SEQUENTIAL MONTE CARLO METHODS FOR SHALLOW WATER TRACKING USING MULTIPLE SENSORS WITH ADAPTIVE FREQUENCY SELECTION

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ABSTRACT

We propose a matched-field processing framework for tracking problems in shallow water environments where the conventional plane-wave assumptions do not hold. Multiple passive acoustic sensors are employed to collect observation data, and sequential Monte Carlo techniques are used for tracking due to the high nonlinearity in the dynamic state formulation. In order to enhance the tracking performance, we design a frequency selection algorithm which adaptively chooses the optimal observation frequency for the sensors at each time instant. The improved tracking performance is demonstrated using simulations.

1. INTRODUCTION

Matched-field processing uses complex acoustic propagation models for underwater signal processing such as shallow water environmental identification, and source localization and tracking. Matched-field tracking (MFT) methods also incorporate the motion of the source, and some examples include environmental source tracking, multi-valued Bartlett processing, ambiguity surface averaging, optimum uncertain field tracking, and optimal minimum variance tracking-before-detect (MV-TBD) [1, 2]. In [1], the source motion model is assumed to be a uniformly moving target that does not change velocity over the integration period of the track. The optimum uncertain field tracking approach introduces a Markov model for the source motion as a means of capturing the stochastic nature of the source dynamics without assuming uniform motion. This approach does assume *a priori* that the source moves only in the neighborhood grid of its previous location between observations. The MV-TBD method follows the same model but exploits the derived equivalence between adaptive minimum variance beamforming and maximum likelihood source localization.

We propose a general MFT framework for shallow water tracking that is based on a widely used motion model and the shallow water sound field representation for the dynamic system formulation. This framework is important in solving more general positioning, navigation, and tracking problems using MFT approaches. Specifically, multiple passive acoustic sensors are distributed at different positions in the water column to collect the observation data in order to correctly track the target in shallow water environments. Due to the highly nonlinear relationship between the observations and the moving target's parameters, we employ unscented Kalman filter and particle filter tracking algorithms. We then develop a dynamic frequency selection algorithm for the sensors to minimize the predicted mean square error (MSE) of the state's estimates in order to enhance the tracking algorithm performance.

The paper is organized as follows. In Section 2, we present the the shallow water environment model when the sensors are stationary and the source moving. In Section 3, we estimate the resulting dynamic state formulation using the unscented Kalman filter and the particle filter. The frequency selection algorithm for agile sensing is presented in Section 4, and in Section 5, we demonstrate its utility in increasing tracking performance using simulations.

2. GENERAL MATCHED-FIELD TRACKING

We consider a stratified waveguide model for a shallow water environment which consists of the sea surface, a water column, multi-layer sediment, and a semi-infinite basement. A point source in the water column radiates an acoustic signal, and the excited sound field is sampled by multiple acoustic sensors distributed at different positions in the water column away from the source.

The waveguide transfer function from the source to the receiver can be obtained from the point source solution to the wave equation; it depends on both the source-receiver configuration and the environmental parameters. Let us assume that we have N independent sensors for collecting data, and that each sensor can observe a single frequency at each time instant. The received scalar observation at the nth sensor at discrete time epoch k can be expressed as

$$y_k^n = c_k^n \, \mathrm{G}\left(\mathbf{R}_k^n, \mathbf{S}_k, \Psi\right) + w_k^n \, . \tag{1}$$

Here, $\mathbf{R}_k^n = [r_k^n \; z_k^n \; f_k^n]$ is the parameter vector of the nth sensor where (r_k^n, z_k^n) are the range and depth coordinates,

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respectively, of the sensor's position, and f_k^n is the observation frequency. The position and the velocity of the moving target is given by $\mathbf{S}_k = [r_k^s \; z_k^s \; \nu_k^{r_s} \; \nu_k^{z_s}]^{\mathbb{T}}$, and Ψ represents the environment parameters (e.g., bathymetry, sound speed profile, attenuation, density). The determined transfer function for the propagation from the source to the nth sensor is given by $\mathrm{G}\left(\mathbf{R}_k^n, \mathbf{S}_k, \Psi\right)$, and w_k^n is additive noise. The time-varying parameters c_k^n incorporate amplitude and phase variability into the transfer function. These parameters can be modeled by a linear regression process with coefficients a^n driven by white noise as $c_k^n = a^n \; c_{k-1}^n + v_k^n$, $n = 1, \cdots, N$. If we define $\mathbf{C}_k = [c_k^1, \cdots, c_k^N]^{\mathbb{T}}$, $\mathbf{V}_k = [v_k^1, \cdots, v_k^N]^{\mathbb{T}}$, $\mathbf{A} = \mathrm{diag}\{a^1, \cdots, a^N\}$, then $\mathbf{C}_k = \mathbf{A}\mathbf{C}_{k-1} + \mathbf{V}_k$. Here we assumed that $\mathbf{V}_k$ is a zero-mean Gaussian random variable and $\mathrm{E}\{\mathbf{V}_k \mathbf{V}_k^{\mathbb{H}}\} = \mathbf{Q}_V$.

The dynamics model is given by $\mathbf{S}_k = \mathbf{F}\mathbf{S}_{k-1} + \mathbf{u}_k$, where $\mathbf{u}_k$ are independent, zero-mean Gaussian samples with covariance matrix $\mathbf{Q}$. Note that these samples model the target acceleration. The matrices $\mathbf{F}$ and $\mathbf{Q}$ are defined in [3]. If we define $\mathbf{X}_k = [\mathbf{S}_k^{\mathbb{T}} \; \mathbf{C}_k^{\mathbb{T}}]^{\mathbb{T}}$ as the tracking state vector and $\mathbf{U}_k = [\mathbf{u}_k^{\mathbb{T}} \; \mathbf{V}_k^{\mathbb{T}}]^{\mathbb{T}}$ as the noise vector, then the state equation for tracking can be represented as

$$\mathbf{X}_k = \mathbf{F}_x \mathbf{X}_{k-1} + \mathbf{U}_k, \tag{2}$$

where $\mathbf{F}_x = \mathrm{diag}\{\mathbf{F}, \mathbf{A}\}$ and $\mathrm{E}\{\mathbf{U}_k \mathbf{U}_k^{\mathbb{H}}\} = \mathbf{Q}_U$.

The corresponding measurement equation for the tracking problem is given by

$$\mathbf{Y}_k = \mathbf{H}\left(\mathbf{X}_k, \mathbf{R}_k, \Psi\right) + \mathbf{W}_k, \tag{3}$$

where $\mathbf{Y}_k = [y_k^1, \cdots, y_k^N]^{\mathbb{T}}$, $\mathbf{R}_k = [\mathbf{R}_k^1, \cdots, \mathbf{R}_k^N]^{\mathbb{T}}$, and $\mathbf{H}(\mathbf{X}_k, \mathbf{R}_k, \Psi) = [c_k^1 \mathrm{G}(\mathbf{R}_k^1, \mathbf{S}_k, \Psi), \cdots, c_k^N \mathrm{G}(\mathbf{R}_k^N, \mathbf{S}_k, \Psi)]^{\mathbb{T}}$. $\mathbf{W}_k$ represents the observation noise with covariance $\mathbf{Q}_W$. Hence, the general MFT problem can be formulated as a dynamic problem using Equations (2) and (3).

As a special case of the shallow water environment model, we consider a simplified ideal model. Specifically, the ocean surface (at $z = 0$) is realistically modeled as an ideal pressure release boundary and the ocean bottom (at $z = D$) as an ideal rigid boundary. A waveguide model corresponding to this scenario is shown in Fig. 1, where Medium I, II and III correspond to air, ocean water and seabed, respectively. At time k, an omnidirectional point source is located in Medium II at the coordinates (r_k^s, z_k^s), and the ocean is D m deep. The environment parameter set is given by $\Psi = \{D, c\}$ where c m/s is the sound speed in Medium II. If we let $\Theta_k = \{\mathbf{R}_k^n, \mathbf{S}_k, \Psi\}$, then following the normal-mode model for shallow water environments [4], the transfer function between the target source and the nth sensor in (3) can be represented as

$$\mathrm{G}(\Theta_k) = \sum_{m=0}^{N_p - 1} C_m\left(\Theta_k\right) M_m\left(\Theta_k\right). \tag{4}$$

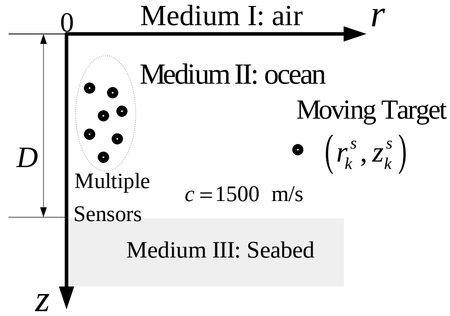

Fig. 1. Waveguide model with point source in Medium II at $r = 0$, $z = z_0$; the ocean is D m deep.

Here, $C_m\left(\Theta_k\right) = e^{-j\frac{\pi}{4}} \frac{1}{2\pi D} \sin(k_m^z z_k^s) \sin(k_m^z z_k^n)$ and $M_m(\Theta_k) = \left(\frac{((f_k^n)^2 - f_m^2)^{\frac{1}{2}}}{c} L_k^n\right)^{-\frac{1}{2}} e^{-j2\pi \frac{L_k^n((f_k^n)^2 - f_m^2)^{\frac{1}{2}})}{c}}$ where $f_m = \frac{(2m+1)c}{4D}$ is the cutoff frequency of the mth mode, N_p is the largest mode number, $L_k^n = r_k^s - r_k^n$ is the range between the target source and the nth sensor, and $k_m^z = \frac{(2m+1)\pi}{2D}$.

If we substitute (4) into (1), we can obtain the measurement equation for the general MFT problem for this simplified model. The following sections of the paper are based on this shallow water model and use (4) for calculating the measurements. For more sophisticated models, we may not be able to calculate the received signal using closed form equations, thus requiring numerical results. However, the methodology presented in this paper can be readily applied to the more sophisticated models as well.

3. MATCHED-FIELD TRACKING ALGORITHMS

We can use nonlinear estimation algorithms, such as the *unscented Kalman filter* (UKF), the particle filter (PF) and the *unscented particle filter* (UPF), to estimate the state of the moving target. This is because the observation vector $\mathbf{Y}_k$ of the sound field is a highly nonlinear function of the target's position, observation frequency, time-varying attenuation factors, and environment parameters. The tracking performance of these nonlinear algorithms is shown in Fig. 2 obtained from numerical simulations using the aforementioned simplified shallow water model with $D = 300$ m and $c = 1,500$ m/s. The sensor network uses three stationary sensors whose parameter sets are $\mathbf{R}_0^1 = [0, 120, 500]$, $\mathbf{R}_0^2 = [10, 180, 500]$ and $\mathbf{R}_0^3 = [7, 240, 500]$, respectively. As we can see in Fig. 2, the best performance is obtained by the UPF. The UKF can also obtain relatively good performance while requiring the minimum computational complexity among these three algorithms.

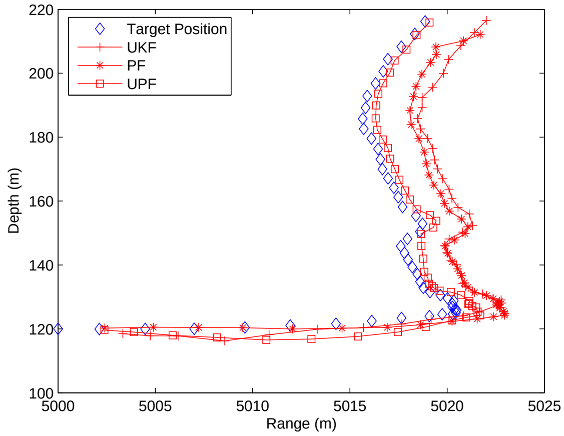

Fig. 2. Tracking performance using the UKF, PF and UPF algorithms.

4. FREQUENCY SELECTION

We can enhance the performance of the tracking algorithms by dynamically selecting the observation frequencies for each sensor to minimize the predicted tracking MSE. The reason for choosing the predicted MSE as our optimization criterion is that we do not have access to either the observation or the target state at the next observation time step. Given the sequence of observations $\mathbf{Y}_{1:k-1}$ from 1 to $k-1$, the predicted MSE for the kth state estimate is given by

$$J(\mathbf{R}_k) = \mathrm{E}_{\mathbf{X}_k,\mathbf{Y}_k|\mathbf{Y}_{1:k-1}}\{(\mathbf{X}_k - \hat{\mathbf{X}}_{k|k})^{\mathbb{T}}(\mathbf{X}_k - \hat{\mathbf{X}}_{k|k})\}, \quad (5)$$

where $\mathrm{E}\{\cdot\}$ is the expectation over $\mathbf{X}_k$ and $\mathbf{Y}_k$ given $\mathbf{Y}_{1:k-1}$, and $\hat{\mathbf{X}}_{k|k}$ is the estimate of $\mathbf{X}_k$ given $\mathbf{Y}_{1:k}$. As the predicted MSE is a function of the sensor parameters, we can minimize it by selecting the parameters at time k [5]. Specifically, assuming that the sensors are immobile, we can minimize the predicted MSE by choosing the appropriate observation frequency $\{f_k^1, \cdots, f_k^N\}$ for the sensors. In this paper, to reduce the computational complexity, we assume that all the sensors employ the same frequency at time k, i.e., $f_k^n = f_k^m = f_k,\ n, m = 1, 2, \cdots, N$.

The predicted MSE can be approximated using the unscented transform (UT). Let $\mathbf{P}_{k-1|k-1}$ denote the error covariance of the estimated state, and $\hat{\mathbf{X}}_{k-1}$ be the estimated state at time $k-1$. Then, the predicted MSE in (5) can be approximated using the UT [6] by following the steps in Table 1. The approximate expected MSE is given by $J(\mathbf{R}_k) = \mathrm{Trace}\{\mathbf{P}_{k|k}\}$. As we assumed that the sensors are stationary and use the same frequency f_k, we can also write $J(\mathbf{R}_k) = J(f_k)$. Then, we use a grid search to find the most appropriate frequency that minimizes $J(f_k)$, i.e., we find $f_k^{min} = min_{f(i)}\{J(f(i))\}$ where $f(i) = f_{min} + \frac{i}{I-1}(f_{max} - f_{min}),\ i = 0, 1, \ldots, I-1$, and we choose f_k^{min} as the observation frequency at time k. Note that, if a particle filter is used to estimate the target's state, then the tracking algorithm with frequency selection is summarized in Table 2.

Predict the error covariance at time k
$\mathbf{P}_{k\|k-1} = \mathbf{F}_x\mathbf{P}_{k-1\|k-1}\mathbf{F}_x^{\mathbb{T}} + \mathbf{Q}_U$
Predict the mean state at time k
$\hat{\mathbf{X}}_{k\|k-1} = \mathbf{F}_x\hat{\mathbf{X}}_{k-1}$
Calculate sigma points and weights:
Set UT parameters λ, L, α and β
$\mathcal{X}_{k\|k-1} = \left[\ \hat{\mathbf{X}}_{k\|k-1}\ \ \hat{\mathbf{X}}_{k\|k-1} \pm \left(\sqrt{(L+\lambda)\mathbf{P}^n_{k\|k-1}}\right)_i\ \right];$
$W_0^{(m)} = \lambda/(L+\lambda)$
$W_0^{(c)} = \lambda/(L+\lambda) + (1-\alpha^2+\beta)$
$W_i^{(m)} = W_i^{(c)} = 1/\{2(L+\lambda)\}$
$\mathcal{Y}_{k\|k-1} = \mathbf{H}(\mathcal{X}_{k\|k-1}, \mathbf{R}_k, \Psi)$
$\hat{\mathbf{y}}_k^- = \sum_{i=0}^{2L} W_i^{(m)}\mathcal{Y}_{i,k\|k-1}$
Predict the mean square error at time k:
$\mathbf{P}_{yy} = \sum_{i=0}^{2L} W_i^{(c)}[\mathcal{Y}_{i,k\|k-1} - \hat{\mathbf{y}}_k^-][\mathcal{Y}_{i,k\|k-1} - \hat{\mathbf{y}}_k^-]^{\mathbb{T}} + \mathbf{Q}_W$
$\mathbf{P}_{xy} = \sum_{i=0}^{2L} W_i^{(c)}[\mathcal{X}_{i,k\|k-1} - \hat{\mathbf{X}}_{k\|k-1}][\mathcal{Y}_{i,k\|k-1} - \hat{\mathbf{y}}_k^-]^{\mathbb{T}}$
$\mathbf{P}_{k\|k} = \mathbf{P}_{k\|k-1} - \mathbf{P}_{xy}\mathbf{P}_{yy}^{-1}\mathbf{P}_{xy}^{\mathbb{T}}$

Table 1. Unscented transform (UT) algorithm.

5. SIMULATION RESULTS

Using the same simulation parameters as in Section 3, we obtained the tracking performance of four algorithms: the UKF, the PF, the UKF with frequency selection (UKF-FS), and the PF with frequency selection (PF-FS). The range of the observation frequencies was chosen to be $200 - 400$ Hz (in increments of 10 Hz). The simulation results are demonstrated in Figures 3, 4 and 5. Figure 3 shows the true trajectory of the target and the corresponding estimated trajectory obtained using the different algorithms. As we can see, the frequency selection yields better tracks, both for the PF and the UKF algorithms. As shown in Fig. 4, that provides the average MSE at each time step, the best performance is obtained by the PF-FS algorithm. Figure 5 shows the selected observation frequencies employed by the UKF-FS and PF-FS algorithms. The tracking results in Fig. 3, the MSE values in Fig. 4 and the frequency values in Fig. 5 are obtained by averaging the results from 100 numerical simulations.

6. CONCLUSION

We proposed a general matched-field processing framework for tracking in shallow water environments. Using observa-

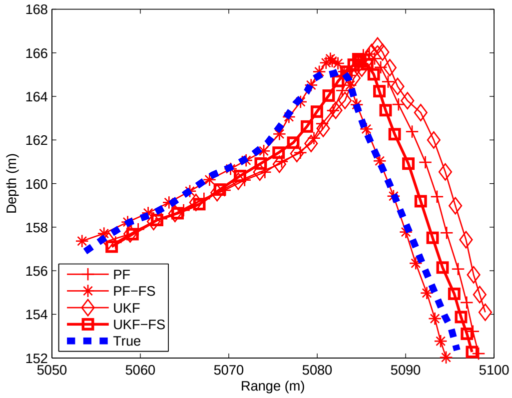

Fig. 3. The tracking performance of the UKF and PF tracking algorithms, compared with the UKF-FS and PF-FS tracking algorithms with frequency selection.

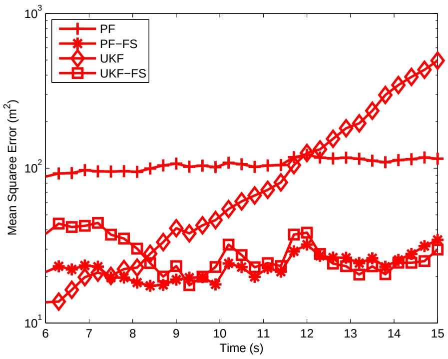

Fig. 4. The average MSE of the UKF and PF tracking algorithm compared with the UKF-FS and PF-FS tracking algorithms with frequency selection.

tions from multiple passive sensors, we used the unscented Kalman filter and the particle filter techniques to estimate the source dynamic state. Furthermore, we developed an algorithm for frequency-agile sensing that adaptively chooses the optimal observation frequencies for the sensors at each observation epoch. Simulations demonstrated that the dynamic selection of frequencies for target tracking improved tracking performance.

7. REFERENCES

[1] H. Bucker, "Matched-field tracking in shallow water," *The Journal of the Acoustical Society of America*, vol. 96, no. 6, pp. 3809–3811, December 1994.

[2] S. L. Tantum, L. W. Nolte, J. L. Krolik, and K. Harmanci, "The performance of matched-field track-before-detect methods using shallow-water pacific data," *The Journal of the Acoustical Society of America*, vol. 112, no. 1, pp. 119–127, July 2002.

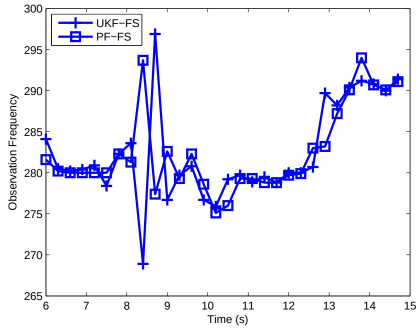

Fig. 5. The observation frequency employed by the UKF-FS and PF-FS tracking algorithms with frequency selection.

for time k do
  for $j = 1 : N$ do
    Draw samples $\hat{\mathbf{X}}_k^j \sim p(\mathbf{X}_k|\mathbf{X}_{k-1})$
    Calculate the weights $w_k^j = p(\mathbf{Y}_k|\mathbf{X}_k)$
  end
  Normalize $w_k^j = w_k^j / \sum_j w_k^j$
  Resample $\hat{\mathbf{X}}_k^j$
  Estimate $\hat{\mathbf{X}}_k$
  Frequency scheduling
  for $i = 1 : I - 1$ do
    $f(i) = f_{min} + \frac{i}{I-1}(f_{max} - f_{min})$
    Calculate $J\left(f(i)\right)$ by the UT in Table 1
  end
  Find i^{min} for the minimum $J(f(i))$
  Set $f_{k+1} = f(i^{min})$
  Calculate $\mathbf{P}_{k+1|k+1}$ by the UT in Table 1 using f_{k+1}
end

Table 2. Particle filter algorithm with frequency selection.

[3] S. P. Sira, D. Morrell, and A. Papandreou-Suppappola, "Waveform design and scheduling for agile sensors for target tracking," in *Proc. Asilomar Conf. on Signals, Syst. and Comput.*, 7-10 Nov. 2004, vol. 1, pp. 820–824.

[4] L. J. Ziomek, *Fundamentals of Acoustic Field Theory and Space-Time Signal Processing*, CRC Press, 1999.

[5] A. S. Chhetri, D. Morrell, and A. Papandreou-Suppappola, "The use of particle filtering with the unscented transform to schedule sensors multiple steps ahead," in *Proc. IEEE Int. Conf. on Acoust., Speech, Signal Processing*, 17-21 May 2004, vol. 2, pp. 301–304.

[6] R. Merwe, A. Doucet, N. Freitas, and E. Wan, "The unscented particle filter," Tech. Rep., Cambridge University Engineering Department, 2000.