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Target Detection in MIMO Radar using Golay Complementary Sequences in the Presence of Doppler

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Abstract— **In this paper, we present a method for detecting a point target using multiple antennas when the relative motion between the receivers and the target induces a non-negligible Doppler shift. As a key illustrative example, we consider a 4×4 system employing a unitary matrix waveform set, e.g., formed from Golay complementary sequences. When a non-negligible Doppler shift is induced by the target motion, the waveform matrix formed from the complementary sequences is no longer unitary, resulting in significantly degraded target range estimates. To solve this problem, we adopt a subspace based approach exploiting the observation that the receive matrix formed from matched filtering of the reflected waveforms has a (non-trivial) null-space. Through processing of the waveforms with the appropriate vector from the null-space, we can significantly improve the detection performance. We provide simulation results to confirm the theoretical analysis.**

I. Introduction

In [1], Howard et al. proposed a new multi-channel radar scheme employing polarization diversity for getting multiple independent views of the target. In this scheme, Golay pairs [2] of phase coded waveforms are used to provide synchronization and Alamouti coding [3] is used to co-ordinate transmission of these waveforms on the horizontal and vertical polarizations. The combination of Golay complementary sequences and Alamouti coding makes it possible to do radar ambiguity polarimetry on a pulse-by-pulse basis, which reduces the signal processing complexity as compared to distributed aperture radar. This scheme [1] has been shown to provide the same detection performance as the single channel radar with significantly smaller transmit energy, or provide detection over greater ranges with the same transmit energy as the single channel radar.

In [4], the 2×2 case was extended to multiple antennas, and more general waveforms families were developed that allowed for perfect separation in case of negligible Doppler. In particular, scheduling for Golay pairs was described for a 4×4 system and it was demonstrated that Golay pairs achieve perfect separation and reconstruction [2]. However, in the presence of Doppler, Golay pairs don't perform very well, and that is one reason why these sequences haven't found a widespread use in radar, since accurate target ranging with these sequences is impossible in the presence of Doppler when conventional processing techniques are employed. In [5], PTM [6]-[10] sequences were used to make the Golay sequence transmissions resilient against Doppler shifts. The method achieves good results for small Doppler shifts, but the number of PRIs needed per transmission of the coded Golay sequence matrix is large, and would require the channel to stay constant over large intervals of time, which makes it a restrictive assumption. In this paper, we describe another Doppler compensation method that exploits the subspace structure of the transmitted waveform matrix. We show that the received waveform matrix can be processed in a way that imparts a specific structure on the subspace that it occupies, and the null-space of this matrix can be used to minimize the effects of Doppler. We develop a processing filter using the null-space of this matrix to alleviate the effects of Doppler in target ranging, and show that the method works over a wide range of target SNRs.

The paper is organized as follows: section 2 provides an introduction to Golay complementary sequences, and their use in Radar detection for the case of negligible Doppler. A signal model incorporating the effects of Doppler is introduced in section 3, which also details our method for compensating the effects of Doppler. Some conclusions and directions for future work at provided in Section 4.

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II. Golay Complementary Sequences and Target Detection

In this section, we describe how Golay codes enable high-resolution detection of targets in the absence of Doppler shift. We make the assumptions that the target round-trip delay of D chip intervals doesn't change

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appreciably during the transmission of N pulses, where $N = 4$ in our case. Without loss of generality, we assume $D = 0$ for ease of explanation.

A. Golay Complementary Sequences

A pair of sequences $s_1(n)$ and $s_2(n)$ of length N_c satisfy the Golay property if the sum of their autocorrelation functions satisfy

$$R_{s_1s_1}(l) + R_{s_2s_2}(l) = \begin{cases} 2N_c & if \quad l = 0 \\ 0 & if \quad l \neq 0 \end{cases} \tag{1}$$

for $l = -N_c - 1, ..., N_c - 1$. If we take the DFT of the above equation, we get

$$|S_1(k)|^2 + |S_2(k)|^2 = 2N_c \tag{2}$$

In [4], we showed that if $s_1(n)$ and $s_2(n)$ are Golay complementary, then so are $s_1^*(-n)$ and $s_2^*(-n)$. Using this fact, we can develop a 4-waveform family using Golay complementary sequences by defining

$$s_3(n) = s_1^*(-n) \tag{3}$$

and

$$s_4(n) = s_2^*(-n) \tag{4}$$

We now describe how this 4-waveform family can be used for target detection in a 4×4 system.

B. Target Detection in the Absence of Doppler

In the case of negligible Doppler, the received signal over 4 PRIs is given by

$$\mathbf{R}(n) = \mathbf{H}^T\mathbf{S}(n) + \mathbf{N}(n) \tag{5}$$

where $\mathbf{S}(n)$ is the transmitted waveform matrix given by [4].

$$\mathbf{S}(n) = \begin{bmatrix} s_1(n) & s_2^*(-n) & s_3(n) & s_4^*(-n) \\ -s_2(n) & s_1^*(-n) & -s_4(n) & s_3^*(-n) \\ -s_3(n) & s_4^*(-n) & s_1(n) & -s_2^*(-n) \\ -s_4(n) & -s_3^*(-n) & s_2(n) & s_1^*(-n) \end{bmatrix} \tag{6}$$

$\mathbf{H}$ is the channel matrix which contains the various round-trip path gains from each transmit antenna to each receive antenna.

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{12} & h_{22} & h_{23} & h_{24} \\ h_{13} & h_{32} & h_{33} & h_{34} \\ h_{14} & h_{42} & h_{43} & h_{44} \end{bmatrix} \tag{7}$$

and $\mathbf{N}(n)$ is the noise matrix. To detect the presence of the target in the delay resolution bin n, We process the received waveform matrix as

$$\mathbf{R}(n) * \mathbf{S}^H(-n) = \mathbf{H}^T\mathbf{S}(n) * \mathbf{S}^H(-n) + \mathbf{N}'(n) \tag{8}$$

where $*$ is the pair-wise convolution of two matrices that follows the same order as matrix multiplication. It can be easily shown [4] that

$$\mathbf{S}(n) * \mathbf{S}^H(-n) = \alpha\mathbf{I}\delta(n) \tag{9}$$

From this, it follows that

$$\mathbf{R}(n) * \mathbf{S}^H(-n) = \alpha\mathbf{H}\delta(n) + \mathbf{N}'(n) \tag{10}$$

In order to detect the presence of a target in the delay resolution bin n, consider the test stastic

$$z(n) = \left\|\mathbf{R}(n) * \mathbf{S}^H(n)\right\|_F^2 \tag{11}$$

where the subscript F stands for Frobenius norm. A plot of $z(n)$ for target SNRs of 5dB and 10dB, respectively are shown in Figure 1. As we can see from the figure, the unitary waveform matrix signal design greatly facilitates high-resolution time-localization of a target when the Doppler shift is negligible.

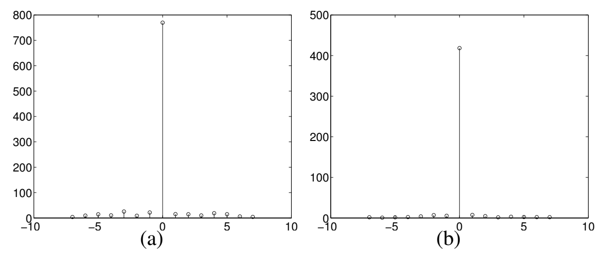

Fig. 1. Plot of $z(n)$ without Doppler for (a) SNR = 5dB (b) SNR = 10dB

III. Doppler Compensation

In this section, we develop a signal model that incorporates the effects of Doppler. We assume that the target is moving at a constant speed, which means that between two successive PRIs, the differential Doppler phase shift is constant.

A. Effects of Doppler

In the presence of Doppler, the received signal may be expressed as

$$\mathbf{R}(n) = \mathbf{H}^T\mathbf{S}(n)\mathbf{D} + \mathbf{N}(n) \tag{12}$$

The Doppler shift matrix $\mathbf{D}$ is given by

$$\mathbf{D} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{j\upsilon} & 0 & 0 \\ 0 & 0 & e^{j2\upsilon} & 0 \\ 0 & 0 & 0 & e^{j3\upsilon} \end{bmatrix} \tag{13}$$

where υ is the Doppler-induced differential phase shift between two successive PRIs. As in the case of negligible Doppler, we process the received waveform matrix as

$$\mathbf{R}(n) * \mathbf{S}^H(-n) = \mathbf{H}^T \mathbf{S}(n)\mathbf{D} * \mathbf{S}^H(-n) + \mathbf{N}'(n) \quad (14)$$

In the presence of a non-negligible Doppler phase shift, the condition in (4) is not satisfied in general, i.e.,

$$\mathbf{S}(n)\mathbf{D} * \mathbf{S}^H(-n) \neq \alpha \mathbf{I}\delta(n) \quad (15)$$

and unambiguous range resolution becomes significantly more difficult. To illustrate this graphically, a plot of $z(n)$ for the same target SNRs of 5dB and 10dB are plotted in Figure 2 for the case of $\upsilon = \pi/3$. For this particular set of round-trip channel gains, the presence of Doppler makes it impossible to detect the target.

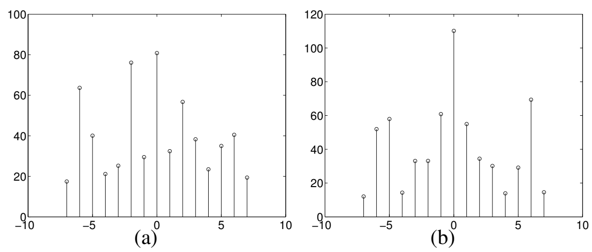

Fig. 2. Plot of $z(n)$ with Doppler ($\upsilon = \pi/3$) for (a) SNR = 5dB (b) SNR = 10dB

B. Doppler Processing

Towards combatting this problem, consider the matrix

$$\hat{\mathbf{R}}(n) = \mathbf{R}(n) * \mathbf{S}^H(-n) \quad (16)$$

Each term of this matrix is a sum of four individual convolution sequences. Next, consider the 4×4 matrix $\mathbf{Y}_i$, given by

$$\mathbf{Y}_i(n) = \left[\begin{array}{cccc} r_{i1}(n) * s_1^*(-n) & r_{i1}(n) * s_1^*(-n) & r_{i1}(n) * s_1^*(-n) & r_{i1}(n) * s_1^*(-n) \\ r_{i2}(n) * s_2(n) & r_{i2}(n) * s_2(n) & r_{i2}(n) * s_2(n) & r_{i2}(n) * s_2(n) \\ r_{i1}(n) * s_1^*(-n) & r_{i1}(n) * s_1^*(-n) & r_{i1}(n) * s_1^*(-n) & r_{i1}(n) * s_1^*(-n) \\ r_{i2}(n) * s_2(n) & r_{i2}(n) * s_2(n) & r_{i2}(n) * s_2(n) & r_{i2}(n) * s_2(n) \end{array} \right] \quad (17)$$

Note that column j above contains the individual convolution sequences that are summed up to yield the ij^{th} term in $\hat{\mathbf{R}}(n)$. Consider the vector

$$\mathbf{w} = \left[\begin{array}{cccc} 1 & 1 & 1 & 1 \end{array} \right]^T \quad (18)$$

In the case of no Doppler, and ignoring the noise, it is easy to verify that

$$\mathbf{w}^H \mathbf{Y}_i(n) = \gamma \mathbf{h}_i \delta(n) \quad (19)$$

where γ is just a scaling constant, and

$$\mathbf{h}_i = \left[\begin{array}{cccc} h_{1i} & h_{2i} & h_{3i} & h_{4i} \end{array} \right] \quad (20)$$

The index i is associated with one of the receive antennas. When Doppler is present, it is likewise easy to verify that

$$\mathbf{w}_D^H \mathbf{Y}_i(n) = \gamma \mathbf{h}_1 \delta(n) \quad (21)$$

where

$$\mathbf{w}_D = \left[\begin{array}{cccc} 1 & e^{j\upsilon} & e^{2j\upsilon} & e^{3j\upsilon} \end{array} \right]^T \quad (22)$$

and this holds for all i. This means that for $n \neq 0$, the matrices $\mathbf{Y}_i$ are singular, and the vector producing the desired output lies in the null-space of these matrices. Also, because of the same waveform inputs, the matrices $\mathbf{Y}_i$ share the same null-space. Thus, we can form a concatenated matrix as

$$\mathbf{Y}_C(n) = \left[\begin{array}{cccc} \mathbf{Y}_1(n) & \mathbf{Y}_2(n) & \mathbf{Y}_3(n) & \mathbf{Y}_4(n) \end{array} \right] \quad (23)$$

It is easy to verify that

$$\mathbf{w}_D^H \mathbf{Y}_C(n) = \gamma \mathbf{h} \delta(n) \quad (24)$$

where $\mathbf{h} = [\mathbf{h}_1 \ \mathbf{h}_2 \ \mathbf{h}_3 \ \mathbf{h}_4]$. Now, since we don't know the true-target delay for which $\mathbf{Y}_C(n)$ is non-singular, we cannot simply find the null-space vector at every n. That is, an approach is need to circumvent the fact $\mathbf{w}_D$ is not in the null-space of $\mathbf{Y}_C(n)$ at the true target delay. Again, WLOG the true target delay is assumed to be $n = 0$. The concatenated matrix $\mathbf{Y}_C(n)$ is formed to exploit the fact that the different matrices share the same null-space vector. To counter the effect of $\mathbf{Y}_C(0)$ on the null-space, instead of working with a single chip interval, we take a length $2q + 1$ lag window and form the matrix

$$\mathbf{X}_C(n) = \left[\begin{array}{ccccc} \mathbf{Y}_C(n-q) & ... & \mathbf{Y}_C(n) & ... & \mathbf{Y}_C(n+q) \end{array} \right] \quad (25)$$

The SVD of $\mathbf{X}_C$ can be written as

$$\mathbf{X}_C(n) = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^H \quad (26)$$

Since $\mathbf{X}_C(n)$ and $\mathbf{X}_C(n)\mathbf{X}_C^H(n)$ share the same singular vectors, we will work with the latter. The idea is to subtract out $\mathbf{Y}_C(0)$ in order to obtain the correct the null space of $\mathbf{R}_{X_C X_C}(n)$. To do this, we compute the SVD of

$$\mathbf{R}_{X_C X_C}(n) - \mathbf{Y}_C(k)\mathbf{Y}_C^H(k) \quad (27)$$

for $n - q \leq k \leq n + q$ and store the singular vector associated with the smallest eigenvalue. Note that out of the $2q + 1$ singular vectors that we store for each n, there is at most one singular vector that corresponds to $\mathbf{Y}_C(0)$ and this happens whenever $0 \in \{n-q, ..., n+q\}$. Again, WLOG the true target delay here is $n = 0$.

The inclusion of $\mathbf{Y}_C(0)$ alters the null-space structure. In order to find which matrix to subtract, since we don't know the true target delay, for each of the $2q+1$ singular vectors, we compute its inner product with the other $2q$ singular vectors, and choose that singular vector that yields the smallest inner product (magnitude) with the rest of the vectors.This process is mathematically described as follows. Let $\mathbf{U}_{min}(n)$ be the matrix with the $2q+1$ singular vectors as columns. The inner product (Grammian) matrix is formed as

$$\mathbf{M}(n) = \mathbf{U}_{\min}^H(n)\mathbf{U}_{\min}(n) \tag{28}$$

We can write $\mathbf{M}(n)$ as

$$\mathbf{M}(n) = \begin{bmatrix} \mathbf{m}_{n-q} \\ \vdots \\ \mathbf{m}_{n+q} \end{bmatrix} \tag{29}$$

The index of the singular vector of interest is obtained as

$$k_{opt} = \arg\min_k \|\mathbf{m}_k\| \tag{30}$$

To check the presence of target in the delay bin n, we process the vector $\mathbf{Y}_C(n)$ as

$$z(n) = \left\| \mathbf{u}_{k_{opt}}^H(n)\mathbf{Y}_C(n) \right\|_F^2 \tag{31}$$

Ideally, the magnitude of $z(n)$ should exhibit a sharp peak at the true target delay and be near zero for all other values of n. In Figure 2, we plot $z(n)$ for target SNRs of 5 and 10dB, respectively, and a differential Doppler phase shift of $\upsilon = \pi/3$, with the window length set to 5 ($q = 2$). We used the same 4×4 unitary waveform matrix as in Figure 1, along with the same round-trip "channel" gains. These results reveal the proposed Doppler processing technique to be promising in terms of compensating for Doppler while still providing high-resolution accurate target delays.

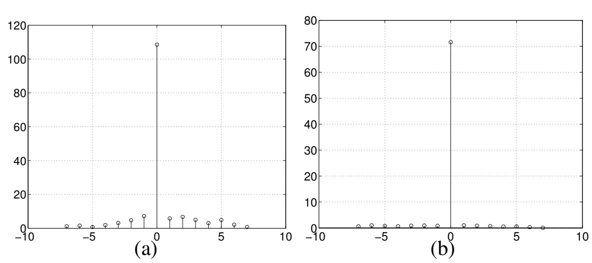

Fig. 3. (a) Plot of $z(n)$ for $SNR = 5dB$ (b) Plot of $z(n)$ $SNR = 10dB$

IV. Conclusions and Future Work

We have developed a technique for doing accurate target ranging in the presence of Doppler using Golay complementary sequences. The technique is based on finding the null-space of the waveform matrix after matched filtering at the receiver, and then using an appropriate vector from the null-space to process the matched filtered received waveforms over multiple PRIs. Simulation results were presented that show how the proposed technique diminishes the effects of Doppler while still facilitating high-resolution, accurate target ranging over a range of target SNRs. Future work includes a theoretical analysis of the detection performance, and extending this technique for multiple targets in the same range cell with different radial velocities. The current technique can handle the latter as long as the targets are separated in time by at least N_c, the length of the transmitted waveform

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