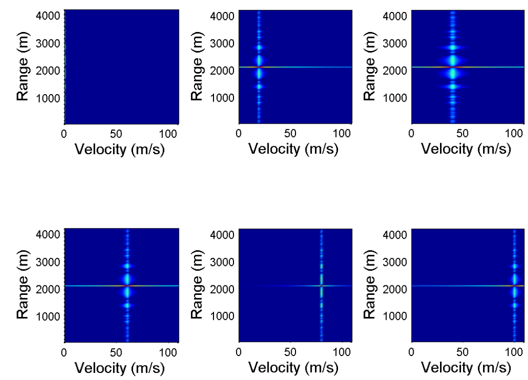

Fig. 7. Ambiguity Function (dB) for Frank waveforms for various velocity targets

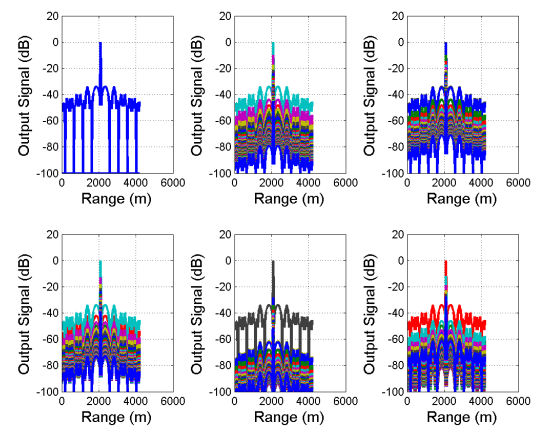

Fig. 8. Range plots for Frank waveforms for various velocity targets

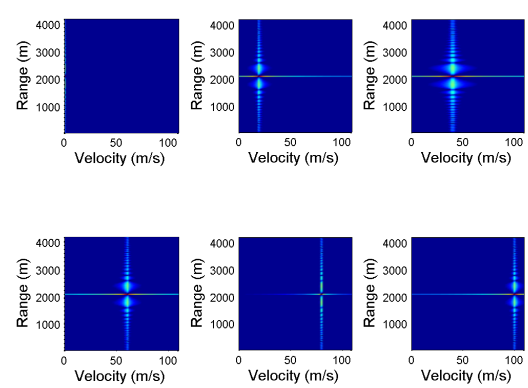

Fig. 9. Ambiguity Function (dB) for P4 waveforms for various velocity targets

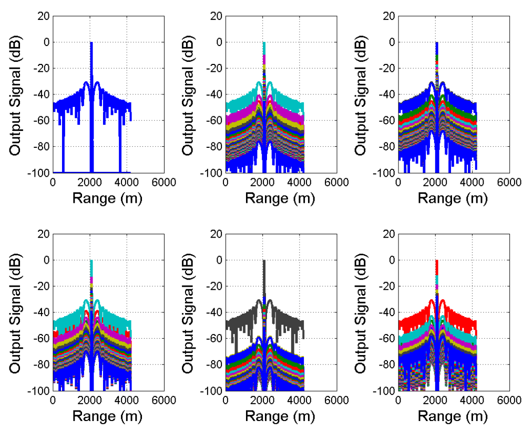

Fig. 10. Range plots for P4 waveforms for various velocity targets