BEAM AND WAVEFORM SCHEDULING APPROACH TO COMBINED RADAR SURVEILLANCE & TRACKING – *THE PARANOID TRACKER.*

S. Suvorova[1], S. D. Howard[2], W. Moran[1]

[1]Department of Electrical and Electronic Engineering, The University of Melbourne, Victoria, 3010, Australia.
[2]Defence Science & Technology Organisation, PO BOX 1500, Edinburgh 5111, Australia.
Email: {suvorova, b.moran}@ee.unimelb.edu.au, stephen.howard@dsto.defence.gov.au

Abstract

In several previous papers, we have applied single and multiple step ahead scheduling techniques to reduce the amount of time an adaptive radar spends on tracking known targets while retaining a given level of track accuracy. This paper reports on our initial study of practical methods for achieving a unification of surveillance and tracking in terms of radar resource management. The method proposed here involves the introduction of permanently existing virtual targets, judiciously placed in the field of view of the radar. The tracker's belief in the existence of the virtual or fictitious targets led us to the name *Paranoid Tracker* for this approach.

1 The Paranoid Tracker

Modern radar systems have considerable flexibility in their modes of operation. In particular, it is often possible to modify the waveform on a pulse-to-pulse basis, and electronically steered phased arrays can quickly point the radar beam in any feasible direction. Such flexibility calls for new methods of scheduling both the waveform and the beam direction so as to optimize the radar performance.

We consider a radar system capable of both rapid beam steering and of waveform switching. The transmit waveform is chosen from a small library of such. One operational requirement of the radar is to detect and track a number of manoeuvring targets while performing surveillance for new potential targets. In several previous papers (see, for example, [9]), we have applied single and multiple step ahead scheduling techniques to reduce the amount of time spent on tracking known targets while retaining a given level of track accuracy. By doing this, we permitted the sensor to spend more time in surveillance for new targets. In a multi-function radar the radar may need to perform tasks other than detection and tracking, such as target identification, or to obtain more precise information about a particular target. So ideally we would like to extend the scheduling techniques we have developed for tracking to the combined surveillance-tracking problem. The idea is to constrain the detection/tracking performance of the radar system to within acceptable bounds and to use beam and waveform scheduling to minimise the proportion of the radar's time needed to achieve this goal.

This paper reports on our initial study of practical methods for achieving this unification of surveillance and tracking in terms of resource management. Previous work along these lines by La Scala, Moran, and Evans [5], was based on an entropic measure of the radar's knowledge of the existence of targets in various states at each point in the range-Doppler-angle space. However, such entropy based methods, although theoretically attractive, are extremely computationally intensive. The method proposed here introduces virtual targets, judiciously placed in the field of view of the radar. In the present study these take the form of long range targets equi-spaced around the perimeter of the radar surveillance area. The information that the tracker has about the virtual targets degrades over time, and this forces a re-visit in the direction of each virtual target if the tracker has not already looked in that direction in maintaining tracks of real targets.

In the current work, tracking is accomplished by means of an LMIPDA tracker as described in [8]. An Integrated Multiple Model approach (IMM) is taken to model for the manoeuvering targets in the tracker. The LMIPDA tracker provides a probability of track existence, permitting a "track-before-detect" methodology to be adopted. "False alarm" tracks are maintained until the probability of track existence falls below a threshold. The aim of the tracker is to maintain the tracks of the existing targets to within a specified accuracy as determined by the absolute value of the determinant of the track error covariance matrix. The control of the surveillance function of the radar is purely through the use of virtual targets. These virtual targets have probabilities of existence that never go to zero, since they always "produce" measurements. The tracker's belief in the existence of the virtual or fictitious targets led us to the name *Paranoid Tracker* for this approach.

Having populated the radar's surveillance region with permanently existing virtual targets, we then use the scheduling algorithms proposed in [9] for tracking alone. These algorithms aim to minimise the total proportion of time spent in tracking. Specifically, at each epoch a target track and a beam direction have to be selected. The scheduler has a list $\Delta = \{\delta_1, \delta_2, \ldots, \delta_L\}$ of "revisit intervals". Each of the numbers δ_k is a number of epochs representing the possible times between measurements of any of the existing targets. It is assumed, for the purposes of scheduling and tracking, that during any of these revisit intervals the target dynamics do not change, though the simulator permits target maneuvers on an epoch by epoch basis.

2 Problem Formulation

We postulate a radar system tracking T targets, where T is a random variable $0 \leq T \leq T_0$ and the tth target is in state $x^t(k)$ at epoch k. We aim to schedule revisit times to targets, both real and virtual.

In IMM-based algorithms the trajectory of the target is assumed to be described at any time by one of $M < \infty$ predefined dynamical models. In this context, we assume that the dynamical models are independent of the target and associated to each is a corresponding state propagation matrix F_m $(m = 1, 2, \ldots, M)$. The recursion equation describing state transitions is

$$x^t(k) = F_m(k)x^t(k-1) + \nu_m^t(k), \qquad (1)$$

where the index m is a possible value of a random variable $M(k)$, the *dynamical model* which takes any discrete value $[1, 2, \cdots, M]$. The Gaussian process noise variables $\nu_1^t(k), \cdots, \nu_M^t(k)$ depend on both target and dynamical model and are independent for different values of each of the indices. The covariance matrix of $\nu_m^t(k)$ is denoted by $Q_m^t(k)$.

In the tracker, the dynamical model of the tth target $M^t(k)$ is assumed to evolve as a Markov Chain with given transition probabilities, denoted by

$$\pi_{m,\ell}^t = P\{M^t(k) = m | M^t(k-1) = \ell\}; m, \ell \in [1, \cdots, M]. \qquad (2)$$

It is assumed that N different *measurement modes* are available for each target, each given by a measurement matrix H_n^t $n = 1, 2 \ldots, N$:

$$z^t(k) = H_n^t(k)x^t(k) + \omega_n^t(k) \qquad (3)$$

where $z^t(k)$ is the measurement to be obtained from the tth target at time k, $\omega_n^t(k)$ is the measurement noise, and $n = n(k)$ is a control variable for the measurement mode. We also permit measurement of only one target at each epoch. The variable $\widetilde{t} = \widetilde{t}(k)$ represents the choice of target to which the beam is steered at the kth epoch. The measurement noise variables $\omega_1^t(k), \cdots, \omega_N^t(k)$ are zero mean white and uncorrelated Gaussian noise sequences with the covariance matrix of $\omega_n^t(k)$ denoted by $R_n^t(k)$. Here we take all of the measurement matrices to be identical, but in each case the noise is waveform dependent.

In cluttered environments, potential measurements result from both targets and clutter scatterers at each scan. Target measurements are unidentified, but assumed to be present with probability of detection $P_D^t(k)$. The set of actual measurements obtained at time k, selected with *gating probability* P_G, is denoted by $z(k)$, and the i-th measurement from this (ordered) set $z(k)$ by $z(i, k)$, where $i = 1, \cdots, m_k$; $m_k \geq 0$ is the number of measurements at time k. This selection of measurements is often referred to as *gating* [1]. The history of all selected measurements up to and including time k is denoted by $Z^k = z(k) \bigcup Z^{k-1}$. Given the dynamics Eq. (1) and measurements Z^k, the tracker aims to estimate posterior probability of target existence $\psi_{k|k}^t$, the state estimate and error covariance, $\hat{x}_{k|k}^t$ $P_{k|k}^t$ respectively for the tth target.

3 Tracking

Two choices are made at each epoch, the target to be measured and the waveform used. The choice of measurement is made using the control variable $n(k)$.

It may be that more than one target is in the beam during a particular radar tansmission, in which case the measurements will be processed using the LMIPDA-IMM algorithm described in [8]. We give an overview of this algorithm here. This is a recursive algorithm combining a multitarget data association algorithm (LMIPDA) with manoeuvring target state estimation implemented using the IMM approach. The IMM component of the tracker acts as a filter bank, one for each possible target trajectory model. For each track t, the filter inputs are:

- *target existence*, modelled by a random variable χ, assumed to evolve as a Markov chain and taking either of two values [7]. $\chi = 1, 0$ according as to whether the target exists or not. The value of χ for each track t is updated with measurements at each scan.
- *predicted state for each IMM model* j, obtained from the previous state by state propagation:
  - *state prediction probability density function (pdf)* $p^t(x_k | M_k = j, Z^{k-1})$, described by its mean $\hat{x}_{k|k-1}^t(j)$ and its error covariance $P_{k|k-1}^t(j)$,
  - *predicted model state probability*

    $$\mu_{k|k-1}^t(j) \triangleq P^t\{M_k = j | Z^{k-1}\}$$

    and
  - *a priori measurement pdf* $p^t(z_k | M_k = j, Z^{k-1})$.

- *measurement set delivered by sensor at time* k, which may be empty.

For each track t, the filter outputs at time k are:

- *a posteriori probability of target existence* $\psi^t_{k|k}$, then used for confirmation or termination of tracks;
- *track state estimate and estimate covariance,* $\hat{x}^t_{k|k}$ *and* $P^t_{k|k}$;
- *filter inputs for time* $k+1$ *for next recursion*, enumerated above, $\psi^t_{k+1|k}$, and, for each IMM model j, $\hat{x}^t_{k+1|k}(j)$, $P^t_{k+1|k}(j)$, $\mu^t_{k+1|k}(j)$ and $p^t(z_{k+1}|M_{k+1}=j, Z^k)$.

The choice of waveforms influences the measurement process through the covariance matrix of the noise $\omega^t_n(k)$ [4]. In this model, the covariance matrix is waveform dependent,

$$R_\phi = TJ_\phi^{-1}T, \tag{4}$$

where J_ϕ is the Fisher information matrix corresponding to the measurement using waveform ϕ and T is the transformation matrix between the time delay and Doppler measured by the receiver and the target range and velocity. It should be pointed out that the use of the Fisher matrix here is an approximation. It really corresponds to the Cramér-Rao lower bound on the estimator for the target from this measurement. It can be shown that the estimator here is *asymptotically efficient* [3] in that the covariance matrix approaches the Cramér-Rao lower bound over a large number of measurements.

4 Scheduling

The scheduling algorithm has a list $\Delta = \{\delta_1, \delta_2, \ldots, \delta_K\}$ of "revisit intervals". Each of the integers δ_k represents the possible number of epochs between measurements of any of the existing targets. It is assumed for the purposes of scheduling and tracking that during any of these revisit intervals the target dynamics do not change, though the simulator permits target maneuvers on an epoch by epoch basis.

The scheduling algorithm determines which target to measure and which waveform to use in the measurement as follows. For each existing target and each waveform the track error covariance $P^t_{k-1|k-1}$ is propagated forward using the dynamic model equations. This is done for each of the potential revisit intervals in the list Δ and each of the waveforms in the library. Evidently the number of combinations grows exponentially in the number of steps ahead, and soon becomes impractical for implementation. In this paper we have restricted our attention to single step ahead scheduling. Then, having obtained the error covariance matrix for all possible combinations of sensor modes, the optimal sensor mode (waveform) is then chosen for each target, including the virtual targets, to be the one which gives the longest re-visit time, while constraining the absolute value of the determinant of the error covariance matrix to be smaller than a prescribed upper limit K. In other words, our objective is the choose the waveform–revisit time pairs ϕ, δ, which satisfy

$$\phi, \delta = \arg\max \Delta, \tag{5}$$

subject to

$$\det(P_{k|k}) \leq K. \tag{6}$$

Once a target is measured, its revisit time is re-calculated. In the absence of measurements the best we can do is to use the current knowledge to predict forward and update the covariance matrix, dynamic model pdf and probability of track existence. The algorithms now becomes as follows:

- *IMM mixing* [2, 6, 8] is conducted as usual;
- *Forward prediction* is then performed separately for each dynamical model. Because the dynamics of the target depends on the revisit time $\delta \in \Delta$ this calculation is performed for each revisit time.
- *Covariance update:* this is normally done with the data, but since we are interested in choosing the best sensor mode at this stage the following calculations are required. If the target does not exists there will be no measurements originating from the target and the error covariance matrix is equal to the *a priori* covariance matrix; if the target exists, is detected, and the measurement is received then the error covariance matrix is updated using the Kalman equation, namely,

$$\begin{aligned} P_{k|k}(j,\delta) &= (1-\psi_{k|k-1}P_D P_G)P_{k|k-1}(j,\delta) \\ &+ \psi_{k|k-1}P_D P_G(I-K(\phi,\delta)H)P_{k|k-1}(j,\delta) \\ &= (I-\psi_{k|k-1}P_D P_G K(\phi,\delta)H)P_{k|k-1}(j,\delta). \end{aligned} \tag{7}$$

where $\psi_{k|k-1}$ is the prior probability of track existence, $P_D P_G$ is the probability that target is detected and its measurement is validated. $K(\phi,\delta)$ is a Kalman gain calculated for each sensor mode; that is, for the waveform ϕ and revisit time δ. Both ϕ and δ take discrete values from waveform library and revisit time set Δ.

$$K(\phi,\delta) = P_{k|k-1}(j,\delta)HS^{-1}(\phi), \tag{8}$$

where S is the innovation covariance matrix, calculated as

$$S = HP_{k|k-1}H^T + R_\phi. \tag{9}$$

- The dynamic model and track existence pdfs are updated in a similar manner. If the target does not exist it produces no measurement; if it does exist, and is detected, then the expected measurement pdf is calculated as follows:

$$\begin{aligned} p_j(z_k) &= p(z_k|\chi_k = 1, M_k = j, Z^k) \\ &= \mathcal{N}(H\hat{x}_{k|k-1}(j); S_j); \\ p(z_k) = p(z_k|\chi_k = 1, Z^k) &= \sum_{i=1}^{M} \mu_{k|k-1}(i) p_i(z_k). \end{aligned} \tag{10}$$

The dynamic model and track existence pdfs are updated :

$$\begin{aligned} \mu_{k|k}(j) &= \frac{p_j(z_k)\mu_{k|k-1}(j)}{\sum_{i=1}^{M} p_i(z_k)\mu_{k|k-1}(i)} \\ \psi_{k|k} &= \frac{p(z_k)\psi_{k|k-1}}{\rho(z_k)(1-\psi_{k|k-1}) + p(z_k)\psi_{k|k-1}}, \end{aligned} \tag{11}$$

where $\rho(z_k) = p(z_k|\chi_k = 0, Z^k)$ is the estimated clutter and other targets density at z_k(see [8]).

- The next step is to combine the estimates for all dynamics models $j = 1, \ldots, M$ into one, using the standard "IMM combination" formulae [2, 6, 8].

Once the error covariance matrix is obtained for all possible combinations of sensor modes, the optimal sensor mode (waveform) is then chosen for each target to be the one which gives the longest re-visit time, while constraining the absolute value of the determinant of the error covariance matrix to be smaller than the prescribed upper limit K. In other words, our objective is

$$\phi, \delta = \arg\max \Delta, \text{ subject to } |\det(P_{k|k})| \leq K. \tag{12}$$

Once a target is measured, its revisit time is re-calculated.

We note that for many manoeuvring targets there may be no solution to the scheduling problem that satisfies the constraints. In this case, the optimum waveform and re-visit time are those with minimum $\det(P_{k|k})$.

5 Simulations

Monte-Carlo simulations were performed to compare the paranoid tracker with the tracker described in [9], which had surveillance of the region scheduled at regular intervals.

The virtual target distribution in the stimulations was as follows. Eight virtual targets were equally spaced across 90 deg of azimuth, such that all of the targets where 15 km away from radar. The virtual targets do not move during the

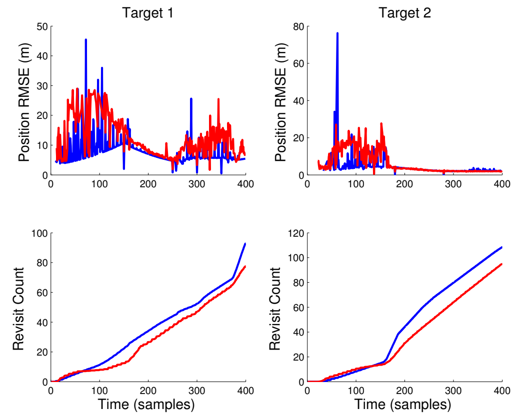

Figure 1: RMSE and re-visit times for each of the two "real" targets in the surveillance region for the first 100 seconds for the reference tracker (blue) and the paranoid tracker (red).

simulation. In this paper we give results only for this configuration. In a future paper we will consider the effect that the number of virtual targets and there distribution have on the surveillance/tracking performance.

The simulation scenario involved tracking of two targets with crossing trajectories. The measures of performance used were the accuracy of tracking by means of RMSE (root mean square error) of target position, the number of re-visits to track targets and the average time used by radar to process the surveillance region. Here "process" is used to mean the collection of measurements from the scene, initiation of new targets, and tracking of existing targets.

Figure 1 shows RMSE and re-visit times for each of the two "real" targets in the surveillance region for 100 seconds. This shows that the amount of time spent on tracking the two targets with the paranoid tracker was significantly smaller than tracking with a regularly scheduled surveillance regime, while tracking errors were maintained within tight bounds.

Figure 2 shows the proportion of time used by the paranoid tracker when compared with the tracker with regularly scheduled surveillance (which used 100 percent of the time). The paranoid tracker saves on average more than 30 percent of the radar processing time. This time can be used by the radar for other functions.

6 Conclusion

We have shown that the introduction of permanently existing virtual targets can be used to combine radar resource management for surveillance and tracking within a single framework. This framework consists of a sensor mode adaptive multitarget tracking algorithm, which seeks to

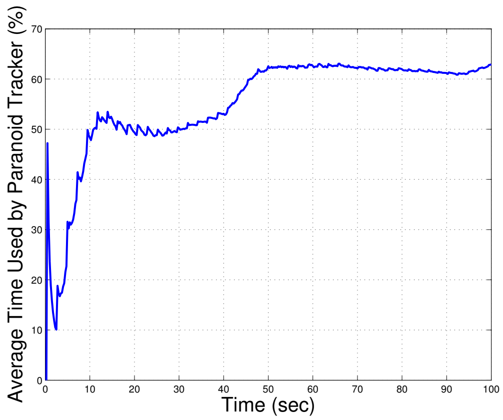

Figure 2: Proportion of the time used by paranoid tracker compared to the tracker with regularly sheduled surveillance.

minimise the proportion of radar time needed to carry out its surveillance and tracking tasks to a certain level of performance, thus releasing time for the radar to perform other tasks.

Future research will involve consideration of the effect that the number of virtual targets and there distribution have on the surveillance/tracking performance.

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