Bat-Inspired Adaptive Design of Waveform and Trajectory for Radar

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***Abstract*—We propose to design jointly the waveform and trajectory of a radar mounted on a moving platform, in order to improve the system performance for tracking maneuvering targets. Inspired by bats, we develop an adaptive algorithm that chooses the optimal pulse repetition interval (PRI) and path of the radar. Our method automatically schedules a low PRI when it recognizes that the target executes a maneuvering action. Simultaneously, it selects the radar trajectory which provides the best estimation of the target parameters. We derive our approach under a framework of sequential Bayesian filtering and implement it with a particle filter. We consider a library of target state models associated with different PRI values and use multiple model to schedule the optimal PRI. We apply the posterior Cramér-Rao bound to measure the system performance and decide on the optimal radar path. We demonstrate the advantages of the adaptive radar scheme using numerical examples.**

I. Introduction

Bats are capable of detecting and catching a fast-moving prey, even in dense cluttered environments. Studies of the echo-location system of these mammals show that bats change the parameters of their transmitted sound during different stages of the target pursuit sequence [1]-[3]. For instance, they reduce the pulse repetition interval (PRI) to refine range information when they are close to the prey. In addition, they adapt their flight path based on the predicted target trajectory [4]. This adaptive behavior provides the bat a high success rate in pursuing and capturing its target. Hence, there is a great interest in devising methods and algorithms that can mimic the bat.

Recent work in the area of biologically inspired systems studied the bat waveform calls and their variations for potential applications in autonomous navigation [5]. Similarly, in [6] the echolocation of a bat specie is examined for developing a sonar system. In [7], VLSI technology is applied to implement a echolocation system based on the computations executed by the bat auditory system. Bat signals have also motivated the design of ultrasound sensors [8]. A mobility aid device for helping blind people was created by imitating the bat [9].

In this paper, we develop a sequential algorithm to jointly design the optimal waveform and trajectory of a moving radar. Inspired by the bat, our proposed method adaptively selects the signal PRI and the radar path to improve the tracking accuracy of maneuvering targets. We use a sequential Bayesian framework to solve jointly the problem of target tracking and radar design. We propose a multiple-model filter to recognize maneuvering actions by the target; and consequently adapt the signal PRI to increase the collected information. We implement the Bayesian filter using a particle filter that is suitable for nonlinear state and measurement models. We also propose a new criterion based on the posterior Cramér-Rao bound to optimally select the radar trajectory. This research extends our recent work on adaptive radar waveform design [10] and path design [11].

This paper is organized as follows. In Section II, we discuss the dynamic state model and the measurement model. In Section III, we develop a multiple-model sequential Monte Carlo method for target tracking. In Section IV, we propose the posterior Cramér-Rao bound as a measure of the tracking performance. In Section V, we derive our algorithm for joint design of the radar waveform and path. Numerical examples and conclusions are given in Section VI and Section VII, respectively.

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II. State and Measurement Models

In this section we present the dynamic state model that describes the target's position and velocity. We also introduce the measurement model and define the statistical assumptions of the process and measurement noise.

A. Target State Model

For the target tracking problem, we define the target state relative to the radar state as

$$\boldsymbol{x}_k = \boldsymbol{x}_{\mathrm{t}k} - \boldsymbol{x}_{\mathrm{r}k} = [x,\ y,\ \dot{x},\ \dot{y}]^{\mathrm{T}}, \tag{1}$$

where $\boldsymbol{x}_{\mathrm{t}k}$ and $\boldsymbol{x}_{\mathrm{r}k}$ are the target and radar state at the k^{th} time step. Here, x, y and $\dot{x}$, $\dot{y}$ represent the target position and velocity relative to the radar in a Cartesian coordinate system. For simplicity, we present a 2D formulation of the problem, which can be easily extended to 3D geometry.

We assume a constat velocity (CV) model for the target state:

$$\boldsymbol{x}_k = F\boldsymbol{x}_{k-1} + \boldsymbol{v}_{k-1}, \tag{2}$$

where F is the transition matrix defined as

$$F = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \otimes I_2, \tag{3}$$

where T is the PRI and I_n is the identity matrix of size n. The process noise $\boldsymbol{v}_k$ represents the random acceleration modeled by a zero mean Gaussian distribution with covariance

$$\Sigma_{\rm a} = \sigma_{\rm a}^2 \begin{bmatrix} T^4/4 & T^3/2 \\ T^3/2 & T^2 \end{bmatrix} \otimes I_2, \tag{4}$$

and where $\sigma_{\rm a}^2$ is the variance of the random acceleration.

B. Multiple Model

To characterize the dynamic of a maneuvering target, we consider a multiple-model state equation

$$\boldsymbol{x}_k = F\boldsymbol{x}_{k-1} + \boldsymbol{v}_{k-1}^{(m)}, \quad m = 1, \ldots, M, \tag{5}$$

where m is the regime (or model) selected at the time step k. The multiple model assumes that the target has a set of allowed accelerations $(\sigma_{\rm a}^{(m)})^2$ and switches between them following a transition probability matrix

$$\Pi = [\pi_{ij}]_{i,j=1,\ldots,M} \tag{6}$$

where π_{ij} is the probability of switching from the i^{th} to j^{th} model. Fig. 1 shows an illustrative example of the tradeoff in choosing models with different acceleration variances. A model with low acceleration variance can accurately estimate targets with constant velocity, but it fails to track maneuvering targets. On the other hand, a model with high acceleration variance can keep track of maneuvering targets, but has higher estimation error.

We note that equation (5) is not a linear system because the state $\boldsymbol{x}_k$ does not depend on the model index m in a linear fashion.

C. Measurement Model

We consider a mono-static radar transmitting narrow-band signals that illuminates a point target located in the far-field region. The target is characterized by the azimuth angle ϕ, range r, and Doppler shift $\omega_{\rm D}$. Then, the output of an array of L sensors receiving the echoes from the target can be expressed as

$$\boldsymbol{y}(t) = \boldsymbol{p}(\phi)s(t-\tau)e^{j\omega_{\rm D}t} + \boldsymbol{e}(t), \quad t = t_1, \ldots, t_N, \tag{7}$$

where $\boldsymbol{p}(\phi) = [e^{j2\pi\boldsymbol{u}^{\rm T}\boldsymbol{r}_1/\lambda}, \ldots, e^{j2\pi\boldsymbol{u}^{\rm T}\boldsymbol{r}_L/\lambda}]^{\rm T}$ is the array response vector, $\boldsymbol{u} = [\cos\phi, \sin\phi]^{\rm T}$ is the planewave direction of arrival vector, $\boldsymbol{r}_l$ is the position of the l-th sensor ($l = 1, \ldots, L$), λ is the wavelength of the carrier signal, $s(t)$ is the transmitted waveform, $\tau = 2r/c$ is the signal delay, and c is the propagation velocity of the signal. The vector $\boldsymbol{e}(t)$ is the additive noise corrupting the measurements; it represents the thermal noise at the sensors and the interference from the environment. We assume that $\boldsymbol{e}(t)$ is a zero-mean white Gaussian process with known covariance $\sigma^2 I_L$; the variable N denotes the number of samples during the observation time T.

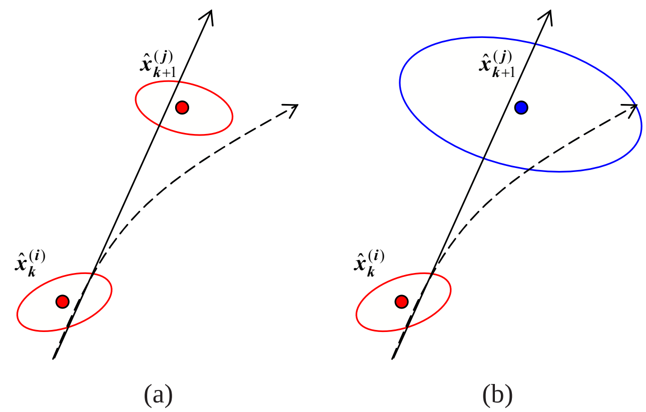

Fig. 1. Tradeoff in choosing models with different acceleration variances. The solid line is the trajectory of a constant-velocity target and the dashed line is the trajectory of a maneuvering target. (a) A model with low acceleration variance produces accurate estimates (small confidence ellipse) but it cannot track a maneuvering target, and (b) a model with high acceleration variance can track a maneuvering target but it has a large confidence ellipse.

The relationship between the target parameters $[\phi,\ r,\ \omega_{\rm D}]$ and the state vector $\boldsymbol{x}$ is given by

$$\begin{aligned} \phi &= \arctan(y/x) \qquad\qquad r = \sqrt{x^2 + y^2} \\ \omega_{\rm D} &= \frac{2\pi}{\lambda r}(\dot{x}x + \dot{y}y) \end{aligned} \tag{8}$$

Therefore, the measurement model is a nonlinear function of the state parameters.

III. Sequential Bayesian Tracking

The problem of target tracking consists of recursively estimating the state $\boldsymbol{x}_k$ based on the measurements $\boldsymbol{y}_{1:k}$ up to the time k. Then, we need to compute the posterior probability density function (pdf) $p(\boldsymbol{x}_k \mid \boldsymbol{y}_{1:k})$. Under the Bayesian inference framework, we can obtain recursive formulas to calculate this pdf when the new measurements $\boldsymbol{y}_k$ are available [12]. However, this recursive procedure cannot be determined analytically when the models are nonlinear or non-Gaussian. Under these conditions, numerical methods are used to find approximate solutions to the optimal Bayesian filter.

A. Multiple-Model Particle Filter

We apply the multiple-model particle filter [13] to implement the former Bayesian algorithm for our tracking problem characterized by a nonlinear state and measurement models. The particle filter algorithm is a sequential Monte Carlo method in which the key idea is to represent the required posterior density function by a set of random samples with associated weights. Then, the parameters of interest are estimated based on these samples and weights.

This particle filter sets an augmented state vector which includes the target's parameters and the model index: $\boldsymbol{z}_k =$

$[\boldsymbol{x}_k^{\mathrm{T}}, m_k]^{\mathrm{T}}$. Let $\{\boldsymbol{z}_k^{(i)}\}$ be a set of random points with associated weights $\{w_k^{(i)}\}$ for $i = 1, \ldots, N_{\mathrm{s}}$, where N_{s} is the number of particles. Then, the posterior density $p(\boldsymbol{z}_k \mid \boldsymbol{y}_{1:k})$ can be approximated as [12]

$$p(\boldsymbol{z}_k \mid \boldsymbol{y}_{1:k}) \approx \sum_{i=1}^{N_{\mathrm{s}}} w_k^{(i)} \delta(\boldsymbol{z}_k - \boldsymbol{z}_k^{(i)}), \tag{9}$$

where the weights are normalized such that $\sum_i w_k^{(i)} = 1$. The samples $\boldsymbol{z}_k^{(i)}$ are easily generated from a proposal distribution $q(\cdot)$ called importance density function. The weights are computed using the principle of importance sampling and depend on the selected importance density. For a sequential filtering case, we can choose an importance density $q(\cdot)$ such that we obtain a recursive weight equation as [14]

$$w_k^{(i)} \propto w_{k-1}^{(i)} \frac{p(\boldsymbol{y}_k \mid \boldsymbol{x}_k^{(i)}, m_k^{(i)}) p(\boldsymbol{x}_k^{(i)} \mid \boldsymbol{x}_{k-1}^{(i)}, m_k^{(i)})}{q(\boldsymbol{x}_k^{(i)} \mid \boldsymbol{x}_{k-1}^{(i)}, m_k^{(i)}, \boldsymbol{y}_k)}. \tag{10}$$

In our particle filter, we choose the importance density to be the transitional prior: $q(\boldsymbol{x}_k \mid \boldsymbol{x}_{k-1}, \boldsymbol{y}_k) = p(\boldsymbol{x}_k \mid \boldsymbol{x}_{k-1}, m_k^{(i)})$. A pseudocode of a generic multiple-model particle filter is shown in [14, Chap. 3].

IV. Tracking Performance Measure

We use the posterior Cramér-Rao bound (PCRB) as a performance measure for the sequential estimation of the target parameters. The PCRB provides a lower bound on the mean-square error (MSE) matrix for the random state vector.

Considering our target tracking problem at the k^{th} time step, we wish to estimate a state trajectory $\boldsymbol{x}_{0:k}$ using the measurements $\boldsymbol{y}_{1:k}$. Then, the Bayesian information matrix (BIM) of the trajectory, whose inverse is the PCRB, is defined as

$$\bar{J}_k \triangleq \mathrm{E}_{\boldsymbol{y}_{1:k}, \boldsymbol{x}_{0:k}} \left[-\triangle_{\boldsymbol{x}_{0:k}}^{\boldsymbol{x}_{0:k}} \log p(\boldsymbol{y}_{1:k},\ \boldsymbol{x}_{0:k}) \right], \tag{11}$$

where $\triangle_\alpha^\beta$ denotes the second order derivative with respect to α and β. The lower right $n_x \times n_x$ block ($n_x = \dim(\boldsymbol{x}_k)$) of $\bar{J}_k^{-1}$ is the PCRB on the estimation of $\boldsymbol{x}_k$; and its inverse is the BIM on the estimation of $\boldsymbol{x}_k$, denoted as J_k.

To derive the optimal path selection criterion, we adopt the recursive equation in [15] to update the BIM J_{k+1}. For the particular case of a linear state model with additive Gaussian noise, this recursive BIM can be written as (see [15])

$$J_{k+1} = \left[\Sigma_{\mathrm{a}} + F J_k(\boldsymbol{\theta}_k)^{-1} F^{\mathrm{T}}\right]^{-1} + \Gamma_{k+1}(\boldsymbol{\theta}_{k+1}), \tag{12}$$

where $\boldsymbol{\theta}_k$ and $\boldsymbol{\theta}_{k+1}$ are the radar parameters at the time step k and $k+1$, respectively, and

$$\Gamma_{k+1} = \mathrm{E}_{\boldsymbol{y}_{k+1}, \boldsymbol{x}_{k+1}} \left[-\triangle_{\boldsymbol{x}_{k+1}}^{\boldsymbol{x}_{k+1}} \log p(\boldsymbol{y}_{k+1} \mid \boldsymbol{x}_{k+1}) \right]. \tag{13}$$

Note that Γ_{k+1} is calculated by averaging over all possible values of $\boldsymbol{y}_{k+1}$. That means we do not need to know the specific values of the next measurements to calculate J_{k+1}.

V. Adaptive Radar Design

In this section we present an algorithm for the optimal design of the radar waveform and trajectory. It is combined with the target tracking algorithm and forms an adaptive approach. The proposed algorithm predicts the target at the $k^{th}+1$ time step using the state models and the estimated parameters at the k^{th} time step. Then, we select those radar actions, such as transmitted waveform and next radar path, which optimize the performance of the system.

A. PRI Scheduling

A radar signal offers a large number of degrees of freedom that can be optimized, including time, frequency, and polarization. In particular we focuss on scheduling adaptively the signal PRI from a set of allowed values. Using the library of state models designed in Section II-B, we assign to each model a different PRI. For those models with high acceleration we set a low PRI. Then, if the tracking filter detects a maneuvering target, it switches to a model with higher acceleration and, at the same time, reduces the signal PRI. As a result, the algorithm increases the amount of measurements from the target when it perform a maneuvering action.

B. Path Design

In order to design the radar trajectory, we attempt to minimize the error in the target state estimation. Then, we propose the weighted trace of the predicted PCRB as a criterion for selecting the optimal radar trajectory:

$$\boldsymbol{\theta}_{k+1}^* = \arg \min_{\boldsymbol{\theta}_{k+1} \in \boldsymbol{\Theta}} \mathrm{Tr}\{W J_{k+1}^{-1}(\boldsymbol{\theta}_{k+1})\} \tag{14}$$

where $\boldsymbol{\Theta}$ denotes a set of allowed values for $\boldsymbol{\theta}_{k+1}$ or a library of all possible radar trajectories, and W is a weighting matrix used to equalize the magnitude of different parameters in the state vector. Additionally, the matrix W can be used to provide different priorities to subsets of parameters by assigning a higher weight.

The proposed criterion function depends not only on the information provided by the state model but also by the measurement model, through the term Γ_{k+1}. To compute this matrix, in general, the expectation in (13) has no closed-form analytical solution and must be solved numerically; for example using Monte Carlo integration. However, this method is computationally intensive and time demanding because the integrand of Γ_{k+1} must be evaluated for every particle. Therefore, in [10] we proposed a suboptimal method that significatively reduces computation time at the expense of accuracy in computing the integral in (13). This procedure has the advantage that it can be merged into the sequential Monte Carlo method for the target tracking.

VI. Numerical Examples

In this section, we use numerical examples to illustrate the behavior of the proposed adaptive radar algorithm. We consider the problem of tracking a moving target in a 2D environment (XY plane). The target moves at a speed of

50m/s on a straight trajectory, makes a smooth turn, and continues on a straight trajectory. The radar transmits linear frequency modulated (LFM) pulses with Gaussian envelop of 10μs length and 10MHz bandwidth. The carrier frequency is 10GHz. The receiver array consists of a uniform circular array of six sensor separated by 0.5λ. For our simulations, we assume that the signal-to-noise ratio (SNR) is 10dB at the receiver, independent of the target range (neglecting the attenuation of the signal due to free-space propagation).

Through the examples we demonstrate the advantages of our adaptive scheme compared with a radar system that has fixed waveform and position. The performance results reported in this section correspond to an average over 100 Monte Carlo simulations.

Example 1. In this example, we first study the advantage of scheduling the optimal PRI. We assume a static radar, located at the origin of the coordinate system, that has the capability of transmitting LFM pulses with different PRI at each time step k. We consider the following library of target state models and associated PRIs:

	Acceleration	PRI
Model 1	$\sigma_{\rm a}^2 = 0.05{\rm m}^2/{\rm s}^4$	$T = 10{\rm s}$
Model 2	$\sigma_{\rm a}^2 = 5{\rm m}^2/{\rm s}^4$	$T = 4{\rm s}$

The transition probability matrix for this library is chosen to be

$$\Pi = \begin{bmatrix} 0.6 & 0.4 \\ 0.4 & 0.6 \end{bmatrix}. \tag{15}$$

Fig. 2 shows an example of the estimated target trajectory for the adaptive scheme. We note that when target follows a straight line trajectory, the system selects Model 1 scheduling the higher PRI. However, as soon as it detects that the target maneuvers, it switches to Model 2, to handle movements with higher acceleration, and selects the lower PRI. This can be also appreciated in Fig. 3 that shows the model probability as a function of time. This figure depicts how one model becomes more likely than the other, and then, it is selected automatically by the tracking filter. Fig. 4 shows the root mean square (RMS) position error for radar systems with adaptive and fixed PRI. When the system transmits fixed PRI of 10s, it can not detect the target turns, resulting in poor tracking performance. If the radar transmits fixed PRI of 10s, it performs better than the adaptive case. However, transmitting continuously signals at high rate implies high computation load and power consumption. Instead, the adaptive radar system efficiently transmits signal with low PRI only when the target is performing a turn.

Example 2. In this new example, we add the adaptive path to the proposed radar scheme. We assume that the radar can maneuver by selecting its velocity vector at each time step k. However, we consider that the radar follows some simple kinematic constraints. We assume that its speed must remains between 50m/s and 200m/s, and the speed can be increased or reduced in 10% at each time step. The radar can modify its bearing angles in steps of 5°, in between -10° and 10°, with respect to its current direction. Fig. 5 shows an example

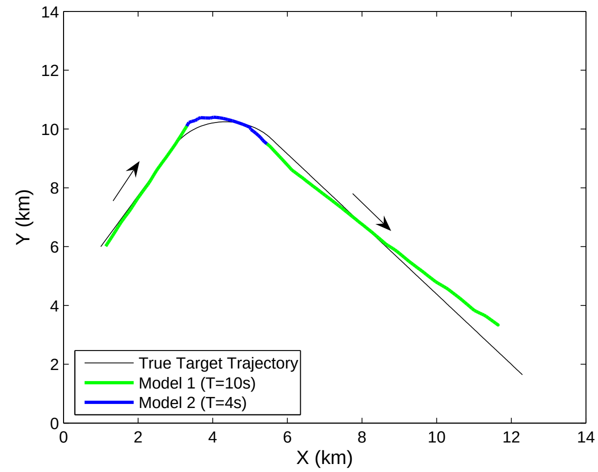

Fig. 2. Example of the estimated target trajectory using the multiple-model algorithm with a library of target state models with different PRI.

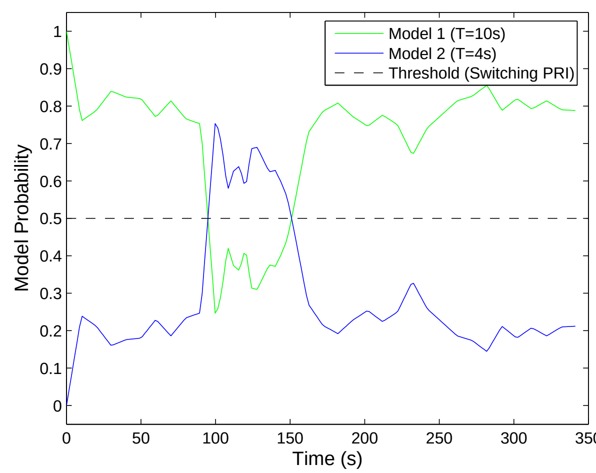

Fig. 3. Example of the estimated model probabilities.

of the designed path of the adaptive radar and the estimated trajectory of the target. Fig. 6 depicts the tracking performance of the radar with joint adaptive PRI and path in comparison to the fixed system using a PRI of 10s. This figure shows that the joint design outperforms the cases in which one of the radar aspects, waveform or path, are fixed.

VII. Conclusions

We developed an adaptive algorithm for joint design of the optimal waveform and trajectory of a moving radar. Our algorithm selects the optimal radar actions, one time step ahead, based on the estimated and predicted target in order to improve the system performance. We proposed a sequential Bayesian framework for solving the target tracking problem. Because the measurement and state models are non-linear, we applied particle filters to implement the tracking algorithm. For designing the waveform, we used a multiple model algorithm

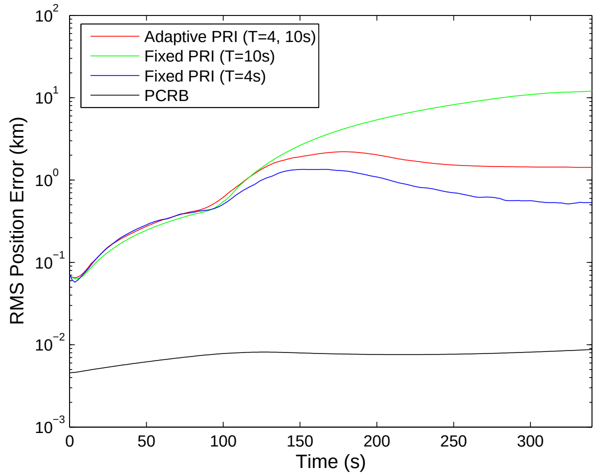

Fig. 4. Position error as a function of time for the case of adaptive radar and fixed radar system.

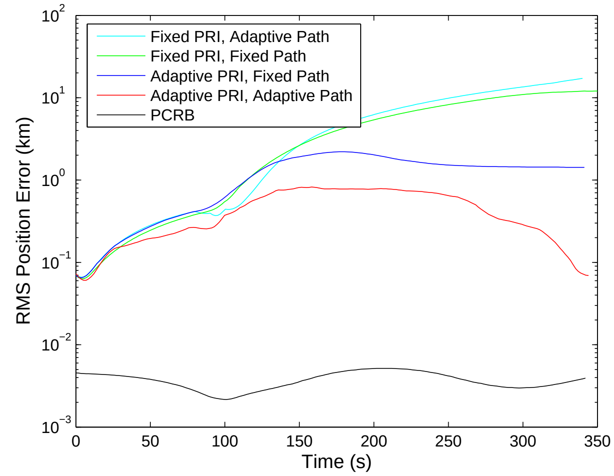

Fig. 6. Position error as a function of time for the case of adaptive radar and fixed radar system.

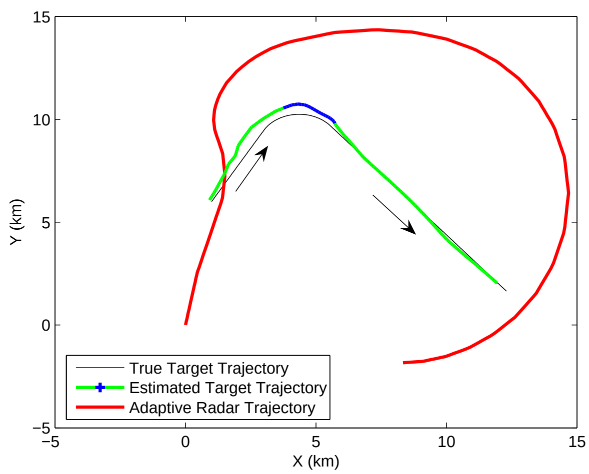

Fig. 5. Example of the estimated radar and target trajectories.

and proposed a library of state models in which a model with high acceleration is associated with a radar signal of low PRI. To plan the optimal trajectory, we devised a cost function based on the posterior Cramér-Rao bound for the estimated target position and velocity. Hence, our proposed adaptive radar algorithm increases the amount of collected information when the target maneuvers and reduces power consumption and computation load when the target moves with constant velocity. Simultaneously, the system decreases the tracking errors by selecting the optimal radar path. Finally, we showed the advantages of the adaptive scheme using examples with simulated data.

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