MULTI STEP AHEAD BEAM AND WAVEFORM SCHEDULING FOR TRACKING OF MANOEUVERING TARGETS IN CLUTTER

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1. INTRODUCTION

Modern radar systems have considerable flexibility in their modes of operation. In particular, it is possible to modify the waveform on a pulse to pulse basis, and electronically steered phased arrays can quickly point the radar beam in any feasible direction. Such flexibility calls for new methods of scheduling both of the waveform and the beam direction so as to optimize the radar performance.

We consider a radar system capable of rapid beam steering and of waveform switching. The transmit waveform is chosen from a small library of such. The operational requirement of the radar is to track a number of manoeuvring targets while performing surveillance for new potential targets. Tracking is accomplished by means of an LMIPDA (Linear Multitarget Integrated Probabilistic Data Association) tracker as described in [1]. Interacting multiple models (IMM) is used to model manoeuvering targets in the tracker. LMIPDA provides a probability of track existence, permitting a "track-before-detect" technique to be adopted. "False alarm" tracks are maintained until the probability of track existence falls below a threshold.

Our aim is to maintain the tracks of the existing targets to within a specified accuracy as determined by the absolute value of the track error covariance matrix. However, this has to be done within the time available given that a full scan has to be performed within a prescribed interval. We give an algorithm for scheduling revisits to measure the targets while maintaining surveillance.

2. PROBLEM FORMULATION

We postulate a radar system tracking T targets where T is a random variable $0 \leq T \leq T_0$ and the tth target is in state $x^t(k)$ at epoch k. In addition the radar undertakes surveillance to discover new targets. This surveillance is assumed to require a certain length of time, say T_{scan} within every interval of length T_{total}. The remainder of the time is spent measuring targets being tracked. We aim to schedule revisit times to targets within these constraints.

A major assumption of IMM-based algorithms is that the trajectory of the target can be described at any time by one of $M < \infty$ pre-defined dynamical models. In this context, we assume that the dynamical models are independent of the target and associated to each is a corresponding state propagation matrix F_m $(m = 1, 2, \ldots, M)$. The recursion for state transition is

$$x^t(k) = F_m(k)x^t(k-1) + \nu_m^t(k), \tag{1}$$

where the index m is a possible value of a random variable $M(k)$, the *dynamical model* which takes any discrete value $[1, 2, \cdots, M]$. Process noise (Gaussian) $\nu_1^t(k), \cdots, \nu_M^t(k)$ depends on both target and dynamical model and is independent between different values of each of these indices. The covariance matrix of $\nu_m^t(k)$ is denoted by $Q_m^t(k)$.

In the tracker, the dynamical model of the tth target $M^t(k)$ is assumed to evolve as a Markov Chain with given transition probabilities, denoted by

$$\pi_{m,\ell}^t = P\{M^t(k) = m | M^t(k-1) = \ell\}; j, \ell \in [1, \cdots, M]. \tag{2}$$

It is assumed that N different *measurement modes* are available for each target, each given by a measurement matrix H_n^t $n = 1, 2 \ldots, N$:

$$z^t(k) = H_n^t(k)x^t(k) + \omega_n^t(k) \tag{3}$$

where here $z^t(k)$ is the measurement to be obtained from the tth target at time k, $\omega_n^t(k)$ is the measurement noise, and $n = n(k)$ is a control variable for the measurement mode. We will also permit measurement of only one target at each epoch. The variable $\widetilde{t} = \widetilde{t}(k)$ represents the choice of target to which the beam is steered at the kth epoch. The measurement noise $\omega_1^t(k), \cdots, \omega_N^t(k)$ are zero mean white and uncorrelated Gaussian noise sequences with the covariance matrix of $\omega_n^t(k)$ denoted by $R_n^t(k)$. In our case all of the measurement matrices are identical, but the noise is waveform dependent.

In cluttered environments, measurements can result from zero or more targets as well as zero or more clutter scatters at each scan. Target measurements are assumed unidentified, but present with probability of detection P_D, not necessary identical for each target (for example, in a radar application they will be range dependent) and not necessarily constant over time. The set of actual measurements obtained at time k, selected with *gating probability* P_G, is denoted by $z(k)$, and the i-th measurement from this (ordered) set $z(k)$ by $z(i,k)$, where $i = 1, \cdots, m_k; \quad m_k \geq 0$ is the number of measurements at time k. This selection of measurements is often referred to as *validation of measurements* or *gating* [2]. Briefly, gating is a way to select a subset of all sensor measurements based on the current estimate of the target state. If the target exists and is detected, the target measurement will be selected with gating probability P_G. The purpose of gating is the reduction of computational time. The history of all selected measurements up to and including time k is denoted by $Z^k = z(k) \bigcup Z^{k-1}$. Given the dynamics Eq. (1) and measurements Z^k, we aim to estimate recursively the *a posteriori* probability of target existence $\psi_{k|k}^t$, the state estimate and error covariance, $\hat{x}_{k|k}^t$ $P_{k|k}^t$ respectively for the tth target.

3. TRACKING

The choice of measurement is made using the control variable $n(k)$. In fact two choices are made at each epoch, the target to be measured and the waveform used. More than one target may be in the beam and then measurements of each target will be updated using the LMIPDA-IMM algorithm described in [1]. We briefly discuss it here. This is a recursive algorithm combining a multi-target data association algorithm (LMIPDA) with manoeuvring target state estimation implemented using IMM. IMM consists of a filter (usually Kalman or similar) bank, one for each possible target trajectory model. For each track t, filter inputs consist of:

- *target existence*, modelled by a random variable χ, assumed to evolve as a Markov chain and taking either of two values [3]. $\chi = 1, 0$ according as the target exists or not. The value of χ for each track t is updated with measurements at each scan.
- *predicted state for each IMM model* j, obtained from the previous state by state propagation:
  - *state prediction probability density function (pdf)* $p^t(x_k|M_k = j, Z^{k-1})$, described by its mean $\hat{x}^t_{k|k-1}(j)$ and its error covariance $P^t_{k|k-1}(j)$,
  - *predicted model state probability*

$$\mu^t_{k|k-1}(j) \triangleq P^t\{M_k = j|Z^{k-1}\}$$

  and

  - *a priori measurement pdf* $p^t(z_k|M_k = j, Z^{k-1})$.
- *measurement set delivered by sensor at time* k, which may be empty.

For each track t, the filter output at time k consists of:

- *a posteriori probability of target existence* $\psi^t_{k|k}$, then used for confirmation or termination of tracks;
- *track state estimate and estimate covariance,* $\hat{x}^t_{k|k}$ *and* $P^t_{k|k}$;
- *filter inputs for time* $k+1$ *for next recursion*, enumerated above, $\psi^t_{k+1|k}$, and, for each IMM model j, $\hat{x}^t_{k+1|k}(j)$, $P^t_{k+1|k}(j)$, $\mu^t_{k+1|k}(j)$ and $p^t(z_{k+1}|M_{k+1} = j, Z^k)$.

The waveforms impinge on the measurement process through the covariance matrix of the noise $\omega^t_n(k)$. We use the basic sensor model proposed in [4]. While this has limitations, it is simple and therefore useful as a starting point for discussion of the problem. In this model, the sensor is characterised by a measurement noise covariance matrix which is waveform dependent

$$R_\phi = T J_\phi^{-1} T, \tag{4}$$

where J_ϕ is the Fisher information matrix corresponding to the measurement using waveform ϕ and T is the transformation matrix between the time delay and Doppler measured by the receiver and the target range and velocity. The Fisher information is given by an expression involving the normalised second order time and frequency moments of the waveform ϕ. It is also expressible in terms of the Hessian of the squared absolute value of the ambiguity function of the waveform at the origin of the range-Doppler plane. This calculation is done in [5]. It should be pointed out that the use of the Fisher matrix here is an approximation. It really corresponds to the Cramér-Rao lower bound on the estimator for the target from this measurement. It can be shown that the estimator here is *asymptotically efficient* (see[6], pp. 38–39) in that the covariance matrix approaches the Cramér-Rao lower bound over a large number of measurements (*loc.cit.*).

4. SCHEDULING

As we have already stated, at each epoch a target track and a beam direction have to be selected. The scheduler has a list $\Delta = \{\delta_1, \delta_2, \dots, \delta_K\}$ of "revisit intervals". Each of the numbers δ_k is a number of epochs representing the possible times between measurements of any of the existing targets. It is assumed for the purposes of scheduling and tracking that during any of these revisit intervals the target dynamics do not change, though the simulator permits target maneuvers on an epoch by epoch basis.

In order to determine which target to measure and which waveform to use, for each existing target and each waveform the track error covariance $P^t_{k-1|k-1}$ is propagated forward using the Kalman update equations and assuming each of the different potential revisit intervals in the list Δ in the dynamics. In the absence of measurements the best we can do is to use the current knowledge to predict forward and update the covariance matrix, dynamic model pdf and probability of track existence. The algorithms now becomes as follows:

- *IMM mixing* [7, 8, 1] is conducted as usual;
- *Forward prediction* is then performed separately for each dynamical model. Because the dynamics of the target depends on the revisit time $\delta \in \Delta$ this calculations are performed for each revisit time.
- *Covariance update:* this is normally done with the data, but since we are interested in choosing the best sensor mode at this stage the following calculations are required. If the target does not exists there will be no measurements originating from the target and the error covariance matrix is equal to the *a priori* covariance matrix, if the target exists, is detected, and the measurement is received then the error covariance matrix is updated using the Kalman equation. The covariance update is calculated using Bayes rule, namely,

$$\begin{aligned} P_{k|k}(j,\delta) &= (1 - \psi_{k|k-1} P_D P_G) P_{k|k-1}(j,\delta) \\ &+ \psi_{k|k-1} P_D P_G (I - K(\phi,\delta)H) P_{k|k-1}(j,\delta) \\ &= (I - \psi_{k|k-1} P_D P_G K(\phi,\delta)H) P_{k|k-1}(j,\delta). \end{aligned} \tag{5}$$

  where $\psi_{k|k-1}$ is the *a priori* probability of track existence, $P_D P_G$ is the probability that target is detected and its measurement is validated. $K(\phi,\delta)$ is a Kalman gain calculated for each sensor mode; that is, for the waveform ϕ and revisit time δ. Both ϕ and δ take discrete values from waveform library and revisit time set Δ.

$$K(\phi,\delta) = P_{k|k-1}(j,\delta) H S^{-1}(\phi), \tag{6}$$

  where S is innovation covariance matrix, calculated as

$$S = H P_{k|k-1} H^T + R_\phi$$

  .

- The dynamic model and track existence pdfs are updated in a similar manner. If the target does not exist it produces no measurement; if it does and is detected the expected measurement pdf is calculated as follows:

$$p_j(z_k) = p(z_k|\chi_k = 1, M_k = j, Z^k) = \mathcal{N}(H\hat{x}_{k|k-1}(j); S_j);$$

$$p(z_k) = p(z_k|\chi_k = 1, Z^k) = \sum_{i=1}^{M} \mu_{k|k-1}(i) p_i(z_k). \tag{7}$$

The dynamic model and track existence pdfs are updated :

$$\mu_{k|k}(j) = \frac{p_j(z_k)\mu_{k|k-1}(j)}{\sum_{i=1}^{M} p_i(z_k)\mu_{k|k-1}(i)} \tag{8}$$

$$\psi_{k|k} = \frac{p(z_k)\psi_{k|k-1}}{\rho(z_k)(1-\psi_{k|k-1}) + p(z_k)\psi_{k|k-1}},$$

where $\rho(z_k) = p(z_k|\chi_k = 0, Z^k)$ is the estimated clutter and other targets density at z_k(see[1]).

- The next step is to combine the estimates for all dynamics models $j = 1, \ldots, M$ into one, using the standard "IMM combination" formulae [7, 8, 1].

The above calculations are performed for all combinations of revisit times in Δ and waveforms in the library. Evidently then the number of combinations grows exponentially in the number of steps ahead, and soon becomes impractical for implementation. Having obtained the error covariance matrix for all possible combinations of sensor modes, the optimal sensor mode (waveform) is then chosen for each target to be the one which gives the longest revisit time, while constraining the absolute value of the determinant of the error covariance matrix to be smaller than the prescribed upper limit K. In other words, our objective is

$$\phi, \delta = \arg\max \Delta, \text{ subject to } |\det(P_{k|k})| \leq K. \tag{9}$$

Scheduling is then done to permit a full scan over the prescribed scan period while also satisfying the constraints imposed by the revisit times obtained by the sensor scheduler. Once a target is measured, its revisit time is re-calculated.

We note that for many manoeuvring targets there may be no solution to the scheduling problem that satisfies the constraints. However, we have not simulated a situation in which this happens.

We have, on the other hand done simple simulations for the case of one-step ahead and two-step ahead scheduling. In the latter case, the revisit times and waveforms are calculated while the target states are propagated forward over two measurements, with the cost function being the absolute value of the determinant of the track error covariance after the second measurement. Only the first of these measurements is done before the revisit calculation is done again for that target, so that the second may never be implemented.

In the next section we present the results of the simulation for two-step versus one-step ahead scheduling.

5. SIMULATION RESULTS

Simulations were performed to compare the effects of no scheduling with random choice of waveform against one-step and two-step ahead beam and waveform scheduling as described in the last section. All three simulations were performed 100 times on the same scenario. In the first case, measurements were taken at each scan with no further measurements beyond the scan measurements permitted. The waveforms were chosen at random from the three waveforms in the library. The simulated scene corresponded to a surveillance area of 15km by 15km contained two maneuvering land targets in stationary land clutter which had small random Doppler to simulate movement of vegetation in wind. The number of clutter measurements at each epoch was generated by samples from a Poisson distribution with mean ~ 5 per scan per sq.km. Target measurements were produced with probability of detection 0.9. The target trajectories were simulated as shown in Figure 1. The target state x^t consisted of target range, target range rate and target azimuth. The targets were performing the following maneuvers: constant velocity, constant acceleration, constant deceleration and coordinated turns with constant angular velocity. In these experiments we used the waveform library consisting of three waveforms: an up-sweep chirp, a down-sweep chirp and an unmodulated pulse. In the scheduling cases, surveillance time used approximately 80 percent of each scan period, the remaining 20% being allocated as described above to the maintenance of tracks of existing targets.

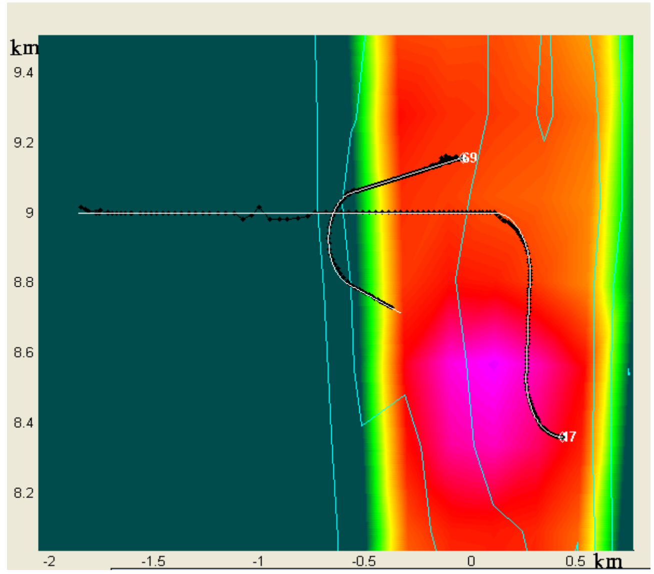

Fig. 1. Scene with Maneuvering Target in Clutter

The outcome of experiments suggests that in the presence of clutter tracking performance can be improved with scheduling and even more with multiple step ahead scheduling as opposed to one step ahead. The results are represented in Figure 2. It should be observed in Figure 2 that RMS error was considerably worse especially during the early part of the simulation for the unscheduled case. In fact the RMS error in the unscheduled case is larger immediately after significant manoeuvres as can be expected. Of course, in this case the revisit time is fixed and is not plotted in the second subplot. One observes, that, for the two-step ahead case, tracking accuracy is improved (top plots) slightly over the one-step ahead case but with a significant reduction in revisit times to maintain those tracks.

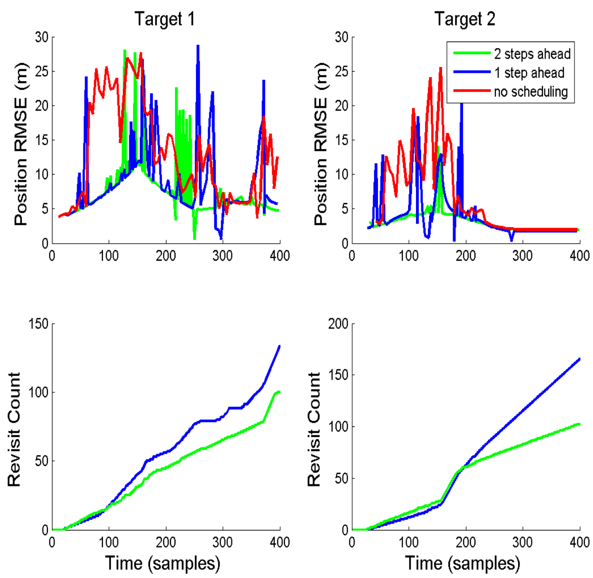

Fig. 2. Root Mean Square Error (RMSE) and Revisit Count for one vs. two step ahead beam and waveform scheduling

6. CONCLUSION

We have described a system for scheduling of waveforms and beam directions of a radar system to detect and track multiple manoeuvring targets, based on the LMIPDA-IMM tracker. We have simulated scenarios using this technique to track multiple manoeuvring targets in simulated clutter data with both one-step ahead and two-step ahead scheduling. Our results indicate that real improvements are obtained in this context by scheduling two steps ahead.

7. REFERENCES

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