Polarimetric MIMO Radar With Distributed Antennas for Target Detection⋆

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***Abstract*—Distributed antenna radar systems provide spatial diversity gain by viewing the target from different angles. Polarimetric radar offers improved performance over conventional radar systems by exploiting polarization diversity. We propose a radar system that offers both spatial and polarimetric diversity gains for detecting stationary point-like targets. The receive antennas of the proposed system use 2D vector sensors, each measuring the horizontal and vertical components of the received electric field separately. We design the Neyman-Pearson detector for this proposed system and analyze its performance. This analysis is used to select the optimal transmit polarizations for this system. Using simulations, we demonstrate the improvement offered by the optimal choice of polarizations. We also show the spatial diversity offered by MIMO radar.**

I. Introduction

The target scattering matrix determines the change in polarization of the transmitted wave when it reflects off the surface of the target [1], [2]. Therefore, knowledge about the target in terms of its scattering matrix helps us design the optimal transmit waveform polarizations for performance improvement over systems transmitting waveforms with fixed polarizations. In [3]–[7], polarimetric design is suggested for use in conventional single antenna radar systems for problems such as detection, estimation and tracking. In [8], radar polarimetry is also used in multiple antenna systems with colocated antennas.

In radar systems, the attenuation experienced by the signal depends on the properties of the target. In a realistic scenario, it is highly likely that this attenuation will be a function of the angle of view of the target. If the angles of view of the target are sufficiently distinct from one another, then it is highly likely that the attenuation coefficients will have very little correlation. Therefore, even if some of the attenuation coefficients are extremely small, it is highly probable that they will be compensated by the others. Multiple Input Multiple Output (MIMO) radar with widely separated (distributed) antennas exploits this property (spatial diversity) by obtaining different views of the target [9], [10].

In this paper (see also [11]) we propose a radar system that combines the advantages of distributed–antenna MIMO systems with the advantages offered by optimally choosing the transmit waveform polarizations. We derive a realistic signal model for this system by varying the scattering matrices with the angle of view. Therefore, this system offers both spatial and polarization diversities. Vector sensors provide significant improvement in performance over scalar sensors [12]. Therefore, we use two–dimensional vector sensors to measure both the horizontal and vertical components of the electric field at each of the receivers. We demonstrated the advantages of retaining the vector measurements without combining them in [11]. In this paper, we examine the problem of target detection and demonstrate the advantages of the proposed system.

II. Signal Model

Before we develop the mathematical model, we describe the target and the radar system. We assume that the target is stationary and is present in the illuminated space. The target is further assumed to be point-like with a scattering matrix that depends on the angle of view. We consider a radar system that has M transmit antennas and N receive antennas with all the antennas widely spaced. Each of the receive antennas employs a two–dimensional vector sensor that measures both the horizontal and vertical components of the received polarized signal separately. Polarimetric models exist for describing the signals received in single-antenna systems [1], [2]. We extend these models to distributed antenna systems in this section.

Define the polarization vector for the i^{th} transmitter to be $\boldsymbol{t}^i = \left[t^i_{\rm h}, t^i_{\rm v}\right]^T$, where each of the entries of the polarization vectors is a complex number. We further assume that $\|\boldsymbol{t}^i\| = 1, \forall i = 1, \ldots, M$. The complex pulse wave shape transmitted from the i^{th} transmitter is $w^i(t)$. We assume that all $w^i(t)$ are orthonormal to each other for all mutual delays between them [9], [10]. In other words, we assume that the cross correlation among these different waveforms is negligible for different lags. At the receiver side, this condition helps us differentiate between the signals transmitted from different antennas.

After transmission, the polarized waveforms travel in space and reflect off the surface of the target towards the receivers

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with altered polarimetric properties. The polarized signal reaching the j^{th} receiver is a combination of all the signals reflecting from the surface of the target towards it. Let $\boldsymbol{y}^j(t)$ be the complex envelope of the signal received by the j^{th} receiver. Note that $\boldsymbol{y}^j(t)$ is a 2–dimensional column vector consisting of the horizontal and the vertical components of the received signal, and it is expressed using a formulation similar to that presented in [13]–[15]:

$$\boldsymbol{y}^j(t) = \sum_{i=1}^{M} a^{ij}\boldsymbol{S}^{ij}\boldsymbol{t}^i w^i(t-\tau^{ij}) + \boldsymbol{e}^j(t), \quad (1)$$

where $\boldsymbol{e}^j(t)$ is the 2–dimensional additive noise, τ^{ij} is the time delay because of propagation and the attenuation is divided into two factors a^{ij} and $\boldsymbol{S}^{ij}$. a^{ij} is that part of attenuation which depends on the properties of the medium, distance between the target and radar, etc. We assume that $\{a^{ij}\}$ are known because the radar has an idea about the region which it is illuminating and the properties of the medium. $\boldsymbol{S}^{ij}$ represents the scattering matrix of the target, which completely describes the change in the polarimetric properties of the signal transmitted from the i^{th} transmitter to the j^{th} receiver. This represents the unknown part of the attenuation.

$$\boldsymbol{S}^{ij} = \begin{bmatrix} s^{ij}_{\text{hh}} & s^{ij}_{\text{hv}} \\ s^{ij}_{\text{vh}} & s^{ij}_{\text{vv}} \end{bmatrix}. \quad (2)$$

To separate the signals coming from different transmitters, the received signal is processed using a series of M matched filters at each receiver. The i^{th} matched filter corresponds to a matching with the i^{th} transmit waveform. We derive the mathematical model for the proposed MIMO radar system by using an approach similar to that presented for the single antenna system in [13]. The signals at the output of the matched filters are normalized by dividing by a^{ij}. Note that normalization changes the variances of the normalized noise term, and hence these variances need not be the same for all transmitter–receiver pairs. The normalized vector output of the i^{th} matched filter at the j^{th} receiver is

$$\boldsymbol{y}^{ij} = \boldsymbol{S}^{ij}\boldsymbol{t}^i + \boldsymbol{e}^{ij}, \quad (3)$$

where the column vector $\boldsymbol{y}^{ij} = \left[y^{ij}_{\text{h}}, y^{ij}_{\text{v}}\right]^T$ consists of the horizontal and vertical components, respectively.

Stacking the elements of the scattering matrix $\boldsymbol{S}^{ij}$ into a vector, we define $\boldsymbol{s}^{ij} = \left[s^{ij}_{\text{hh}}, s^{ij}_{\text{hv}}, s^{ij}_{\text{vh}}, s^{ij}_{\text{vv}}\right]^T$. There are MN such vectors, and arranging them into a single vector gives us a $4MN$x1 dimensional column vector:

$$\boldsymbol{s} = \left[\left(\boldsymbol{s}^{11}\right)^T, \ldots, \left(\boldsymbol{s}^{1N}\right)^T, \ldots, \left(\boldsymbol{s}^{M1}\right)^T, \ldots, \left(\boldsymbol{s}^{MN}\right)^T\right]^T. \quad (4)$$

Similarly, stacking the normalized outputs of the matched filters and also the corresponding additive noise components into column vectors, we define

$$\boldsymbol{y} = \left[\left(\boldsymbol{y}^{11}\right)^T, \ldots, \left(\boldsymbol{y}^{1N}\right)^T, \ldots, \left(\boldsymbol{y}^{M1}\right)^T, \ldots, \left(\boldsymbol{y}^{MN}\right)^T\right]^T, \quad (5)$$

$$\boldsymbol{e} = \left[\left(\boldsymbol{e}^{11}\right)^T, \ldots, \left(\boldsymbol{e}^{1N}\right)^T, \ldots, \left(\boldsymbol{e}^{M1}\right)^T, \ldots, \left(\boldsymbol{e}^{MN}\right)^T\right]^T. \quad (6)$$

Define a set of matrices

$$\boldsymbol{P}^i = \begin{bmatrix} t^i_{\text{h}} & t^i_{\text{v}} & 0 & 0 \\ 0 & 0 & t^i_{\text{h}} & t^i_{\text{v}} \end{bmatrix}, \quad (7)$$

$\forall i = 1, \ldots, M$, each corresponding to a particular transmitter.

Using the above definitions, we express the measurement vector $\boldsymbol{y}$ using the following mathematical model:

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{e}, \quad (8)$$

where

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{P}^1 & \cdots & \mathbf{0} & \cdots & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \cdots & \boldsymbol{P}^1 & \cdots & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \cdots & \boldsymbol{P}^M & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \cdots & \mathbf{0} & \cdots & \boldsymbol{P}^M \end{bmatrix}. \quad (9)$$

$\mathbf{0}$ is a zero matrix of dimensions 2x4. Terms $\boldsymbol{y}$ and $\boldsymbol{e}$ are $2MN$x1 dimensional observation and noise vectors respectively. Thus, we have reduced our mathematical model to the well–known linear form. We now look at the statistical assumptions made on these terms.

We assume that the noise terms present in $\boldsymbol{e}$ are uncorrelated and that $\boldsymbol{e}$ follows proper complex Gaussian distribution. Define the covariance matrix of $\boldsymbol{e}$ as $\boldsymbol{\Sigma_e}$ and we assume it is known. $\boldsymbol{\Sigma_e}$ is diagonal. Its diagonal entries need not be the same because of the normalization performed at the output of each of the matched filters, as mentioned earlier. The matrix $\boldsymbol{H}$ is a $2MN$x$4MN$ dimensional design matrix whose constituent elements depend on the transmit waveform polarizations. We assume that the vector $\boldsymbol{s}$, which contains elements from all the scattering matrices, is a random vector following proper complex Gaussian distribution with a $4MN$x$4MN$ covariance matrix given by $\boldsymbol{\Sigma_s}$. We further assume that $\boldsymbol{\Sigma_s}$ is known. Furthermore, we assume that $\boldsymbol{s}$ and $\boldsymbol{e}$ are independent. In [11], we suggested a method to estimate the covariance matrices if they are unknown.

III. Problem Formulation

The problem of detecting the target reduces to the following binary hypothesis testing problem:

$$H_0 \;:\; \boldsymbol{y} = \boldsymbol{e}, \quad (10)$$

$$H_1 \;:\; \boldsymbol{y} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{e}. \quad (11)$$

Therefore, under the null hypothesis, $\boldsymbol{y}$ will have complex Gaussian distribution with zero mean and covariance matrix $\boldsymbol{\Sigma_e}$. Under the alternative hypothesis, the independence of $\boldsymbol{s}$ and $\boldsymbol{e}$ implies that $\boldsymbol{y}$ will follow complex Gaussian distribution with zero mean and covariance matrix given by $\boldsymbol{C}+\boldsymbol{\Sigma_e}$, where $\boldsymbol{C} = \boldsymbol{H}\boldsymbol{\Sigma_s}\boldsymbol{H}^H$ denotes the covariance matrix of $\boldsymbol{H}\boldsymbol{s}$.

IV. Detector

A. Test Statistic

Under the above–mentioned hypotheses, the probability density functions of the observation vector are given as

$$f(\boldsymbol{y}|H_0) \propto \frac{1}{|\boldsymbol{\Sigma_e}|} e^{-\boldsymbol{y}^H \boldsymbol{\Sigma_e}^{-1} \boldsymbol{y}}, \tag{12}$$

$$f(\boldsymbol{y}|H_1) \propto \frac{1}{|\boldsymbol{\Sigma_e} + \boldsymbol{C}|} e^{-\boldsymbol{y}^H (\boldsymbol{\Sigma_e} + \boldsymbol{C})^{-1} \boldsymbol{y}}. \tag{13}$$

The Neyman-Pearson lemma states that the likelihood ratio test is the most powerful test for any given size [16]. The likelihood ratio is given as

$$\frac{f(\boldsymbol{y}|H_0)}{f(\boldsymbol{y}|H_1)} = \frac{|\boldsymbol{\Sigma_e} + \boldsymbol{C}|}{|\boldsymbol{\Sigma_e}|} e^{-\boldsymbol{y}^H \left(\boldsymbol{\Sigma_e}^{-1} - (\boldsymbol{\Sigma_e} + \boldsymbol{C})^{-1}\right) \boldsymbol{y}}. \tag{14}$$

Computing the logarithm of the above expression and ignoring the known constants, we clearly see that $\boldsymbol{y}^H \left(\boldsymbol{\Sigma_e}^{-1} - (\boldsymbol{\Sigma_e} + \boldsymbol{C})^{-1}\right) \boldsymbol{y}$ is our test statistic and we compare it with a threshold before selecting a hypothesis:

$$\boldsymbol{y}^H \left(\boldsymbol{\Sigma_e}^{-1} - (\boldsymbol{\Sigma_e} + \boldsymbol{C})^{-1}\right) \boldsymbol{y} \quad \gtrless_{H_0}^{H_1} \quad k, \tag{15}$$

where the threshold k is chosen based on the size specified for the test.

B. Performance Analysis

In order to analyze the performance of the above–mentioned detector, we need to know the distribution of the test statistic under both hypotheses. The test statistic is a quadratic form of the complex Gaussian random vector $\boldsymbol{y}$. It is well known in statistics that a quadratic form $\boldsymbol{z}^T \boldsymbol{U} \boldsymbol{z}$ of a real Gaussian random vector $\boldsymbol{z}$ with covariance matrix $\boldsymbol{B}$ will follow Chi-square distribution if and only if the matrix $\boldsymbol{U}\boldsymbol{B}$ is idempotent [17]. Using this result, we infer that our test statistic does not necessarily follow Chi-square distribution for all feasible choices of $\boldsymbol{\Sigma_e}$ and $\boldsymbol{C}$ because we did not impose any constraint on $\boldsymbol{\Sigma_s}$. Hence, it is difficult to find the exact probability density function (pdf) for it. In order to study the pdf of our test statistic, we first begin with an assumption that $\boldsymbol{C}$ is diagonal. Later, we will extend this approach to the non-diagonal case by applying proper diagonalization.

Define the l^{th} diagonal element of $\boldsymbol{C}$ as c^l and that of $\boldsymbol{\Sigma_e}$ as v^l. Then, the test statistic reduces to

$$\sum_{i=1}^{M} \sum_{j=1}^{N} \left(\frac{\left|y_{\mathrm{h}}^{ij}\right|^2}{v^{2(i-1)N+2j-1}} - \frac{\left|y_{\mathrm{h}}^{ij}\right|^2}{v^{2(i-1)N+2j-1} + c^{2(i-1)N+2j-1}} \right) + \sum_{i=1}^{M} \sum_{j=1}^{N} \left(\frac{\left|y_{\mathrm{v}}^{ij}\right|^2}{v^{2(i-1)N+2j}} - \frac{\left|y_{\mathrm{v}}^{ij}\right|^2}{v^{2(i-1)N+2j} + c^{2(i-1)N+2j}} \right),$$

where $y_{\mathrm{h}}^{ij}, y_{\mathrm{v}}^{ij}$ are always independent Gaussian random variables under both hypotheses for all transmitter–receiver pairs because of the diagonal assumption of $\boldsymbol{\Sigma_e}$ and $\boldsymbol{C}$. Therefore, the test statistic is a weighted sum of independent Chi-square random variables and it does not necessarily follow the Chi-square distribution. Its actual distribution depends on the weights. The pdf of a sum of independent random variables is obtained by performing multiple convolutions among the constituent pdfs. However, in this case, it is difficult to find the exact solution. Hence, we shall look for approximations to the actual pdf.

In [18], the distribution of the weighted sum of Chi squares is studied. If π_q are real positive constants and N_q are independent standard normal random variables $\forall q = 1, \cdots, K$, then the pdf of the Gamma approximation of $R = \sum_{q=1}^{K} \pi_q {N_q}^2$ is given as

$$f_R(r, \alpha, \beta) = r^{\alpha - 1} \frac{e^{-\frac{r}{\beta}}}{\beta^\alpha \Gamma(\alpha)}, \tag{16}$$

where the parameters α and β are given as

$$\alpha = \frac{1}{2} \left(\frac{\left(\sum_{q=1}^{K} \pi_q\right)^2}{\sum_{q=1}^{K} \pi_q^2} \right), \tag{17}$$

$$\beta = \left(\frac{1}{2} \left(\frac{\sum_{q=1}^{K} \pi_q}{\sum_{q=1}^{K} \pi_q^2} \right) \right)^{-1}. \tag{18}$$

Γ is the gamma function defined as $\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t} dt$.

Under the null hypothesis, y_{h}^{ij} and y_{v}^{ij} have zero mean and variances $v^{(2(i-1)N+2j-1)}$ and $v^{(2(i-1)N+2j)}$, respectively. Hence, applying the above approximation with appropriate weights, the parameters of the Gamma distribution are

$$\alpha_{H_0} = \left(\frac{\left(\sum_{l=1}^{2MN} \frac{c^l}{v^l + c^l}\right)^2}{\sum_{l=1}^{2MN} \left(\frac{c^l}{v^l + c^l}\right)^2} \right), \tag{19}$$

$$\beta_{H_0} = \left(\frac{\sum_{l=1}^{2MN} \frac{c^l}{v^l + c^l}}{\sum_{l=1}^{2MN} \left(\frac{c^l}{v^l + c^l}\right)^2} \right)^{-1}. \tag{20}$$

Similarly the parameters of the Gamma approximation under the alternative hypothesis can be computed (see [11]). Note that so far we have assumed a diagonal structure for matrix $\boldsymbol{C}$ in the above discussion. However, we still need to find expressions for the pdf of the test statistic when $\boldsymbol{C}$ is not diagonal. Diagonalization will be used to extend the analysis even for the case of non–diagonal matrices [19]. Since $\boldsymbol{\Sigma_e}$ and $\boldsymbol{C}$ are covariance matrices, $\left(\boldsymbol{\Sigma_e}^{-1} - (\boldsymbol{\Sigma_e} + \boldsymbol{C})^{-1}\right)$ will be a Hermitian matrix, which therefore decomposes into $\boldsymbol{D}^H \boldsymbol{\Lambda} \boldsymbol{D}$, where $\boldsymbol{\Lambda}$ is a diagonal matrix consisting of eigenvalues as the diagonal elements and $\boldsymbol{D}$ contains the corresponding orthonormal eigenvectors. The test statistic now becomes $(\boldsymbol{D}\boldsymbol{y})^H \boldsymbol{\Lambda} (\boldsymbol{D}\boldsymbol{y})$. If we show that $\boldsymbol{D}\boldsymbol{y}$ has a diagonal covariance matrix under both hypotheses, then our analysis extends to the case in which $\boldsymbol{C}$ is not diagonal also, with appropriate adjustments made to the parameters of the Gamma approximation. Under H_0, $\boldsymbol{D}\boldsymbol{y}$ is a complex Gaussian random vector with a covariance matrix $\mathrm{Cov}_{H_0}(\boldsymbol{D}\boldsymbol{y}) = \boldsymbol{D}\boldsymbol{\Sigma_e}\boldsymbol{D}^H$, which is diagonal because $\boldsymbol{\Sigma_e}$ is diagonal and $\boldsymbol{D}$ has orthonormal vectors. Similarly, under H_1, $\boldsymbol{Dy}$ is a complex normal random vector with covariance matrix

$$\begin{aligned}\mathrm{Cov}_{H_1}(\boldsymbol{Dy}) &= \boldsymbol{D}(\boldsymbol{\Sigma_e}+\boldsymbol{C})\boldsymbol{D}^H,\\ &= \left(\boldsymbol{D}(\boldsymbol{\Sigma_e}+\boldsymbol{C})^{-1}\boldsymbol{D}^H\right)^{-1},\\ &= \left(\boldsymbol{D}\left((\boldsymbol{\Sigma_e}+\boldsymbol{C})^{-1}-\boldsymbol{\Sigma_e}^{-1}+\boldsymbol{\Sigma_e}^{-1}\right)\boldsymbol{D}^H\right)^{-1},\\ &= \left(\boldsymbol{D}\boldsymbol{\Sigma_e}^{-1}\boldsymbol{D}^H-\boldsymbol{\Lambda}\right)^{-1},\end{aligned}$$

which is diagonal. Hence, under both hypotheses, the test statistic is a weighted sum of Chi square random variables even when matrix $\boldsymbol{C}$ is not diagonal. The only difference is that the weights will now be different, and they are defined by the diagonalization process.

After approximating the pdf using the Gamma density, the probability of detection (P_{D}) and the probability of false alarm (P_{FA}) are defined as follows:

$$P_{\mathrm{D}} = \int_k^{\infty} t^{\alpha_{H_1}-1}\frac{e^{-\frac{t}{\beta_{H_1}}}}{\beta_{H_1}^{\alpha_{H_1}}\Gamma(\alpha_{H_1})}dt, \tag{21}$$

$$P_{\mathrm{FA}} = \int_k^{\infty} t^{\alpha_{H_0}-1}\frac{e^{-\frac{t}{\beta_{H_0}}}}{\beta_{H_0}^{\alpha_{H_0}}\Gamma(\alpha_{H_0})}dt, \tag{22}$$

where the parameters $\alpha_{H_0}, \beta_{H_0}, \alpha_{H_1}$, and β_{H_1} are as mentioned earlier. For a given value of P_{FA}, the value of the threshold k is calculated easily using the above expression because functions for evaluating the above expressions exist in MATLAB. After finding the threshold, P_{D} is calculated accordingly. Note that the value of the threshold and P_{D} depends on matrix $\boldsymbol{C}$, which in turn depends on the polarizations of the transmitted waveforms. Hence, the performance of the detector is related to the transmit waveform polarizations.

C. Optimal Design

In order to find the optimal design, we perform a grid search over the possible waveform polarizations across all the transmit antennas with the help of the above expressions for P_{D} and P_{FA}. The optimal design corresponds to the transmit polarizations that give the maximum P_{D} for a given P_{FA}.

V. Numerical Results

We consider a system with two transmitters and two receivers under the same target detection scenario as described so far. We choose the covariance matrix of $\boldsymbol{s}$ to be of the following form:

$$\boldsymbol{\Sigma_s} = \begin{bmatrix} \boldsymbol{\Sigma_s^{11}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma_s^{12}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{\Sigma_s^{21}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{\Sigma_s^{22}} \end{bmatrix}, \tag{23}$$

where $\boldsymbol{\Sigma_s^{ij}}$ represents the covariance matrix of the random vector $\boldsymbol{s}^{ij}$ and $\mathbf{0}$ is a 4x4 dimensional zero matrix. Each of these matrices were chosen as follows:

$$\boldsymbol{\Sigma_s^{11}} = \begin{bmatrix} 0.3 & 0.1\epsilon & 0.1\epsilon & 0.1\epsilon \\ 0.1\epsilon^* & 0.2 & 0.1\epsilon & 0.1\epsilon \\ 0.1\epsilon^* & 0.1\epsilon^* & 0.4 & 0.1\epsilon \\ 0.1\epsilon^* & 0.1\epsilon^* & 0.1\epsilon^* & 0.5 \end{bmatrix}, \tag{24}$$

$$\boldsymbol{\Sigma_s^{12}} = \begin{bmatrix} 0.5 & 0.05\epsilon & 0.05\epsilon & 0.05\epsilon \\ 0.05\epsilon^* & 0.3 & 0.05\epsilon & 0.05\epsilon \\ 0.05\epsilon^* & 0.05\epsilon^* & 0.4 & 0.05\epsilon \\ 0.05\epsilon^* & 0.05\epsilon^* & 0.05\epsilon^* & 0.3 \end{bmatrix}, \tag{25}$$

$$\boldsymbol{\Sigma_s^{21}} = \begin{bmatrix} 0.4 & 0.1\epsilon & 0.1\epsilon & 0.1\epsilon \\ 0.1\epsilon^* & 0.3 & 0.1\epsilon & 0.1\epsilon \\ 0.1\epsilon^* & 0.1\epsilon^* & 0.2 & 0.1\epsilon \\ 0.1\epsilon^* & 0.1\epsilon^* & 0.1\epsilon^* & 0.4 \end{bmatrix}, \tag{26}$$

$$\boldsymbol{\Sigma_s^{22}} = \begin{bmatrix} 0.4 & 0.05\epsilon & 0.05\epsilon & 0.05\epsilon \\ 0.05\epsilon^* & 0.4 & 0.05\epsilon & 0.05\epsilon \\ 0.05\epsilon^* & 0.05\epsilon^* & 0.2 & 0.05\epsilon \\ 0.05\epsilon^* & 0.05\epsilon^* & 0.05\epsilon^* & 0.5 \end{bmatrix}, \tag{27}$$

where $\epsilon = 1+\sqrt{-1}$. The complex elements of the noise vector $\boldsymbol{e}$ are assumed to be uncorrelated with variance $\sigma^2 = 0.2$. We vary the value of P_{FA} to plot the optimal ROC curve by performing a grid search using the analytical results derived earlier. Next, we obtain the reference curves for our results by computing the ROC curves assuming that all the transmit antennas are horizontally or vertically polarized. These plots are presented in Figure 1, and a significant improvement in performance is clearly visible.

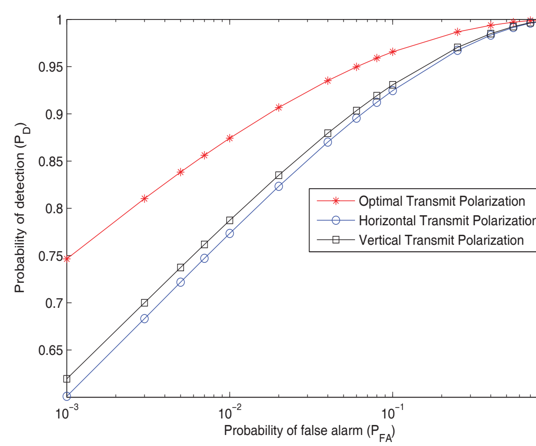

Fig. 1. ROC curves demonstrating the improvement offered by the optimal choice of polarizations when $\sigma^2 = 0.2$.

We proceed with our analysis for this numerical example. First, we fix P_{FA} to be equal to 0.02. For this value of P_{FA}, we wish to check the improvement offered by the optimal design for different values of the noise variance. We plot the optimal P_{D} as a function of σ^2. We also plot P_{D} as a function of σ^2 for the case in which only horizontal or vertical polarizations are used. The improvement in performance offered by the optimal design is clear from Figure 2.

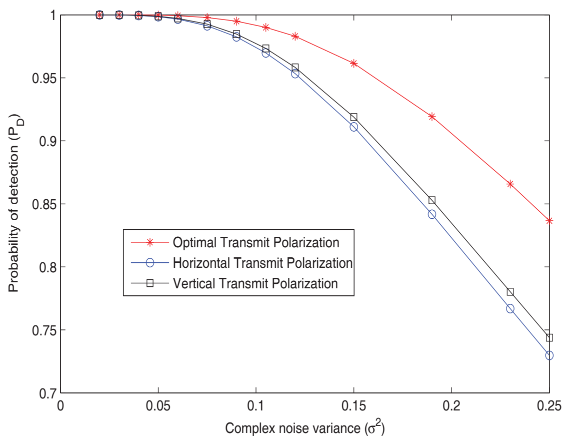

Fig. 2. Probability of detection (P_{D}) as a function of the complex noise variance when $P_{\mathrm{FA}} = 0.02$.

Now, we plot the ROC curves for SISO radar with optimal transmit polarizations to show the gain in performance because of the multiple widely separated antennas. For the SISO system, we consider only the first transmit and receive antennas in our above mentioned example. Therefore, the covariance matrix of the scattering vector $\boldsymbol{s}$ becomes $\boldsymbol{\Sigma_s} = \boldsymbol{\Sigma_s^{11}}$. In order to make a fair comparison, we transmit more power than the power transmitted per antenna while using MIMO radar. It is clear from Figure 3 that 2X2 polarimetric MIMO radar system significantly outperforms its SISO counterpart even when the SISO system uses four times the transmit power used by each antenna in the 2X2 system.

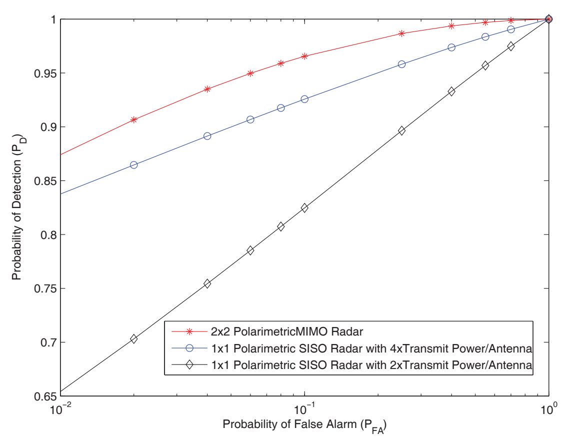

Fig. 3. ROC curves demonstrating the improvement offered by employing multiple widely separated antennas compared with single input single output systems when $\sigma^2 = 0.2$.

VI. Conclusion

We have proposed a radar system that combines the advantages of MIMO radar with distributed antennas and polarimetric radar at the same time. The proposed system uses two–dimensional vector sensors at each of the receivers, measuring both the horizontal and vertical components of the received signal. We dealt with the problem of target detection for such a system. We designed the well–known Neyman-Pearson detector for this problem and also analyzed the performance of the detector by obtaining approximate expressions for the probabilities of false alarm and detection. Using numerical examples, we demonstrated the performance improvement offered by the proposed system.

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