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Complementary Codes based Channel Estimation for MIMO-OFDM Systems

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***Abstract*— We present a pilot-assisted method for estimating the frequency selective channel in a MIMO-OFDM (Multiple Input Multiple Output - Orthogonal Frequency Division Multiplexing) system. The pilot sequence is designed using the DFT (Discrete Fourier Transform) of the Golay complementary sequences. Novel exploitation of the perfect autocorrelation property of the Golay codes, in conjunction with OSTBC (Orthogonal Space-Time Block Code) based pilot waveform scheduling across multiple OFDM frames, facilitates simple separation of the channel mixtures at the receive antennas. The DFT length used to transform the complementary sequence into the frequency domain is shown to be a key critical parameter for correctly estimating the channel. NMSE (Normalized Mean Squared Error) between the actual and the estimated channel is used to characterize the estimation performance.**

I. Introduction

OFDM is a multi-carrier modulation scheme that has gained considerable popularity over the past decade because of its ability to combat frequency-selective fading normally encountered in a multi-path wireless environment. OFDM converts the frequency selective channel into flat-fading sub-channels, thereby significantly reducing the receiver complexity by eliminating the need for using equalization at the receiver. [2]. MIMO systems were first introduced in [4]-[5]. Under certain conditions [5], the capacity of MIMO systems is shown to increase linearly with $\min\{N_T, N_R\}$ where N_T and N_R are the number of antennas at the transmitter and the receiver, respectively.

MIMO-OFDM systems provide performance gains because they combine the diversity and multiplexing gains of MIMO with the resilience of OFDM against multi-path fading. In order to achieve these performance improvements, accurate CSI (Channel State Information) is required at the receiver. A number of different pilot assisted methods have been proposed in the literature for estimating the channel, such as [6]-[8]. In a pilot assisted scheme for channel estimation, known pilot sub-carriers are inserted in the transmitted signal to sample the multi-path channel. At the receiver, these samples are then used to obtain a representation of the entire channel through linear or higher order interpolation. Pilots essentially sample the frequency response of the channel. If the sampling is done according to the Nyquist criterion, the entire frequency response of the channel can be reconstructed from the samples obtained via the pilots.

The difficulty in channel estimation for multi-antenna systems lies in the fact that at every receive antenna, we get a signal coming from different transmit antennas, and hence different channels. Therefore, we need to estimate the individual channels given mixtures of the channels. A technique for estimating the channel in a 2×2 transmit diversity system has been proposed in [1], which is an iterative technique. It requires the initialization of the channel estimates at the receiver by sending the complementary codes sequences – no data – during the first two OFDM symbol periods followed by a successive interference cancelation procedure that kind of goes back and forth between data estimation and channel estimation. This is because the method in [1] is not based on pilot tones as the Golay codes are simply added to the OFDM frame. In this paper, we present a simpler and computationally efficient method to obtain the sampled channel estimates using pilots without any need for initialization symbols or extra processing at the receiver. Our method uses Golay complementary sequences [9] to design pilots in the frequency domain. Complementary sequences have the property that the sum of their autocorrelation produces a Dirac-delta function. The use of a combination of complementary sequences and OSTBC [3] like coding allows us to separate channels from different transmitting antennas at the receiving antenna.

The paper is organized as follows: section II introduces the channel model, complementary sequences and relevant results about waveforms separation in the radar literature. Section III describes the proposed channel estimation paradigm. Simulation results are presented in section IV and section V provides a short discussion and conclusions.

II. Preliminaries

In this section, we introduce the multi-path channel model, Golay complementary sequences and some relevant literature from radar waveform design.

A. Multipath Channel Model

We consider a multi-path channel with $B_c < B_T$, Where B_T is the transmission bandwidth and B_c is the coherence bandwidth. B_c can be expressed in terms of the maximum delay spread $\tau_{\max}$ as

$$B_c \approx \frac{1}{\tau_{\max}} \tag{1}$$

We will assume that the channel is slowly fading and is quasi-static over $n \geq 2$ consecutive symbols. With these

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assumptions, the sampled channel from transmit antenna i to receive antenna j can be represented in the time-domain as

$$h_{ij}(t) = \sum_{l=0}^{L-1} \beta_{ij}^{(l)} \delta(t - t_l) \quad (2)$$

where $\beta_{ij}^{(l)}$ are i.i.d complex Gaussian random variables with variance σ_h^2. L is the maximum sampled channel impulse response (CIR) length which captures all the necessary variation in the continuous channel impulse response.

B. Golay Complementary Sequences

A pair of sequences $e_1[n]$ and $e_2[n]$ of length N_c satisfy the Golay property if the sum of their autocorrelation functions satisfy

$$R_{e_1 e_1}[l] + R_{e_2 e_2}[l] = \begin{cases} 2N & if \quad l = 0 \\ 0 & if \quad l \neq 0 \end{cases} \quad (3)$$

for $l = -N_c - 1, ..., N_c - 1$. These sequences can be constructed for length $2^N 10^K 26^M$ with $N, K, M \in \{\mathrm{N} \cup \{0\}\}$. If we take the DFT of the above equation, we get

$$|E_1[k]|^2 + |E_2[k]|^2 = 2N \quad (4)$$

In order for the above relation to reflect the DFT of (3), The DFT length needs to be chosen properly. This is because $R_{e_i e_i}[n]$ represents a linear convolution of $e_i[n]$ and $e_i^*[-n]$, i.e.

$$R_{e_1 e_1}[n] = e_i[n] * e_i^*[-n] \quad (5)$$

However, we observe that

$$IDFT_{N_{DFT}} \left\{ |E_i[k]|^2 \right\} = e_i[n] \circledast e_i^*[-n] \quad (6)$$

gives the circular convolution of $e_i[n]$ and $e_i^*[-n]$. Therefore, in order to get (5) from (6), the DFT length N_{DFT} needs to be chosen such that

$$N_{DFT} \geq 2N_c - 1 \quad (7)$$

This is equivalent to zero padding the original sequence $e_i[n]$ by $N_c - 1$ zeros so that their circular convolution represents the linear convolution of the original sequence $e_i[n]$.

This property will play a key role in our channel estimation strategy, as we later show. Also, we observe that if $e_1[n]$ and $e_2[n]$ satisfy the Golay property, then $e_1^*[-n]$ and $e_2^*[-n]$ also form a complementary sequence pair.

C. Complementary Sequences and Radar Waveform Design

In [11] and [12], instantaneous radar polarimetric techniques were proposed for 2×2 and 4×4 systems using complementary sequences. The idea is to schedule the complementary sequence according to the OSTBC scheme in order to estimate the radar returns for improved detection performance.

In [11], the target scattering matrix, given by

$$\mathbf{H} = \begin{bmatrix} h_{11}\delta[n-D] & h_{12}\delta[n-D] \\ h_{21}\delta[n-D] & h_{22}\delta[n-D] \end{bmatrix} \quad (8)$$

where D is the target delay, was estimated by using the OSTBC complementary sequences of length N given by

$$\mathbf{E} = \begin{bmatrix} e_1[n] & -e_2^*[-n] \\ e_2[n] & e_1^*[-n] \end{bmatrix} \quad (9)$$

where h_{ij} represents the radar return from the transmitting antenna i to receiving antenna j and the columns and rows of $\mathbf{E}$ represent the transmitting antennas and symbol periods respectively. For simplicity of exposition, we will assume that $D = 0$. The received waveform can be represented as

$$\mathbf{R} = \mathbf{H} * \mathbf{E} + \mathbf{N} \quad (10)$$

where $\mathbf{N}$ is the white noise matrix. The target scattering matrix can be estimated by observing that

$$\mathbf{E} * \mathbf{E}^H = \alpha \mathbf{I} \delta[n] \quad (11)$$

where α is a constant given by

$$\alpha = |E_1[k]|^2 + |E_2[k]|^2 = 2N \quad (12)$$

and $*$ represents the matrix convolution. The matrix $\mathbf{E}^H$ can be written as

$$\mathbf{E}^H = \begin{bmatrix} e_1^*[-n] & e_2^*[-n] \\ -e_2[n] & e_1[n] \end{bmatrix} \quad (13)$$

If we process the received waveform matrix with $\mathbf{E}^H$, we see that

$$\mathbf{R} * \mathbf{E}^H = \alpha \mathbf{H} + \mathbf{N}' \quad (14)$$

where

$$\mathbf{N}' = \mathbf{N} * \mathbf{E}^H \quad (15)$$

From this, we can see that the channel can be estimated without the receiver having any prior information about the channel. This property will be central to the idea of channel estimation presented in the next section.

III. Pilot Design and Channel Estimation

We consider a 2×2 MIMO-OFDM system with N sub-carriers of which N_p are pilot sub-carriers. Each pair of transmit-receive antennas encounters a length L multipath channel. The system cyclically extends every OFDM symbols before transmission by appending the trailing N_g samples to the beginning of the symbol. We assume $N_g \geq L$ so that all the sub-carriers remain mutually orthogonal at the receiver [2]. The N_p point pilot sequence is designed in the frequency domain as described in the following section. We present a development without considering the effects of receiver noise. We will show that if certain conditions are met, the channel estimation is exact. Therefore, the only source of degradation is the receiver noise. We will later characterize the noisy estimation performance using the NMSE criterion.

A. Frequency Domain Pilot Design

The pilot sequence is designed in the frequency domain using the DFT of complementary sequences. The N_p point DFT of the complementary sequence is given by

$$\tilde{E}_i[k] = DFT_{N_p}\{e_i[n]\} \quad (16)$$

We use $\tilde{E}[k]$ to emphasize the fact that this represents the pilot sequence with $k = 0, 1, ..., N_p$ and is the sequence carried by the pilot sub-carriers, as shown in Figure 2. The advantage of designing the pilot sequence in the frequency domain lies in the fact that the sequence is carried by orthogonal sub-carriers. The relative magnitudes of these sub-carriers change because each sub-carrier, upon passing through the channel, is multiplied by the corresponding value of the channel frequency response at that frequency. However, with the assumption that the guard interval N_g is longer than the maximum length of the sampled channel impulse response L, the sub-carriers still remain orthogonal at the receiver, and we can recover the pilot sequence without any interference from the data. If $H_{ij}[k]$ represents the N-point DFT of the sampled channel impulse response between i^{th} transmit antenna and the j^{th} receive antenna, we define the sampled frequency response at the pilot frequencies as

$$\tilde{H}_{ij}[k] = H_{ij}[kN/N_p] \qquad k = 0, 1, ..., N_p - 1 \tag{17}$$

The transmitted and received signals over the two OFDM symbol intervals are shown in Figure 1. The received sequence, after removing the cyclic prefix and taking the DFT, can be written as

$$\mathbf{R}[k] = \left[\begin{array}{cc} \mathbf{Y}_1[k] & \mathbf{Y}_2[k] \end{array} \right] \tag{18}$$

where

$$\begin{aligned} \mathbf{Y}_1[k] &= \left[\begin{array}{c} Y_{11}[k] \\ Y_{21}[k] \end{array} \right] \\ &= \left[\begin{array}{cc} H_{11}[k] & H_{21}[k] \\ H_{12}[k] & H_{22}[k] \end{array} \right] \left[\begin{array}{c} E_1[k] \\ E_2[k] \end{array} \right] \end{aligned} \tag{19}$$

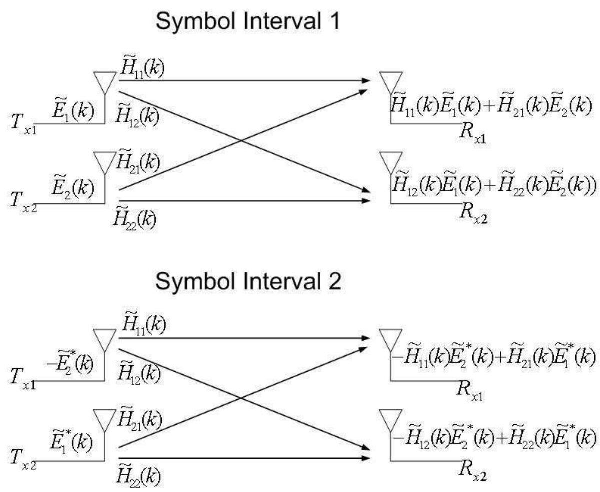

Fig. 1. Transmitted and received signals over the two contiguous OFDM symbol periods

and

$$\begin{aligned} \mathbf{Y}_2[k] &= \left[\begin{array}{c} Y_{12}[k] \\ Y_{22}[k] \end{array} \right] \\ &= \left[\begin{array}{cc} H_{11}[k] & H_{21}[k] \\ H_{12}[k] & H_{22}[k] \end{array} \right] \left[\begin{array}{c} -E_2^*[k] \\ E_1^*[k] \end{array} \right] \end{aligned} \tag{20}$$

We know that the data sub-carriers do not play any part in the channel estimation because they are orthogonal to the pilot sub-carriers. Therefore, we only consider the sub-carrier indices pertaining to the pilot sub-carriers, i.e.

$$\tilde{Y}_i[k] = Y_i[kN/N_p] \qquad \tilde{k} = 0, 1, ..., N_p - 1 \tag{21}$$

With this information, the received matrix can also be written as

$$\begin{aligned} \tilde{\mathbf{R}}[k] &= \left[\begin{array}{cc} \tilde{\mathbf{Y}}_1[k] & \tilde{\mathbf{Y}}_2[k] \end{array} \right] \\ &= \tilde{\mathbf{H}}[k]\tilde{\mathbf{E}}[k] \end{aligned} \tag{22}$$

where

$$\tilde{\mathbf{E}}[k] = \left[\begin{array}{cc} \tilde{E}_1[k] & -\tilde{E}_2^*[k] \\ \tilde{E}_2[k] & \tilde{E}_1^*[k] \end{array} \right] \tag{23}$$

In order to separate the different channels at the receiver, we need to process the receiver waveform with a matrix $\mathbf{A}$ such that

$$\tilde{\mathbf{R}}[k]\mathbf{A} = \alpha\tilde{\mathbf{H}}[k] \tag{24}$$

This implies that we need

$$\tilde{\mathbf{E}}[k]\mathbf{A} = \alpha\mathbf{I} \tag{25}$$

This can be achieved by exploiting (4), from which we can directly infer that $\mathbf{A} = \tilde{\mathbf{E}}^H[k]$ gives the desired result, since

$$\begin{aligned} \tilde{\mathbf{E}}[k]\tilde{\mathbf{E}}^H[k] &= \left[\begin{array}{cc} \tilde{E}_1[k]\tilde{E}_1^*[k] + \tilde{E}_2^*[k]\tilde{E}_2[k] & \tilde{E}_1[k]\tilde{E}_2^*[k] - \tilde{E}_2^*[k]\tilde{E}_1[k] \\ \tilde{E}_2[k]\tilde{E}_1^*[k] - \tilde{E}_1^*[k]\tilde{E}_2[k] & \tilde{E}_2[k]\tilde{E}_2^*[k] + \tilde{E}_1^*[k]\tilde{E}_1[k] \end{array} \right] \\ &= \alpha\mathbf{I} \end{aligned} \tag{26}$$

Therefore, we have that

$$\tilde{\mathbf{R}}[k]\tilde{\mathbf{E}}^H[k] = \alpha \left[\begin{array}{cc} \tilde{H}_{11}[k] & \tilde{H}_{21}[k] \\ \tilde{H}_{12}[k] & \tilde{H}_{22}[k] \end{array} \right] \tag{27}$$

More specifically, $\tilde{H}_{11}[k]$ can be estimated as

$$\begin{aligned} \hat{\tilde{H}}_{11}[k] &= \tilde{Y}_{11}[k][]\tilde{E}_1^*[k] - \tilde{Y}_{12}[k]\tilde{E}_2[k] \\ &= \tilde{H}_{11}[k] \left(\tilde{E}_1[k]\tilde{E}_1^*[k] + \tilde{E}_2[k]\tilde{E}_2^*[k]\right) \\ &+ \tilde{H}_{21}[k] \left(\tilde{E}_1[k]\tilde{E}_1^*[k] - \tilde{E}_2[k]\tilde{E}_2^*[k]\right) \\ &= \tilde{H}_{11}[k] \end{aligned} \tag{28}$$

Similarly, we have that

$$\begin{aligned} \hat{\tilde{H}}_{21}[k] &= \tilde{Y}_{11}[k]\tilde{E}_2^*[k] + \tilde{Y}_{12}[k]\tilde{E}_1[k] \\ &= \tilde{H}_{21}[k] \left(\tilde{E}_1[k]\tilde{E}_1^*[k] + \tilde{E}_2[k]\tilde{E}_2^*[k]\right) \\ &+ \tilde{H}_{11}[k] \left(\tilde{E}_1[k]\tilde{E}_2^*[k] - \tilde{E}_2^*[k]\tilde{E}_1[k]\right) \\ &= \tilde{H}_{21}[k] \end{aligned} \tag{29}$$

$$\begin{aligned}\hat{\tilde{H}}_{12}[k] &= \tilde{Y}_{21}[k]\tilde{E}_1^*[k] - \tilde{Y}_{22}[k]\tilde{E}_2[k] \\ &= \tilde{H}_{12}[k] \left(\tilde{E}_1[k]\tilde{E}_1^*[k] + \tilde{E}_2[k]\tilde{E}_2^*[k]\right) \\ &+ \tilde{H}_{22}[k]\left(\tilde{E}_2[k]\tilde{E}_1^*[k] - \tilde{E}_1^*[k]\tilde{E}_2[k]\right) \\ &= \tilde{H}_{12}[k]\end{aligned} \tag{30}$$

$$\begin{aligned}\hat{\tilde{H}}_{22}[k] &= \tilde{Y}_{21}[k]\tilde{E}_2^*[k] + \tilde{Y}_{22}[k]\tilde{E}_1[k] \\ &= \tilde{H}_{22}[k] \left(\tilde{E}_1[k]\tilde{E}_1^*[k] + \tilde{E}_2[k]\tilde{E}_2^*[k]\right) \\ &+ \tilde{H}_{12}[k]\left(\tilde{E}_1[k]\tilde{E}_2^*[k] - \tilde{E}_2^*[k]\tilde{E}_1[k]\right) \\ &= \tilde{H}_{22}[k]\end{aligned} \tag{31}$$

As we can see from these equations, the complementary sequences allow us to estimate all the different channels at the pilot sub-carrier frequencies without incurring any extra computational overhead. The noisy channel estimates can be represented as

$$\hat{\tilde{H}}_{ij}[k] = \tilde{H}_{ij}[k] + \tilde{n}[k] \tag{32}$$

Where $\tilde{n}[k]$ represents the noise sequence at the sub-carrier frequencies. In order to obtain the actual channel impulse response $\tilde{h}_{ij}[n]$ via IDFT of the estimated sampled channel frequency response $\hat{\tilde{H}}_{ij}[k]$,i.e.

$$\hat{\tilde{h}}_{ij}[n] = IDFT_{N_p}\left\{\hat{H}_{ij}[k]\right\} \tag{33}$$

certain conditions need to be met, as we now explain.

B. DFT Length

We saw in the previous section that the channel estimation involves a product of three terms, $E_i[\tilde{k}]$, $H_{ij}[\tilde{k}]$, and $E_j^*[\tilde{k}]$. This operation represents a three-fold circular convolution in the time domain, i.e.

$$\tilde{E}_i[k]\tilde{H}_{ij}[k]\tilde{E}_j^*[k] = DFT\left\{e_i[n] \circledast h_{ij}[n] \circledast e_j^*[-n]\right\} \tag{34}$$

In order to get the actual $h_{ij}[n]$, which is the channel impulse response, the relationship between N_p and N_c, which are the lengths of the pilot sequence and the complementary sequence, is very critical.

We know that if two sequences of length M and L are linearly convolved, the resulting sequence is of length $M + L - 1$. In order to get the same sequence through DFT processing, we need to take the $M+L-1$ point DFT of both the sequences (by zero padding the two sequences to make them of length $M + L - 1$), multiply the DFTs together, and take the IDFT to get the $M + L - 1$ point linear convolution. This idea can be applied to our three-fold convolution by choosing the initial DFT length to be at least $2*N_c+L-1$ where N_c is length of the complementary sequences and L is the channel length. However, since convolution satisfies both commutativity and associativity, we can first focus our attention on the convolution of the complementary sequences. The sum of the $2N_c - 1$ point linear convolution of the complementary sequences is a Dirac delta function delayed by N_c, i.e. $\delta[n - N_c - 1]$. The convolution of channel with this function is

$$h_{ij}[n] * \delta[n - N_c - 1] = h_{ij}[n - N_c - 1] \tag{35}$$

From this, we see that the resulting three-fold convolution is the sampled channel impulse response delayed by $N_c - 1$. Note that this resulting $2N_c + L - 2$ point sequence has $N_c - 1$ zeros before and after the L point channel sequence. This tells us that if we were to reduce the initial DFT length to $N_c + L - 1$, this would have the effect of aliasing the trailing $N_c - 1$ zeros in the three-fold linear convolution with the leading $N_c - 1$ zeros. What we get is an $N_c + L - 1$ point sequence which represents the actual sampled channel response delayed by $N_c - 1$. We can use the fact that this $N_c + L - 1$ sequence still has $N_c - 1$ zeros to further reduce the initial DFT length by observing that if we reduce the initial DFT length to N_c, the resulting three fold convolution would be the initial $N_c - 1$ zeros being aliased with the values of the sampled channel response, which can also be thought of as a circular shift of the channel response. Since we know N_c and the maximum channel length L, we know the amount of circular shift present in the estimated channel, and we can circularly shift it back to get the actual channel response. Therefore, to estimate the channel correctly, we can establish a relationship between the number of pilots N_p, which also represents the initial DFT length used to transform the complementary sequences into the frequency domain, the length of the complementary sequences N_c, and maximum length of the sampled channel impulse response L, given by

$$N_p \geq N_c \geq L \tag{36}$$

Note that pilots represent an overhead in a communication system, and therefore, we need to minimize the number of pilots needed to achieve a desired performance level. We have reduced the minimum required number of pilot symbols, N_p, per OFDM frame to be L which is the minimum required by any OFDM system for channel estimation purposes.

C. Pilot Generation

In the previous section, we noted that we modulate the pilot sub-carriers with the DFT of the complementary sequence. Since the DFT of the same complementary sequences is going to be used for generating the pilot sequence, we can take the N_p point DFT of the complementary sequences and store that sequence for use with every symbol. This N_p point sequence is then mapped to the equi-spaced pilot sub-carriers. This operation is shown in Figure 2.

The DFT block shown in Figure 2 need not operate in real time, since we would be using the same set of complementary sequences for performing the channel estimation. Therefore, we can pre-compute and store the N_p point DFT of the complementary sequences.

1) Alternate Implementation: Another time-domain equivalent of this operation leads to an alternate method of generating the same pilot sequence in the time-domain. Consider taking the N_p point DFT of the complementary sequences where N_p satisfies (36). If we

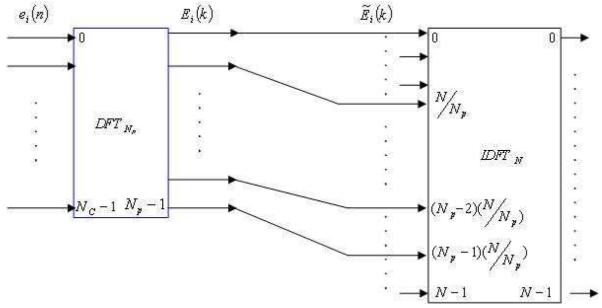

Fig. 2. Pilot sequence generation

take the IDFT of this transformed sequence, we get the original complementary sequence padded with $L-1$ zeros at the end. i.e.

$$\widehat{e}_i[n] = \begin{cases} e_i[n] & n = 0, 1, ..., N_c - 1 \\ 0 & n = N_c, N_c + 1, ..., N_p - 1 \end{cases} \quad (37)$$

WLOG, let us assume that $N/N_p = j \in \mathrm{N}$. This means if we only consider the pilot sequence and assume that the data sub-carriers are zero, we see that the pilot-only OFDM symbol has $N/N_p - 1$ zeros between the adjacent pilot symbols. The OFDM symbol in the frequency domain can be represented as

$$\tilde{S}[Nk/N_p] = \tilde{E}_i[k] \quad (38)$$

If we take the N-point IDFT of this sequence, we see that we get

$$s_i[n + mN_p] = \widehat{e}_i[n] \quad (39)$$

for $n = 0, 1, ..., N_p - 1$, $m = 0, 1, ..., (N/N_p) - 1$. This equation is the basis for an alternate implementation of the system using the DFT of complementary sequences to modulate the pilot sub-carriers. Instead of using the DFT of the complementary sequences, we can add $s_i[n]$ to the time domain OFDM symbol directly: the data sub-carriers are modulated by the data sequence and are passed through the DFT block, and then $s_i[n]$ is added to the time-domain OFDM data sequence. The time-domain OFDM data sequence is obtained by taking the IDFT of the data sequence mapped to the data sub-carriers, and not mapping anything to the pilot sub-carriers in the frequency domain.

IV. Simulation Results

Having laid the foundations of the central idea behind this channel estimation technique, we now present some simulations results. We consider an OFDM system with $N = 256$ sub-carriers and $N_p = 16$ equi-spaced pilot sub-carriers. The channel is a unit variance Rayleigh fading channel with uniform power delay profile and $L = 5$ taps. We use real valued complementary sequence of length $N_c = 10$, given by

$$\begin{aligned} e_1[n] &= \{1, 1, -1, 1, -1, 1, -1, -1, 1, 1\} \\ e_2[n] &= \{1, 1, -1, 1, 1, 1, 1, 1, -1, -1\} \end{aligned} \quad (40)$$

Our performance measure is the normalized MSE between the actual channel impulse response and the estimated channel response, i.e.

$$J_{MSE}(SNR_{ave}) = \frac{\sum_{l=0}^{L-1} \left| h_{ij}[l] - \hat{h}_{ij}[l] \right|^2}{\sum_{l=0}^{L-1} \left| h_{ij}[l] \right|^2} \quad (41)$$

SNR_{ave} is the received SNR averaged over the two antennas and symbol periods given by

$$SNR_{ave} = \frac{1}{4} \sum_{i=1}^{2} \sum_{j}^{2} SNR_{ij} \quad (42)$$

Where SNR_{ij} is the SNR at the i^{th} receive antenna in the j^{th} symbol interval. We showed in the previous section that we can get the exact channel estimates in the case where there is no noise in the system. Therefore, the performance is limited only by the receiver noise. Figure 3 shows the $NMSE$ plotted against SNR_{ave}. The $NMSE$ decreases monotonically as the receiver SNR increases, thereby validating our claim that the estimation performance is limited only by the receiver noise.

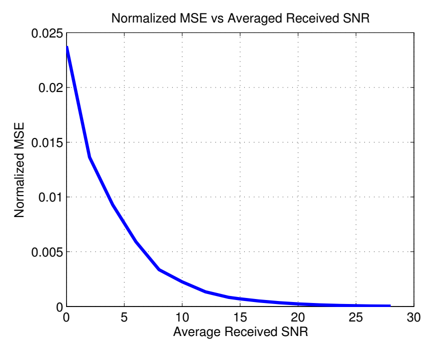

Fig. 3. Plot of NMSE against the average SNR

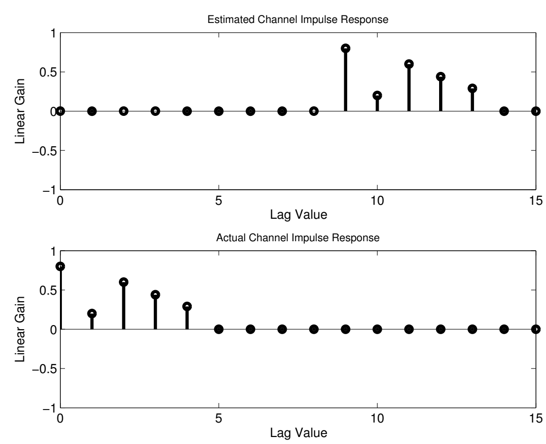

Fig. 4. Estimated and Actual CIR

In Figure 4, we show the estimated and the actual channel impulse response for the channel from the transmit antenna 1 to receive antenna 1 in the case where there is no noise and we have $N_p \geq N_c + L$. The impulse responses are

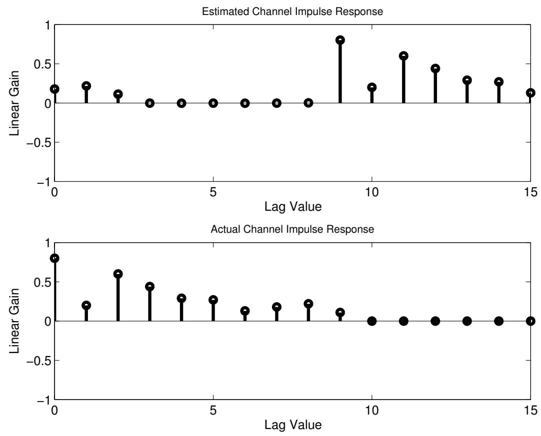

Fig. 5. Estimated and Actual CIR for $N_p < N_c + L - 1$

exactly the same, except that the estimated channel response is delayed by N_c samples. This happens because the autocorrelation on the complementary sequences is a delta function with a delay of N_c, and since the estimation of the channel impulse response involves the convolution of the actual channel response with the autocorrelation function of the complementary sequences, we observe a delay of N_c samples.

In Figure 5, the estimated and the actual channel impulse responses are shown for the case where $N_p < N_c + L - 1$. Specifically, we have $N_p = 16$, $N_c = 10$, and $L = 10$ and there is no noise. We can see from this figure that the estimated channel impulse response is a time-domain aliased version of the actual channel impulse response, where the time-domain aliasing manifests itself as a circular shift of the channel impulse response. However, since we have $N_p \geq N_c$, if we left shift the sequence by $N_c + L - 1 - Np = 3$, we have the same situation as the one shown in Figure 5, where the estimated channel is just a delayed version of the actual channel.

V. Conclusions

We have introduced a new channel estimation technique that uses Golay code based pilot waveforms. The OSTBC based scheduling of these waveforms facilitates simple separation of the channel mixtures in a MIMO environment when certain constraints on the DFT size used to transform the Golay codes in the frequency domain are met. In systems where the maximum sampled channel length does not exceed the cyclic prefix, it has been shown that the performance of this scheme is limited only by the receiver noise if we assume perfection timing and frequency synchronization. Simulation results have been provided to confirm the analytical results.

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