SPATIO-TEMPORAL SCHEDULING OF COMPLEMENTARY SEQUENCES WITH APPLICATION TO MIMO-OFDM

Chad Lau, Michael Zoltowski

School of Electrical and Computer Engineering
Purdue University, West Lafayette, IN

Robert Calderbank

Department of Electrical Engineering
Princeton University, Princeton, NJ

ABSTRACT

In this paper, a new method of space-time processing is proposed for Orthogonal Frequency Division Multiplexing (OFDM) using complementary sequences derived from the rows of the DFT matrix. The autocorrelative properties of the complementary sequences allows multiple complex data signals at the transmitter with an arbitrary number of antennas to be perfectly separated at the receiver without prior channel knowledge while achieving full-rate. This new method is proposed and derived for multiple MIMO-OFDM systems with multipath fading; at the receiver, symbol estimation is effected via maximum likelhihood estimation (ML).

***Index Terms*—** MIMO systems, matched filters, Discrete Fourier Transforms

1. INTRODUCTION

In multipath wireless scenarios subject to Rayleigh fading, spatial diversity offers an often needed performance gain. As suggested by [5], adding antennas or otherwise augmenting the physical complexity of the receiver may be impractical or infeasible for many applications, in which case it becomes necessary to consider transmit diversity, which can be achieved via space-time processing without knowledge of the channel at the transmitter.

One popular method of achieving full diversity gain is Orthogonal Space-time Block Coding (OSTBC). In particular, Alamouti's famous paper [4] presents an OSTBC for complex designs with $M = 2$ transmit antennas. This design has been proved to be optimal in the sense that no other OSTBCs for *complex* designs exist for $M > 2$ [5]. Anonther drawback of OSTBCs is that channel knowledge is required at the receiver for both separation of the incoming data and the detection scheme.

In this paper, OFDM data signals are passed through filters prior to transmission and appropriately matched-filtered at the receiver using complementary sequences, which possess the desirable property of having autocorrelations which sum to a delta function. As will be shown, these sequences can be easily derived from the rows of the DFT matrix. Because intersymbol interference (ISI) can render the autocorrelative property of the applied complementary sequences ineffective, Orthogonal Frequency Division Multiplexing (OFDM) systems become prime candidates for their use; if chosen longer than the maximum channel delay spread, the cyclic extension of the transmitted signal completely removes all effects of ISI, thus merely necessitating trivial single-tap equalization at the reciever. The autocorrelative properties of the complementary DFT-based sequences then allow data separation at the receiver *without* the need of channel knowledge.

This paper thus presents a novel method designing MIMO-OFDM systems such that complex OFDM data may be transmitted full-rate and perfectly separated at the reciever without channel knowledge.

This work was supported in part by the Air Force Office of Scientific Research under MURI Grant # FA9550-05-1-0443 and by the National Science Foundation under grant # 0914915-CCF.

2. DATA SEPARATION

This section begins by introducing a simple, ideal example of a MIMO-OFDM system capable of perfectly separating M data streams, then extends this model is to a practical and realizable system as well as derives the DFT-based complementary sequences which make this model possible.

2.1. Ideal System

First consider a MIMO-OFDM system employing N orthogonal subcarriers which transmits M different data sequences on its M antennas; antenna i solely transmits components of OFDM signal $x_i[n]$, $1 \leq i \leq M$, over the course of M time frames as illustrated in 1 ($x_i[n]$ is the cyclically extended IDFT of complex data sequences $s_i[n]$). During each time slot, antenna i transmits $1/M$ of the total bandwidth of $x_i[n]$ via an ideal subband filter. The figure shows that each antenna transmits a separate portion of $X(\omega)$ during each time slot such that after M transmissions, the entire spectrum is transmitted.

The figure also shows that for any particular time slot, each antenna transmits over a separate frequency range. Thus, by employing the appropriate matched filters, the reciever can easily resolve the individual components of each signal for each receive antenna. However, realization of the ideal subband filters requires the use of infinite-length sinc functions in the time domain. As will now be shown, a simple conversion to sequnces derived from the rows of the DFT matrix yields a simple and practical system for the perfect separation of data.

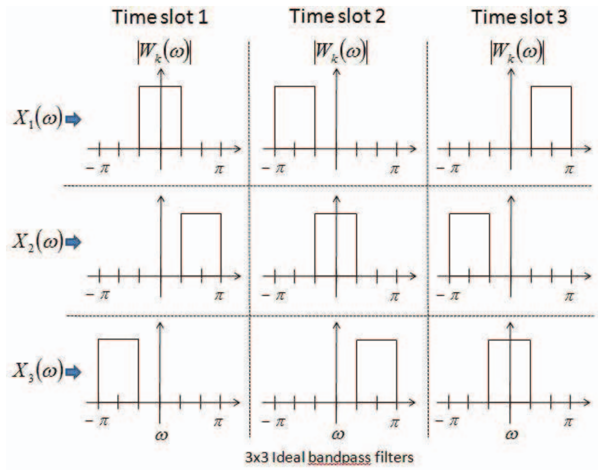

Fig. 1. Ideal thirdband filters as applied in 3×3 MIMO-OFDM system. All filtered signals for each time slot are completely separable in frequency.

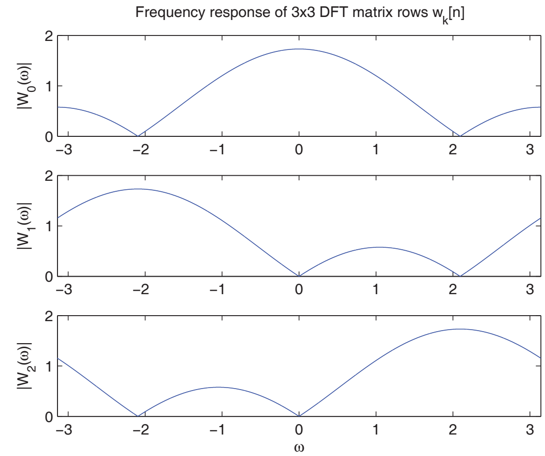

Fig. 2. Magnitude of the frequency response of DFT-based waveforms $w_k[n] = (1/\sqrt{3})e^{-j\frac{2\pi kn}{3}}$, $k, n = 0, 1, 2$.

2.2. DFT-based complementary sequences

Instead of infinite-length filters as in the previous model, the proposed system utilizes the rows of an $M \times M$ DFT matrix. This section will define the DFT matrix and show its application in the proposed system.

The M-point DFT of an OFDM-modulated signal (post N-pt. IFFT and cyclic extension) $x[n]$, $n = 0, \ldots, M-1$, is defined as

$$X[k] = \frac{1}{\sqrt{M}} \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi kn}{M}} = \mathbf{W}_M \mathbf{x}. \quad (1)$$

which, as shown, can also be written as a matrix-vector product [1], where $\mathbf{W}_M$ is the $M \times M$ DFT matrix, an orthogonal matrix whose rows $w_k[n]$, $k = 0, \ldots, M-1$ are orthogonal complex sinewaves given by

$$w_k[n] = \frac{1}{\sqrt{M}} e^{-j\frac{2\pi kn}{M}}, k = 0, \ldots, M-1. \quad (2)$$

Fig. 2 displays the frequency responses for the orthogonal sinewaves $w_k[n]$, $n = 0, 1, 2$ from the above $M = 3$ example. Whereas the ideal bandpass filters from the above example each isolated exactly one third of the total spectrum (see Fig. 1), the rows of the 3×3 DFT matrix act as 'crude' third-band filters, centered at $\omega = \pi, \pm\frac{\pi}{3}$, with large sidelobes spilling over into the unwanted frequency range. In the frequency domain, the rows of 3×3 DFT matrix yield sidelobes which peak at one third the maximum magnitude of the mainlobe, offering nearly 9.5 dB of stopband attenuation; for larger $M \geq 10$, minimum stopband attenuation is about 13 dB [3].

An interesting result occurs if the ideal subband filters in the above example are replaced by these DFT-based filters. Despite the excessive rolloff that the DFT-based filters exhibit, these filters are nonetheless capable of perfectly separating each of the data sequences $x_i[n]$ as are the ideal filters. The reason for this amazing result lies in the properties of DFT matrix. In [2], Golay defines complementary sequences as a pair of sequences whose autocorrelations sum to scaled delta function; the sidelobes of the autocorrelation functions perfectly cancel. It will now be shown that this defintion can be generalized to a set of $M \geq 2$ sequences by using the M rows of $\mathbf{W}_M$.

Using (2), the autocorrelation functions of the M sequences $w_k[n]$ are

$$\begin{aligned} r_{ww}^{(k)}[m] &= \frac{1}{M} \sum_{n=-\infty}^{\infty} e^{-j\frac{2\pi kn}{M}} w_R[n] e^{j\frac{2\pi k(n-m)}{M}} w_R^*[n-m] \\ &= \frac{1}{M} e^{-j\frac{2\pi km}{M}} \underbrace{\sum_{n=-\infty}^{\infty} w_R[n] w_R[n-m]}_{\Lambda[m]} \\ &= \frac{1}{M} \Lambda[m] e^{-j\frac{2\pi km}{M}} \end{aligned}$$

where $w_R[n] = u[n] - u[n-M]$ is the rectangular window of length M and $\Lambda[m] = M - |m|$ is the corresponding triangular window function. Summing across all rows k,

$$\sum_{k=0}^{M-1} r_{ww}^{(k)}[m] = \frac{1}{M} \Lambda[m] \sum_{k=0}^{M-1} (e^{-j\frac{2\pi m}{M}})^k = M\delta[m]. \quad (3)$$

Equation (3) thus thus allows us to generalize the definition of complementary sequences given in [2] to a pair of sequences to a set of $M \geq 2$ sequences by using the rows of the $M \times M$ DFT matrix. The following section derives the signal processing required so that these DFT-based complementary sequences may be used to perfectly separate independent MIMO-OFDM data streams at the receiver.

2.3. Perfect data separation

This section will detail the signal processing at both the transmitter and receiver required to perfectly separate independent MIMO-OFDM data streams.

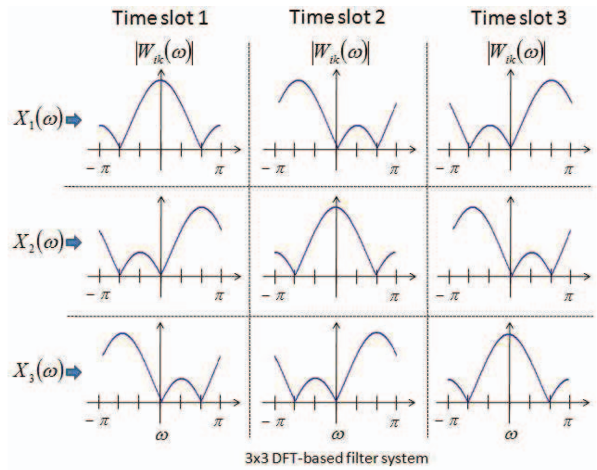

Fig. 3. Nonideal DFT-based filters as applied in 3×3 MIMO-OFDM system. Despite nonideal filter shape, all filtered signals for each time slot remain completely separable in frequency.

First consider the order in which the filters $w_k[n]$ will be applied across all M antennas and time slots. The following displays the waveform scheduling for the transmitter of a general case:

$$\begin{matrix} x_1[n] \rightarrow \\ x_2[n] \rightarrow \\ \vdots \\ x_M[n] \rightarrow \end{matrix} \underbrace{\begin{bmatrix} w_0[n] & w_1[n] & \dots & w_{M-1}[n] \\ w_{M-1}[n] & w_0[n] & \dots & w_{M-2}[n] \\ & & \ddots & \\ w_1[n] & w_2[n] & \dots & w_0[n] \end{bmatrix}}_{\mathbf{S}_{T_x}} \tag{4}$$

According to (4), antenna $i = 1$ applies filter $w_k[n]$ during time slot k, $k = 0, \dots, M-1$, and for the remaining antennas $i = 2, \dots, M$, the scheduling takes on a circulant form; the remaining rows of (4) are circularly shifted to the right. This circulant filter scheduling is also evident in Figs. 1 and 3. It is important to note that the elements of (4) are *sequences* to be convolved with the data sequences $x_i[n]$. Specifically, each row corresponds to the order of the filters applied on each antenna (per time slot), and (4) indicates the data sequence transmitted by each row. Also of importance are the spectral efficiency implications: as opposed to infinite-length ideal filters, the OFDM data sequences are now convolved with length-M sequences. The cases considered in this paper include $M = 2, 3, 4$; convolution of a length $N >> M$ OFDM signal with such a short sequence has negligible impact on the spectral efficiency of the system.

Each receive antenna sees a mixture of the filtered data signals on each time slot k, which is easily determined for time slot k by summing along column k of $\mathbf{S}_{T_x}$, convolved with their appropriate data sequences. For example, in the $M = 3$ case, the recieved signal in the first time slot is given by

$$\begin{aligned} r[n] = (&x_1[n] * w_0[n] + x_2[n] * w_2[n] \\ &+ x_3[n] * w_1[n]) * h[n] + v[n] \end{aligned} \tag{5}$$

where $*$ denotes convolution, $h[n]$ is the channel impulse response, and v[n] is an additive i.i.d noise sequence with variance σ^2. Note that this is only true when the channel is quasistatic, i.e. the channel does not vary over the M time frames. In the ideal case, each of these components are perfectly separable in frequency; in the DFT-based system, this is not the case, and separating and reconstructing the data sequences requires additional signal processing. Equation (6) shows the waveform matched-filtering implemented at the receiver.

$$\begin{matrix} \text{Time} \downarrow \\ \text{Slot} \end{matrix} \underbrace{\begin{bmatrix} w_0^*[-n] & w_{M-1}^*[-n] & \dots & w_1^*[-n] \\ w_1^*[-n] & w_0^*[-n] & \dots & w_2^*[-n] \\ & & \ddots & \\ w_{M-1}^*[-n] & w_{M-2}^*[-n] & \dots & w_0^*[-n] \end{bmatrix}}_{\mathbf{S}_{R_x}} \tag{6}$$

It is evident from (6) that the elements (sequences) of $\mathbf{S}_{R_x}$ are transposed, complex conjugtes, and time reversals of the elements of the transmitter matrix $\mathbf{S}_{T_x}$, i.e. $\mathbf{S}_{R_x}$ contains filters matched to those of the $\mathbf{S}_{T_x}$. At the receiver, waveforms indicated in (6) are applied on each antenna; the i-th column of $\mathbf{S}_{R_x}$ is applied to the received signals on any antenna across M time slots in order to extract the contribution of the i-th transmit antenna to the superimposed received signal.

Without loss of generality, suppose the OFDM data sequence $x_1[n]$ for an $M \times 1$ MIMO-OFDM system is to be reconstructed. For the above claim to be valid, the filters contained in the first column of $\mathbf{S}_{R_x}$ should solely extract components of $x_1[n]$ and cancel all other contributions from the remaining data sequences. In each time slot, filtered versions of each data sequence are transmitted according to (4), so by linearity, we can examine the contribution of each data sequence $x_i[n]$ independently. Applying the first column of $\mathbf{S}_{R_x}$ and only considering the contributions of $x_1[n]$, the output of the receiver is (neglecting noise)

$$\begin{aligned} y_1^{(1)}[n] &= x_1[n] * h_{11}[n] * (w_0[n] * w_0^*[-n] + \\ &+ \dots + w_{M-1}[n] * w_{M-1}^*[-n]) \\ &= M x_1[n] * h_{11}[n] \end{aligned} \tag{7}$$

due to the complementary nature of the DFT sequences as shown in (3). It must now be shown that applying the first column of $\mathbf{S}_{R_x}$ with any and all of the remaining signal contributions must yield the zero function. By examining (4) and (6), it can be shown that the received signal contribution from $x_i[n]$, $i \neq 1$, filtered by the first column of $\mathbf{S}_{R_x}$ is the following sum of cross-correlation functions:

$$y_1^{(i)}[n] = M x_i[n] * h_{i1}[n] * \sum_{k=0}^{M-1} \underbrace{(w_{k+M-(i-1)}[n] * w_k^*[-n])}_{c_k^{(i)}[n]} \tag{8}$$

$$\begin{aligned} c_k^{(i)}[n] &= e^{-j\frac{2\pi}{M}(k+M-(i-1))n} w_R[n] * e^{j\frac{2\pi}{M}kn} w_R[-n] \\ &= \sum_{l=-\infty}^{\infty} e^{-j\frac{2\pi}{M}(k+M-(i-1))l} e^{j\frac{2\pi}{M}k(l-n)} w_R[l] w_R[l-n] \end{aligned} \tag{9}$$

Summing $c_k^{(i)}$ over all $k = 0, \ldots, M-1$,

$$\sum_{k=0}^{M-1} c_k^{(i)} = \underbrace{\sum_{k=0}^{M-1} e^{-j\frac{2\pi}{M}kn}}_{\delta[n]} \sum_{l=-\infty}^{\infty} w_R[l] w_R[l-n] e^{-j\frac{2\pi}{M}(M-(i-1))}$$
$$= \sum_{l=-\infty}^{\infty} w_R[l] e^{-j\frac{2\pi}{M}((i-1)-M)l}$$
$$= \text{DTFT}\{w_R[l]\}\,|_{\omega=\frac{2\pi}{M}((i-1)-M)}, \tag{10}$$

yielding the Discrete Time Fourier Transform (DTFT) of the rectangular window $w_R[l]$ evaluated at frequency $\omega = \frac{2\pi}{M}((i-1)-M)$, for all cases $i = 2, \ldots, M$. However,

$$\text{DTFT}\{w_R[l]\} = \sum_{l=0}^{M-1} e^{-j\omega l} = e^{-j\frac{(M-1)}{2}\omega} \frac{\sin\left(\frac{M}{2}\omega\right)}{\sin\left(\frac{\omega}{2}\right)} \tag{11}$$

has nulls at each integer multiple of $\omega = \frac{2\pi}{M}$, and substituing into (8)

$$y_1^{(i)}[n] = M x_i[n] * h_{i1}[n] * 0 = 0 \;\forall\, n \tag{12}$$

for all $i \neq 1$, thus verifying the claim.

Identical waveform matched-filtering (applying column i of $\mathbf{S}_{R_x}$ to extract received contribution from transmit antenna i) is applied on all antennas $1 \leq j \leq M_r$ (M_r receive antennas) to obtain the following measurements $y_j^{(i)}$

$$y_j^{(i)} = M x_i[n] * h_{ji}[n] + \tilde{v}_j[n] \tag{13}$$

where $\tilde{v}_j[n]$ is an additive i.i.d. noise sequence with variance $M\sigma^2$. Note that because each of the $y_j^{(i)}[n]$ are the result of a sum of convolved sequences across M time frames, M noise sequences are also included in the computation and the power of the overall noise term is thus increased.

A key feature of the proposed system is its ability to perfectly separate the data at the receiver. It is important to observe that the while the system separates out each data sequence in the form of (13) without the need for any *a priori* channel knowledge. This feature separates the proposed method from popular ST-coding techniques such as Orthogonal Space-Time Block Coding (OSTBC). OSTBC techniques, such as Alamouti's famous result, require channel knowledge in order to separate the data streams, then again require channel knowledge to estimate the transmitted data [4, 5]. As a result, error in the channel estimates will lead to errors in both the separation and detection of the received data. However, in the proposed method, data separation occurs *prior* to channel estimation, and the quality of the channel estimates only affects the ML detection scheme.

2.4. ML Estimation

At the receiver, after stripping off the effects of the cyclic prefix and taking the N-point DFT of each resulting sequence, the following measurements in the frequency domain are found:

$$Y_j^{(i)}[k] = M H_{ji}[k] s_i[k] + \tilde{V}_j[k] \tag{14}$$

where $k = 0, \ldots, N-1$ denotes the k-th OFDM subcarrier, and $s_i[k]$ the data symbol from stream i which modulates the k-th subcarrier; the maximum likelihood technique now jointly estimates the data symbols on a per-tone basis:

$$\hat{\mathbf{s}}[k] = \min_{\mathbf{s}[k]} \left| \tilde{\mathbf{Y}}[k] - M\mathbf{H}[k]\tilde{\mathbf{s}}[k] \right|^2 \tag{15}$$

where the $2MM_r \times 1$ vector $\tilde{\mathbf{Y}}[k]$ contains the real and imaginary parts of each of the measurements from (14), $\tilde{\mathbf{s}}[k]$ contains the real and imaginary parts of the symbols modulating the k-th subcarrier, and the $2MM_r \times 2M$ matrix $\mathbf{H}[k]$ contains the channel corresponding channel information from (14), i.e. (15) is a purely real-valued linear system of equations consisting of the real and imaginary parts of (14).

2.5. Additional diversity techniques

Utilizing the proposed DFT-based technique will help mitigate co-channel interference from multiple transmit antennas, but the simplicity of the technique makes it a an excellent vehicle for additional diversity techniques. In this paper, the proposed DFT-based system also employs Linear Constellation Precoding (LCP) as described in [7, 6]. Briefly, LCP involves applying a unitary precoding matrix $\boldsymbol{\Theta}$ at the transmitter, which can be generalized for any number of transmit antennas M:

$$\boldsymbol{\Theta} = \mathbf{W}_M^T \text{diag}\left(1, \alpha, \alpha^2, \ldots, \alpha^{M-1}\right) \tag{16}$$

where $\alpha = e^{j2\pi/P}$, and P is a nonunique integer which may be chosen according to a heuristic provided in [6]. When applied to the data, the transmitter outputs

$$\mathbf{s}_{sp}[k] = \boldsymbol{\Theta}\mathbf{s}[k], \tag{17}$$

and (16) indicates that the precoder $\boldsymbol{\Theta}$ first phase-rotates the constellation of each data sequence, then modulates the rotated data via the M-point DFT matrix $\mathbf{W}_M^T$. These actions thus effectively spread the data in the spatial domain and are proven to achieve the full diversity gain MM_r [6].

3. RESULTS

Simulations of the proposed DFT-based technique and fractional rate OSTBC were performed and compared on both MISO and MIMO systems employing $M = 2$, 3, and 4 transmit and receive antennas. QPSK data was transmitted on $N = 256$ orthogonal subcarriers across normalized circularly-symmetric Gaussian channels and normalized such that the total power transmitted over M time slots was unity. All of the DFT-based processing in (4) and (6) is performed between the actions of appending and removing the cyclic prefix. Additionally, each of the DFT-based systems also employ unitary linear constellation precoding (LCP) in order

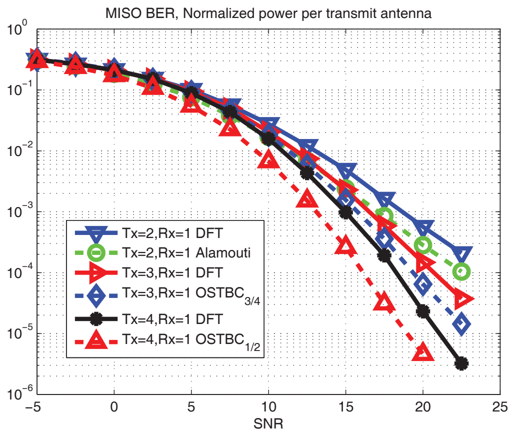

Fig. 4. Comparison of BER results for various MISO systems employing the proposed DFT-based technique and non full-rate complex Orthogonal Space-Time Block Coding (OSTBC).

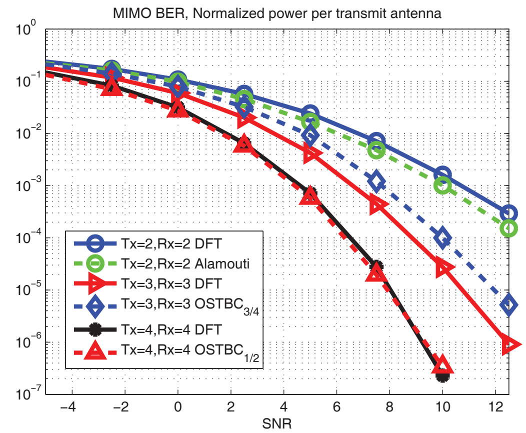

Fig. 5. Comparison of BER results for various MIMO systems employing the proposed DFT-based technique and non full-rate complex Orthogonal Space-Time Block Coding (OSTBC).

to achieve full diversity. Figs. 4 and 5 display the MISO and MIMO BER results, respectively.

The curves in Fig. 4 indicate that DFT-based MISO systems, i.e. $M = 2, 3, 4$, $M_r = 1$, approach those of MISO systems employing full-rate Alamouti coding (2×2 OSTBC) half-rate OSTBC for $M = 3, 4$. Although OSTBC shows a slightly better BER performance, it is unable to transmit complex data at full-rate for $M > 2$ transmit antennas. The proposed DFT-based technique, however, transmits complex data *full-rate* for systems of any arbitrary size M. The graph shows that for $M = 2$, $R_x = 1$, the two full-rate systems achieve similar BER results, and Alamouti coding yields about a 1.7 dB gain on the DFT-based system. However, since both systems achieve a full diversity gain $MM_r = M$, the curves exhibit identical slope for high SNR. As M is increased, the gain of OSTBC becomes slightly larger, approaching 2 dB for $M = 4$. Again, it is important to note that the proposed full-rate DFT-based system is capable of a performance which approaches OSTBC, which is only capable of transmitting complex data at fractional rates.

Regarding MIMO systems (see Fig. 5), the difference in performance between DFT processing and full-rate OSTBC (Alamouti, $M = 2$) becomes less pronounced; the remaining results differ markedly from the corresponding MISO cases (as the diversity order MM_r is increased). For $M = 3$, it is seen that the DFT-based filtering technique outperforms three-fourths-rate OSTBC (identical diversity gain), and increasing M to 4 yields identical performances between the DFT filtering technique and half-rate OSTBC. Thus the proposed technique performs either as well as or better than MIMO OSTBC techniques for $M > 2$.

4. CONCLUSION

This paper presents a novel scheme in which complementary sequences generated from the DFT matrix are used to efficiently achieve perfect separation of data without channel knowledge, thus allowing full-rate complex data transmission for MIMO-OFDM with an arbitrary number of transmit antennas. Simulations reveal that it is possible to approach BER performance of $M = 2$ MISO and MIMO-OFDM systems employing Alamouti coding orthogonal space-time block coding. Additionally, simulations show that the proposed system achieves a full-rate performance close to or better than to that of fractional-rate OSTBC schemes for MISO and MIMO-OFDM systems with $M = 2, 3, 4$.

5. REFERENCES

[1] K.R. Rao and P.C. Yip. *The Transform and Data Compression Handbook*. CRC Press, Boca Raton, FL, 2001.

[2] M.J.E. Golay. Complementary Series. *IRE Trans. Inform. Theory*, IT-7:82–87, 1961.

[3] P.P. Vaidyanathan. *Multirate Systems and Filter Banks*. P T R Prentice Hall, Inc., Englewood Cliffs, NJ, 1993.

[4] S.M. Alamouti. A simple transmit diversity technique for wireless communications. *IEEE J. on Select Areas in Communications*, 16(8), 1998.

[5] V. Tarokh, H. Jafarkhani, and A.R. Calderbank. Space-Time Block Codes from Orthogonal Designs. *IEEE. Trans. on Commun. Theory*, 45(5):1456–1467, July 1999.

[6] Y. Xin, Z. Wang, and G.B. Giannakis. Linear constellation precoding for OFDM with maximum multipath diversity and coding gains. *IEEE Trans. on Wireless Communications*, 2(2), 2003.

[7] Z. Liu, Y. Xin, and G.B. Giannakis. Linear constellation precoding for OFDM with maximum multipath diversity and coding gains. *IEEE Trans. on Communications*, 51(3), 2003.