ADAPTIVE POLARIZED WAVEFORM DESIGN FOR TARGET TRACKING USING ELECTROMAGNETIC VECTOR SENSORS

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ABSTRACT

We develop an adaptive waveform design method for target tracking. In our method, at each time step, we optimally select the parameters, including the polarization, of the transmitted signal waveform to improve the tracking accuracy. An array of electromagnetic vector sensors is employed to fully recover the polarimetric information from the reflected signals. We derive our approach under a framework of sequential Bayesian filtering. We apply a sequential Monte Carlo method to manipulate the nonlinear and non-Gaussian state and measurement models. We design a criterion for the waveform optimization based on a posterior Cramér-Rao bound.

Index Terms— Adaptive waveform design, target tracking, sequential Bayesian filtering, electromagnetic vector sensors, sequential Monte Carlo methods

1. INTRODUCTION

In this paper we address the problem of developing adaptive waveform design methods for target tracking using electromagnetic vector sensors. The proposed methods are derived under a framework of sequential (or recursive) Bayesian filtering.

In an ordinary active sensing system, parameters of the transmitted waveform are fixed during the sensing phase. However, target states, including target position, velocity, and scattering coefficients, will change dynamically during the sensing process. Furthermore, in some scenarios environments may also change dynamically, for example, there exist time-varying clutter or other interferences. Hence, the fixed waveform might not match the operational scenarios, which degrades the sensing performance. Therefore, the purpose of our work is to adaptively design the transmitted waveform in response to the target's dynamic states and the time-varying environmental conditions. Hence, we can achieve a better tracking performance compared with the conventional target tracking systems where the transmitted waveforms are fixed.

Another advantage of our waveform design method is that we employ the freedom provided by the polarization of the transmitted signal to design our tracking system, whereas in usual waveform design methods only the shape of the transmitted waveform is controlled. As we know, optimally selecting the polarization state of the transmitted waveform can mitigate the multipath interference and improve the performance of the sensing system in detection, tracking and target identification. Therefore, by exploiting the polarimetric aspects of the reflected signals we can further improve the parameter estimation accuracy and the resolution of the targets.

Some other work on adaptive waveform design for target tracking is presented in [1]-[4]. In [1] and [2], the state and measurement models are assumed to be linear and Gaussian; hence, a Kalman filter is used for target tracking and the criterions of minimum mean square tracking error and minimum validation gate volume are used for optimal waveform design. In [3]-[4], the methods are applied to a nonlinear, Gaussian measurement model and a linear, Gaussian state model, and the authors select the optimal waveform parameters by minimizing a mean square tracking error. In our proposed approach, we track a target using a sequential Monte Carlo method that is suitable for nonlinear and non-Gaussian situations. We also design a new criterion based on the posterior Cramér-Rao bound to optimally select the waveform parameters.

2. DYNAMIC STATE AND MEASUREMENT MODELS

In this section we first create a dynamic state model that can be used to track not only the target position and velocity, but also the target scattering coefficients which are important parameters to identify a target. We then derive a measurement model in which we transmit a polarized waveform and receive the reflected signal using an electromagnetic (EM) vector array. An EM vector sensor can fully exploit the polarization information from the received signal by measuring the six components of the EM field. It has been shown that employing vector sensors improves the estimation of the signal direction of arrival (DOA) and the resolution of closely spaced signal arrivals [5].

2.1. Target Dynamic State Model

We denote by S_{t} the complex scattering matrix representing the polarization change of the transmitted signal upon its refection on the target:

$$S_{\mathrm{t}} = \begin{bmatrix} s_{\mathrm{hh}} & s_{\mathrm{hv}} \\ s_{\mathrm{vh}} & s_{\mathrm{vv}} \end{bmatrix}. \tag{1}$$

The variables s_{hh} and s_{vv} are co-polar scattering coefficients, whereas s_{hv} and s_{vh} are cross-polar coefficients. For the mono-static radar case, $s_{\mathrm{hv}} = s_{\mathrm{vh}}$. Then, we represent the target state at time step k as

$$\boldsymbol{x}_k = \begin{bmatrix} \boldsymbol{\rho}_k^{\mathrm{T}} & \boldsymbol{s}_k^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \tag{2}$$

where $\boldsymbol{\rho} = [x \ y \ z \ \dot{x} \ \dot{y} \ \dot{z}]^{\mathrm{T}}$ represents the target position and velocity in a Cartesian coordinate system, and $\boldsymbol{s} = [\mathrm{Re}\{s_{\mathrm{hh}} \ s_{\mathrm{hv}} \ s_{\mathrm{vv}}\} \ \mathrm{Im}\{s_{\mathrm{hh}} \ s_{\mathrm{hv}} \ s_{\mathrm{vv}}\}]^{\mathrm{T}}$ is the target scattering coefficient vector.

We assume i) the target velocity and scattering coefficients are nearly constant, and ii) the target position and velocity are statistically independent to the scattering coefficients. Then, we obtain a

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linear target dynamic state model given by

$$\boldsymbol{x}_k = F\boldsymbol{x}_{k-1} + \boldsymbol{v}_{k-1} = \begin{bmatrix} F_\rho & \mathbf{0} \\ \mathbf{0} & F_s \end{bmatrix} \boldsymbol{x}_{k-1} + \boldsymbol{v}_{k-1} \qquad (3)$$

where F_ρ is the transition matrix for state vector $\boldsymbol{\rho}$ as

$$F_\rho = \begin{bmatrix} I_3 & T_{\mathrm{PRI}} I_3 \\ \mathbf{0} & I_3 \end{bmatrix} \qquad (4)$$

where I_n denotes the identity matrix of size n, and T_{PRI} is the pulse repetition intervel (PRI); $F_s = I_6$ is the transition matrix for $\boldsymbol{s}$; $\boldsymbol{v}_k$ is the independent process noise, assumed to be zero-mean Gaussian distributed with covariance matrix

$$Q = \begin{bmatrix} Q_\rho & \mathbf{0} \\ \mathbf{0} & Q_s \end{bmatrix} \qquad (5)$$

where Q_ρ and Q_s denote the covariance matrixes for states $\boldsymbol{\rho}$ and $\boldsymbol{s}$, respectively. Both Q_ρ and Q_s are assumed to be known.

The assumption that the target scattering coefficients are nearly constant is suitable for the situation that the target is far away from the sensor array and the target position change during the tracking period is not large compared with the distance between the target and the sensor array. In general, the dynamic model for the scattering coefficients is a nonlinear function with respect to other states, e.g., the target position. This results in a nonlinear state model.

2.2. Measurement Model

We consider a target characterized by azimuth ϕ, elevation ψ, range r, Doppler shift ω_{D}, and scattering matrix S_{t}. These parameters are related to the states $\boldsymbol{x}$. To uniquely identify the polarimetric aspect of the target, the polarization diversity of the transmitted waveform is required and the complete EM information of the signal reflected from the target has to be processed [5]. To provide these measurements, we assume the receiver of the active sensing system is an array of EM vector sensors where each sensor measures the six components of the EM field.

Consider an array of M vector sensors receiving the signal returned from a target at high elevation. The complex envelop of the measurements can be expressed as

$$\boldsymbol{y}(t) = A(\phi,\psi) S_{\mathrm{t}} \boldsymbol{\xi}(t-\tau) e^{j\omega_{\mathrm{D}} t} + \boldsymbol{e}(t), \quad t = t_1, \ldots, t_N, \quad (6)$$

where $A(\phi,\psi) = \boldsymbol{p}(\phi,\psi) \otimes V(\phi,\psi)$ is the array response; $[\phi\ \psi]^{\mathrm{T}}$ is the bearing angle vector; $\boldsymbol{p}(\phi,\psi) = [\exp\{j2\pi \boldsymbol{k}^{\mathrm{T}} \boldsymbol{r}_1/\lambda\}, \ldots, \exp\{j2\pi \boldsymbol{k}^{\mathrm{T}} \boldsymbol{r}_M/\lambda\}]^{\mathrm{T}}$ represents the phase of the planewave arriving from the direction given by $\boldsymbol{k} = [\cos\phi\cos\psi,\ \sin\phi\cos\psi,\ \sin\psi]^{\mathrm{T}}$ at position $\boldsymbol{r}_m$ of the m-th sensor ($m = 1, \ldots, M$), λ is the signal wavelength; and $V(\phi,\psi)$ is the response of a single vector sensor given by [5]

$$V(\phi,\psi) = \begin{bmatrix} -\sin\phi & -\cos\phi\sin\psi \\ \cos\phi & -\sin\phi\sin\psi \\ 0 & \cos\psi \\ -\cos\phi\sin\psi & \sin\phi \\ -\sin\phi\sin\psi & -\cos\phi \\ \cos\psi & 0 \end{bmatrix}. \qquad (7)$$

The polarized transmitted wave $\boldsymbol{\xi}(t)$ is a narrowband signal which can be represented by a complex vector [5]

$$\boldsymbol{\xi}(t) = \begin{bmatrix} \xi_{\mathrm{h}}(t) \\ \xi_{\mathrm{v}}(t) \end{bmatrix} = s(t) \begin{bmatrix} \cos\alpha\cos\beta + j\sin\alpha\sin\beta \\ -\sin\alpha\cos\beta + j\cos\alpha\sin\beta \end{bmatrix} \qquad (8)$$

The angles α and β are the orientation and ellipticity of the polarization ellipse depicted by the electric field vector. The function $s(t)$ represents the scalar complex envelop of the transmitted signal. The time delay $\tau = 2r/c$, where c is the wave propagation velocity and r is the distance from the target to the sensor array. The vector $\boldsymbol{e}(t)$ is the additive noise corrupting the sensor measurements. N denotes the number of samples during the interval T_{PRI}.

Since $\boldsymbol{\xi}(t)$ is the transmitted signal, the waveform design problem consists of selecting the envelop $s(t)$ and the polarization angles α and β in (8). We denote these waveform parameters by $\boldsymbol{\theta}$.

It can be verified that the relation between the target parameters $[\phi,\ \psi,\ r,\ \omega_{\mathrm{D}}]$ and the states $\boldsymbol{x}$ is given by

$$\begin{aligned} \phi &= \arctan(y/x) & \psi &= \arctan(z/\sqrt{x^2+y^2}) \\ r &= \sqrt{x^2+y^2+z^2} & \omega_{\mathrm{D}} &= 2\omega_{\mathrm{c}}\sqrt{\dot{x}^2+\dot{y}^2+\dot{z}^2}/c \end{aligned}$$

where ω_{c} is the carrier frequency. Therefore, we obtain a nonlinear relation between the measurements and the states at time step k as

$$\boldsymbol{y}_k(t) = \tilde{\boldsymbol{h}}(t, \boldsymbol{x}_k; \boldsymbol{\theta}_k) + \boldsymbol{e}_k(t), \quad t = t_1, \ldots, t_N \qquad (9)$$

where $\tilde{\boldsymbol{h}}(t,\boldsymbol{x}) = A(\phi,\psi) S_{\mathrm{t}} \boldsymbol{\xi}(t-\tau) e^{j\omega_{\mathrm{D}} t}$. When we lump $\{\boldsymbol{y}_k(t),\ t = t_1, \ldots, t_N\}$ together into a vector, we obtain the measurement model as

$$\begin{aligned} \boldsymbol{y}_k &= \begin{bmatrix} \boldsymbol{y}_k(t_1) \\ \vdots \\ \boldsymbol{y}_k(t_N) \end{bmatrix} = \begin{bmatrix} \tilde{\boldsymbol{h}}(t_1, \boldsymbol{x}_k; \boldsymbol{\theta}_k) \\ \vdots \\ \tilde{\boldsymbol{h}}(t_N, \boldsymbol{x}_k; \boldsymbol{\theta}_k) \end{bmatrix} + \begin{bmatrix} \boldsymbol{e}_k(t_1) \\ \vdots \\ \boldsymbol{e}_k(t_N) \end{bmatrix} \\ &= \boldsymbol{h}(\boldsymbol{x}_k; \boldsymbol{\theta}_k) + \boldsymbol{e}_k. \end{aligned} \qquad (10)$$

3. TARGET TRACKING USING PARTICLE FILTERS

We develop a target tracking approach using sequential Monte Carlo methods (particle filters). The proposed method can be applied to nonlinear and non-Gaussian state and measurement models. In our tracking problem, the potential dimension of the state can be large, which results in difficult direct important sampling. Hence, we adopt a Gibbs sampler to draw samples from an importance density function. This provides more efficient sampling.

The sequential Monte Carlo method is a technique for implementing a recursive Bayesian filter by Monte Carlo simulation. The key idea is to represent a posterior density function (belief) by a set of random samples with associated weights and to compute estimates based on these samples and weights. Let $\{\boldsymbol{x}_k^{(i)}, w_k^{(i)},\ i = 1, \ldots, N_{\mathrm{s}}\}$ denote a random measure that characterizes the posterior density function $p(\boldsymbol{x}_k \mid \boldsymbol{y}_{1:k})$ where $\{\boldsymbol{x}_k^{(i)}\}$ is a set of support points with associated weights $\{w_k^{(i)}\}$ for $i = 1, \ldots, N_{\mathrm{s}}$. The weights are normalized such that $\sum_i w_k^{(i)} = 1$.

For a sequential filtering case, we can choose an importance density $q(\cdot)$ such that we obtain a weight update equation as

$$w_k^{(i)} \propto w_{k-1}^{(i)} \frac{p(\boldsymbol{y}_k \mid \boldsymbol{x}_k^{(i)}) p(\boldsymbol{x}_k^{(i)} \mid \boldsymbol{x}_{k-1}^{(i)})}{q(\boldsymbol{x}_k^{(i)} \mid \boldsymbol{x}_{k-1}^{(i)}, \boldsymbol{y}_k)}, \qquad (11)$$

and the belief $p(\boldsymbol{x}_k \mid \boldsymbol{y}_{1:k})$ can be approximated as

$$p(\boldsymbol{x}_k \mid \boldsymbol{y}_{1:k}) \approx \sum_{i=1}^{N_{\mathrm{s}}} w_k^{(i)} \delta(\boldsymbol{x}_k - \boldsymbol{x}_k^{(i)}) \qquad (12)$$

where $\boldsymbol{x}_k^{(i)}$ are sampled from $q(\boldsymbol{x}_k \mid \boldsymbol{x}_{k-1}^{(i)}, \boldsymbol{y}_k)$.

Now considering our target tracking problem, from the dynamic state model (3) we observe that if we track the target position, velocity, and scattering coefficients simultaneously, the dimension of the parameter space is very large. Hence, if we directly draw samples from the importance density $q(\boldsymbol{x}_k \mid \boldsymbol{x}_{k-1}^{(i)}, \boldsymbol{y}_k)$, it is typically inefficient. Therefore, we apply a Markov chain Monte Carlo (MCMC) method, a class of iterative simulatation-based methods, to sample from the importance density.

In our developed particle filter, we propose to choose the importance density to be the transitional prior $p(\boldsymbol{x}_k \mid \boldsymbol{x}_{k-1}^{(i)})$, and we use a MCMC algorithm, Gibbs sampler, to draw samples from $p(\boldsymbol{x}_k \mid \boldsymbol{x}_{k-1}^{(i)})$. According to the state model (3), we partition the components of the state as $\boldsymbol{x}_k = [\boldsymbol{\rho}_k^{\mathrm{T}}, \ \boldsymbol{s}_k^{\mathrm{T}}]^{\mathrm{T}}$. Then, we derive a Gibbs sampling algorithm to draw samples $\boldsymbol{x}_k^{(i)} \sim p(\boldsymbol{x}_k \mid \boldsymbol{x}_{k-1}^{(i)})$ at time step k in a particle filter. Such a Gibbs sampling is described as follows.

- Initialization, $j = 0$. Set randomly or deterministically

$$\boldsymbol{x}_k^{(i,0)} = \left[\left(\boldsymbol{\rho}_k^{(i,0)} \right)^{\mathrm{T}} \quad \left(\boldsymbol{s}_k^{(i,0)} \right)^{\mathrm{T}} \right]^{\mathrm{T}}.$$

- Iteration j, $j = 1, \ldots, M$, M is a large number.
  - Sample $\boldsymbol{\rho}_k^{(i,j)} \sim p(\boldsymbol{\rho}_k \mid \boldsymbol{s}_k^{(i,j-1)}, \boldsymbol{x}_{k-1}^{(i)})$.
  - Sample $\boldsymbol{s}_k^{(i,j)} \sim p(\boldsymbol{s}_k \mid \boldsymbol{\rho}_k^{(i,j)}, \boldsymbol{x}_{k-1}^{(i)})$.
- Installation of $\boldsymbol{\rho}_k^{(i,M)}$ and $\boldsymbol{s}_k^{(i,M)}$ into $\boldsymbol{x}_k^{(i)}$

$$\boldsymbol{x}_k^{(i)} = \left[\left(\boldsymbol{\rho}_k^{(i,M)} \right)^{\mathrm{T}} \quad \left(\boldsymbol{s}_k^{(i,M)} \right)^{\mathrm{T}} \right]^{\mathrm{T}}.$$

Then, the obtained $\boldsymbol{x}_k^{(i)}$ is a sample from $p(\boldsymbol{x}_k \mid \boldsymbol{x}_{k-1}^{(i)})$.

Under a special case, as in our proposed dynamic state (3), that the partitions $\boldsymbol{\rho}$ and $\boldsymbol{s}$ are statistically independent of each other, the Gibbs sampling is simplified as

- Sample $\boldsymbol{\rho}_k^{(i)} \sim p(\boldsymbol{\rho}_k \mid \boldsymbol{\rho}_{k-1}^{(i)})$.
- Sample $\boldsymbol{s}_k^{(i)} \sim p(\boldsymbol{s}_k \mid \boldsymbol{s}_{k-1}^{(i)})$.

Then, we install $\boldsymbol{x}_k^{(i)} = \left[\left(\boldsymbol{\rho}_k^{(i)} \right)^{\mathrm{T}}, \ \left(\boldsymbol{s}_k^{(i)} \right)^{\mathrm{T}} \right]^{\mathrm{T}}$.

4. OPTIMAL WAVEFORM DESIGN

In this section we design an optimal waveform design method for target tracking. It is combined with the above target tracking algorithm and forms an adaptive waveform design approach. In order to realize this optimization, at time step k, we create a criterion that represents the estimation performance at time step $k+1$ when employing specific waveform parameters. Then, we select the waveform parameters that optimize this performance criterion.

For random parameters, as in our sequential Bayesian filtering for target tracking, a lower bound analogous to the Cramér-Rao bound (CRB) in a nonrandom parameter estimation exists and is derived in [6], usually referred to as posterior CRB (PCRB). That is, the PCRB provides a lower bound on the mean square error (MSE) matrix for the estimation of random parameters. This bound is independent of the specific estimation methods. Hence, we create our waveform selection criterion based on the PCRB.

4.1. Optimal Waveform Selection Based on PCRB

Consider that in our target tracking problem based on the state model (3) and measurement model (10), at time step k, we want to estimate a state trajectory $\boldsymbol{x}_{0:k}$ using the measurements $\boldsymbol{y}_{1:k}$. We denote $\boldsymbol{X}_k = [\boldsymbol{x}_0^{\mathrm{T}}, \ldots, \boldsymbol{x}_k^{\mathrm{T}}]^{\mathrm{T}}$. Then, the trajectory Bayesian information matrix (BIM), whose inverse is the PCRB, is defined as

$$\bar{J}_k \triangleq \mathrm{E}_{\boldsymbol{y}_{1:k},\, \boldsymbol{x}_{0:k}} \left[-\triangle_{\mathbf{X}_k}^{\mathbf{X}_k} \log p(\boldsymbol{y}_{1:k}, \ \boldsymbol{x}_{0:k}) \right] \tag{13}$$

where $\triangle_{\boldsymbol{\psi}}^{\boldsymbol{\eta}}$ denotes the second-order partial derivative with respect to $\boldsymbol{\psi}$ and $\boldsymbol{\eta}$. The lower right $n_x \times n_x$ block ($n_x = \dim(\boldsymbol{x})$) of $\bar{J}_k^{-1}$ is the PCRB for estimating $\boldsymbol{x}_k$, and its inverse is the BIM for estimating $\boldsymbol{x}_k$, denoted as J_k.

To derive the optimal waveform selection criterion, we adopt a recursive equation in [6] to update BIM J_{k+1} as follows

$$J_{k+1} = \Omega_k - \left(D_k^{12} \right)^{\mathrm{T}} \left(J_k + D_k^{11} \right)^{-1} D_k^{12} + \Gamma_{k+1}, \tag{14}$$

where

$$D_k^{11} = \mathrm{E}_{\boldsymbol{x}_k,\, \boldsymbol{x}_{k+1}} \left[-\triangle_{\boldsymbol{x}_k}^{\boldsymbol{x}_k} \log p(\boldsymbol{x}_{k+1} \mid \boldsymbol{x}_k) \right] \tag{15a}$$

$$D_k^{12} = \mathrm{E}_{\boldsymbol{x}_k,\, \boldsymbol{x}_{k+1}} \left[-\triangle_{\boldsymbol{x}_k}^{\boldsymbol{x}_{k+1}} \log p(\boldsymbol{x}_{k+1} \mid \boldsymbol{x}_k) \right] \tag{15b}$$

$$\Omega_k = \mathrm{E}_{\boldsymbol{x}_k,\, \boldsymbol{x}_{k+1}} \left[-\triangle_{\boldsymbol{x}_{k+1}}^{\boldsymbol{x}_{k+1}} \log p(\boldsymbol{x}_{k+1} \mid \boldsymbol{x}_k) \right] \tag{15c}$$

$$\Gamma_{k+1} = \mathrm{E}_{\boldsymbol{y}_{k+1},\, \boldsymbol{x}_{k+1}} \left[-\triangle_{\boldsymbol{x}_{k+1}}^{\boldsymbol{x}_{k+1}} \log p(\boldsymbol{y}_{k+1} \mid \boldsymbol{x}_{k+1}) \right]. \tag{15d}$$

For our target tracking problem, the waveform parameters $\boldsymbol{\theta}$ only appear in the measurement model (10). Hence, in the BIM recursive equation (14), only the matrix Γ_{k+1} is related to the waveform parameters $\boldsymbol{\theta}_{k+1}$. Therefore, we design our criterion to select the optimal waveform parameters $\boldsymbol{\theta}_{k+1}$ only using Γ_{k+1}.

In our sequential optimal waveform selection, we employ not only the information provided by the state and measurement models, but also the measurement history $\boldsymbol{y}_{1:k}$. Hence, we modify the matrix Γ_{k+1} to include the measurement history and design the optimal waveform based on a new matrix $\widetilde{\Gamma}_{k+1}$ as follows

$$\widetilde{\Gamma}_{k+1} = \mathrm{E}_{\boldsymbol{y}_{k+1},\, \boldsymbol{x}_{k+1} \mid \boldsymbol{y}_{1:k}} \left[-\triangle_{\boldsymbol{x}_{k+1}}^{\boldsymbol{x}_{k+1}} \log p(\boldsymbol{y}_{k+1} \mid \boldsymbol{x}_{k+1}) \right] \tag{16}$$

Since $\widetilde{\Gamma}_{k+1}$ is a matrix, we can use its determinant or trace as a cost function to create the waveform selection criterion. For example, if we use the trace of $\widetilde{\Gamma}_{k+1}$, denoted as $\mathrm{Tr}\{\cdot\}$, as the cost function, then we determine the optimal parameters for the next transmitted waveform as

$$\boldsymbol{\theta}_{k+1}^{*} = \arg \max_{\boldsymbol{\theta}_{k+1} \in \boldsymbol{\Theta}} \mathrm{Tr}\{\widetilde{\Gamma}_{k+1}(\boldsymbol{\theta}_{k+1})\}. \tag{17}$$

where $\boldsymbol{\Theta}$ is a set of the allowed values for $\boldsymbol{\theta}_{k+1}$. It can also denote a library of all possible waveforms. The obtained parameter $\boldsymbol{\theta}_{k+1}^{*}$ will be used for the transmitted waveform at time step $k+1$.

4.2. Computation of the Bayesian Information Matrix $\widetilde{\Gamma}_{k+1}$

To compute the matrix $\widetilde{\Gamma}_{k+1}$, in general, the expectation in (16) has no closed-form analytical solution. We propose to use Monte Carlo integration to solve the expectation integral. It can be verified that the matrix $\widetilde{\Gamma}_{k+1}$ can be calculated as

$$\widetilde{\Gamma}_{k+1} = \mathrm{E}_{\boldsymbol{x}_{k+1} \mid \boldsymbol{y}_{1:k}} [\Theta_{k+1}] \tag{18a}$$

$$\Theta_{k+1} = \mathrm{E}_{\boldsymbol{y}_{k+1} \mid \boldsymbol{x}_{k+1}} \left[-\triangle_{\boldsymbol{x}_{k+1}}^{\boldsymbol{x}_{k+1}} \log p(\boldsymbol{y}_{k+1} \mid \boldsymbol{x}_{k+1}) \right]. \tag{18b}$$

Note that Θ_{k+1} is the standard Fisher information matrix (FIM) for estimating the state vector $\boldsymbol{x}_{k+1}$ based on the observations $\boldsymbol{y}_{k+1}$.

For a sequential Monte Carlo method, we assume at time step k, we obtain N_s samples and its associated weights from the posterior $p(\boldsymbol{x}_k \mid \boldsymbol{y}_{1:k})$ as $\{\boldsymbol{x}_k^{(i)},\ w_k^{(i)};\ i = 1, \ldots, N_\mathrm{s}\}$. Then, the expectation in (18a) can be computed by the following two steps:

- For $i = 1, \ldots, N_\mathrm{s}$, draw samples $\boldsymbol{x}_{k+1}^{(i)} \sim p(\boldsymbol{x}_{k+1} \mid \boldsymbol{x}_k^{(i)})$.
- The matrix $\widetilde{\Gamma}_{k+1}$ is approximated as

$$\widetilde{\Gamma}_{k+1} \approx \sum_{i=1}^{N_\mathrm{s}} w_k^{(i)} \Theta_{k+1}\left(\boldsymbol{x}_{k+1}^{(i)}\right). \tag{19}$$

In order to calculate (18b), for each $\boldsymbol{x}_{k+1}^{(i)}$, we draw N_y identically independently distributed (IID) samples $\{\boldsymbol{y}_{k+1}^{(j)};\ j = 1, \ldots, N_\mathrm{y}\}$ from the likelihood function $p(\boldsymbol{y}_{k+1} \mid \boldsymbol{x}_{k+1}^{(i)})$. Then, we approximate the FIM $\Theta_{k+1}\left(\boldsymbol{x}_{k+1}^{(i)}\right)$ as

$$\Theta_{k+1}\left(\boldsymbol{x}_{k+1}^{(i)}\right) \approx \frac{1}{N_\mathrm{y}} \sum_{j=1}^{N_\mathrm{y}} \Lambda\left(\boldsymbol{y}_{k+1}^{(j)}, \boldsymbol{x}_{k+1}^{(i)}\right), \tag{20}$$

where we define the matrix function

$$\Lambda(\boldsymbol{y}_{k+1}, \boldsymbol{x}_{k+1}) = -\triangle_{\boldsymbol{x}_{k+1}}^{\boldsymbol{x}_{k+1}} \log p(\boldsymbol{y}_{k+1} \mid \boldsymbol{x}_{k+1}). \tag{21}$$

Therefore, we approximate $\widetilde{\Gamma}_{k+1}$ using Monte Carlo integration as

$$\widetilde{\Gamma}_{k+1} \approx \frac{1}{N_\mathrm{y}} \sum_{i=1}^{N_\mathrm{s}} \sum_{j=1}^{N_\mathrm{y}} w_k^{(i)} \Lambda\left(\boldsymbol{y}_{k+1}^{(j)}, \boldsymbol{x}_{k+1}^{(i)}\right). \tag{22}$$

5. NUMERICAL EXAMPLES

We use an array of two vector sensors to track a moving target in a 2D (x-y) environment. The two vector sensors are located along the y-axis, separated by 0.5λ ($\lambda = 0.3$ m). We only track the position and velocity of the moving target, hence, the state vector is $\boldsymbol{x} = [x\ \dot{x}\ y\ \dot{y}]^\mathrm{T}$. For the envelop of the transmitted signal, we consider a linear frequency modulated pulse with Gaussian envelop as

$$s(t) = (\pi\eta^2)^{-1/4} \exp\left[-\left(\frac{1}{2\eta^2} - jb\right) t^2\right] \tag{23}$$

where we set $\eta = 1\mu$s and $b = 135 \times 10^9 \mathrm{s}^{-2}$. The polarization angles belong to the intervals $\alpha \in [-90°, 90°]$ and $\beta \in [-45°, 45°]$. We adaptively select the polarization angles at each time step. The initial state of the target is $\boldsymbol{x}_0 = [4000\mathrm{m}\ 50\mathrm{m/s}\ 8000\mathrm{m}\ -200\mathrm{m/s}]$ and the scattering coefficients vary dynamically at each time step. The measurement noise is a zero-mean complex-Gaussian distributed vector, and the signal-to-noise ratio is 1 dB.

We compare the tracking performance of the adaptive waveform selection and fixed waveform schemes in the numerical examples. The comparison results of position tracking and velocity tracking are shown in Fig. 1 and 2, respectively. From these results, we observe that both schemes track the target position and velocity very well. However, in the adaptive waveform selection scheme, since we optimally choose the waveform parameters at each time step, its tracking performance is much better than the fixed waveform scheme.

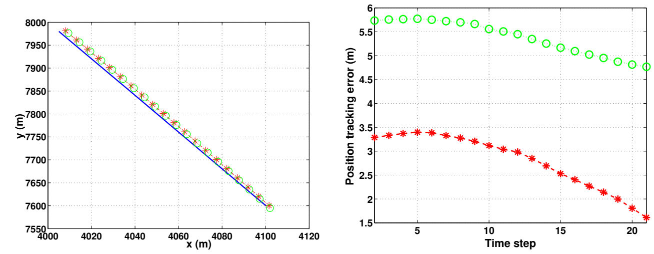

Fig. 1. (a) True and estimated target moving trajectory; (b) position estimation error (solid: true state; star: adaptively selected waveform; circle: fixed waveform).

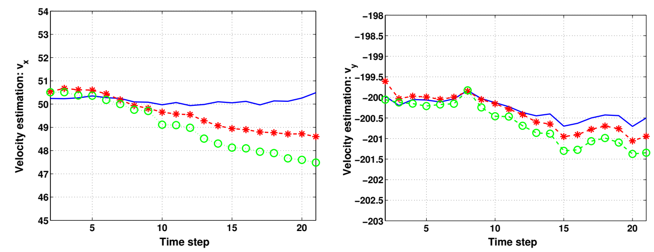

Fig. 2. True and estimated target velocity (solid: true state; star: adaptively selected waveform; circle: fixed waveform).

6. CONCLUSIONS

We developed an adaptive waveform design method for target tracking using EM vector sensors. We exploited the polarimetric aspect of the transmitted waveform, hence, we can further improve the tracking accuracy. We proposed a sequential Monte Carlo method for target tracking. This method is suitable for nonlinear and non-Gaussian state and measurement models. We designed a cost function based on the posterior Cramér-Rao bound and applied a Monte Carlo method to compute it. Numerical examples demonstrated the advantages of the adaptive waveform design scheme.

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