MIMO RADAR DETECTION UNDER PHASE SYNCHRONIZATION ERRORS

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ABSTRACT

We consider the problem of target detection for coherent multi-input multi-output (MIMO) radar with widely separated antennas in the presence of phase synchronization mismatch between the transmitter and receiver pairs. First, we introduce a data model using von-Mises distribution to represent the phase error terms. Then we employ expectation-maximization algorithm to estimate the error distribution parameter and target returns as well as the noise variance. We develop a generalized likelihood ratio test (GLRT) target detector using these estimates and demonstrate the effect of phase uncertainties on the detection performance using Monte Carlo simulations.

Index Terms— MIMO Radar Signal Processing, Signal Detection, Phase error.

1. INTRODUCTION

Multiple-input multiple-output (MIMO) radar systems with widely separated antennas are generally categorized into two processing modes: (i) non-coherent and (ii) coherent, depending whether the phase information is ignored or included, respectively [1]. For the non-coherent mode, with random target assumption, the performance gain in target detection and estimation is considered in various studies, see [1, Chapters 8 and 9] and references therein.

In addition to the unknown deterministic target parameters assumption, the coherent MIMO radar assumes the perfect knowledge of the orientation, location and local oscillator characteristics of all the antennas (hence the perfect knowledge of phase information). In this mode, the MIMO systems are shown to improve target parameter estimation, the resolution of target localization and target detection in non-homogeneous clutter [1], [2].

In this paper, we develop a generalized likelihood-ratio test (GLRT) target detector for a coherent MIMO radar system in the presence of phase synchronization mismatch between the transmitter and receiver pairs. In practice, this mismatch may occur due to imperfect knowledge of the locations and local oscillator characteristics of the antennas (each antenna may have their own oscillator independent of the others) [3]. We model these errors randomly using von-Mises distribution taking values between $-\pi$ and $+\pi$ [4]. The von-Mises distribution is used in applications of directional statistics and generalizes the uniform distribution (with zero shape parameter the von-Mises reduces to uniform distribution) [5]. We model the uncertainty in the phase error using different shape parameters and demonstrate its negative effect on the detection performance. We observe that an increase in the shape parameter (increasing the concentration of the phase error distribution around the mean value, zero in our case), decreases the uncertainty (the entropy), which corresponds to an increase in the detection performance. Note that the von-Mises distribution fits the fact that every phase error takes values that can be represented in modulo 2π and the errors more than 2π are ambiguous.

The rest of the paper is organized as follows. In Section 2, we introduce our parametric measurement and statistical models. In Section 3, we first present an expectation-maximization (EM) algorithm [6] to estimate the target, phase error (shape parameter) and the noise parameters and then formulate the GLRT for the target detection. This is a detection problem of random signals with unknown parameters, and except some special cases there is no solution to such estimation problems unless the EM algorithm is used [7]. In Section 4, we use Monte Carlo simulations to analyze the detection performance. Finally we provide concluding remarks in Section 5.

2. RADAR MODEL

In this section, we develop the measurement and statistical models for a coherent MIMO radar system to detect a target in a range cell of interest (COI). We will use these models to present an algorithm, within a generalized multivariate analysis of variance (GMANOVA) framework [8] in the presence of phase error when the signal and noise parameters are unknown.

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2.1. Measurement Model

We consider a two dimensional (2D) system with M transmitters and N receivers. Define $(x_{\mathrm{Tx}_m}, y_{\mathrm{Tx}_m})$, $m = 1, ..., M$, and $(x_{\mathrm{Rx}_n}, y_{\mathrm{Rx}_n})$, $n = 1, ..., N$, as the locations of the transmitters and receivers, respectively. We also assume a stationary point target located at (x_0, y_0) and having radar cross section (RCS) values changing w.r.to the angle aspect (e.g., multiple scatterers, which cannot be resolved by the transmitted signals, with (x_0, y_0) as the center of gravity) [1]. Define the complex envelope of the signal from the m^{th} transmitter is $\beta_m s_m(t)$, $m = 1, \ldots, M$, such that $|\beta_m|^2$ as the transmitted energy with $\sum_{m=1}^{M} |\beta_m|^2 = E$ (E is constant for any M) and $\int_{T_s} |s_m(t)|^2 \mathrm{d}t = 1,\ m = 1, \ldots, M$, with T_s as the signal duration. We write the lowpass equivalent of the received signal at the n^{th} receiver following [1]:

$$r_n(t) = \sum_{m=1}^{M} \alpha_{nm}\sigma_{nm}\beta_m s_m(t-\tau_{nm})e^{-j(\psi_{nm}+\theta_{nm})} + e_n(t), \tag{1}$$

where

- $\alpha_{nm} = \sqrt{\dfrac{G_{tx}G_{rx}\lambda^2}{(4\pi)^3 R_m^2 R_n^2}}$ is the channel parameter from the m^{th} transmitter to the n^{th} receiver, with G_{tx} and G_{rx} as the gains of the transmitting and receiving antennas, respectively; λ as the wavelength of the incoming signal; R_m and R_n as the distances from transmitter and receiver to target, respectively
- σ_{nm} is the square root of the target RCS seen by the m^{th} transmitter and n^{th} receiver pair
- $\tau_{nm} = (R_m + R_n)/c$, and c is the speed of the signal propagation in the medium
- $\psi_{nm} = 2\pi f_c \tau_{nm}$, with f_c as the carrier frequency
- θ_{nm} is the phase error between the n^{th} receiver and m^{th} transmitter
- $e(t)$ is additive noise.

To enable the data separation at the receiver side arriving from the different transmitters, we assume low–cross-correlation transmitted signals. The design of signals with these properties is a challenging research subject [9], but for simplification of the problem and demonstration of our methods and analysis, we assume that the required signal criteria are met (this assumption is commonly made for MIMO radar, see [1, Chapters 8 and 9] and references therein.) Hence, we apply matched-filtering and obtain the output of the n^{th} receiver corresponding to the i^{th} transmitter :

$$r_{ni} = \beta_i \alpha_{ni}\sigma_{ni}e^{-j(\psi_{ni}+\theta_{ni})} + e_{ni}, \tag{2}$$

where $r_{ni} = \int_{\tau_{ni}}^{\tau_{ni}+T_s} r_n(t)s_i^*(t-\tau_{ni})\mathrm{d}t$ and $e_{ni} = \int_{\tau_{ni}}^{\tau_{ni}+T_s} e_n(t)$ $s_i^*(t-\tau_{ni})\mathrm{d}t$.

Then, combining the received data corresponding to the transmitted signal $s_i(t)$ for one pulse, we obtain

$$\boldsymbol{r_i} = \boldsymbol{A}_i \boldsymbol{X}_i \boldsymbol{\phi}_i + \boldsymbol{e}_i, \tag{3}$$

where $\boldsymbol{r_i} = [r_{1i},\ \ldots,\ r_{Ni}]^T$, $\boldsymbol{A}_i = \beta_i \mathrm{diag}(\alpha_{1i}e^{-j\psi_{1i}}, \ \ldots,\ \alpha_{Ni}e^{-j\psi_{Ni}})$, $\boldsymbol{X}_i = \mathrm{diag}(\sigma_{1i},\ \ldots,\ \sigma_{Ni})$, $\boldsymbol{\phi}_i = [e^{-j\theta_{1i}},\ \ldots,\ e^{-j\theta_{Ni}}]^T$ and $\boldsymbol{e}_i = [e_{1i},\ \ldots,\ e_{Ni}]^T$. We stack the receiver outputs corresponding to all the signals into an $NM \times 1$ vector

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{X}\boldsymbol{\phi} + \boldsymbol{e}, \tag{4}$$

where

$$\begin{aligned}
\boldsymbol{y} &= [\boldsymbol{r}_1^T,\ \ldots\ \boldsymbol{r}_M^T]^T \\
\boldsymbol{A} &= \mathrm{blkdiag}(\boldsymbol{A}_1,\ \ldots,\ \boldsymbol{A}_M) \\
\boldsymbol{X} &= \mathrm{blkdiag}(\boldsymbol{X}_1,\ \ldots,\ \boldsymbol{X}_M) \\
\boldsymbol{\phi} &= [\boldsymbol{\phi}_1^T,\ \ldots\ \boldsymbol{\phi}_M^T]^T \\
\boldsymbol{e} &= [\boldsymbol{e}_1^T,\ \ldots\ \boldsymbol{e}_M^T]^T.
\end{aligned}$$

We transmit K pulses and assume that the target is stationary during this observation time; then

$$Y = [\boldsymbol{y}(1)\ \boldsymbol{y}(2)\ \cdots\ \boldsymbol{y}(K)]_{NM\times K} = \boldsymbol{A}\boldsymbol{X}\Phi + \boldsymbol{E}, \tag{5}$$

where $\Phi = [\boldsymbol{\phi}(1)\ \cdots\ \boldsymbol{\phi}(K)]_{NM\times K}$, and $\boldsymbol{E} = [\boldsymbol{e}(1)\ \boldsymbol{e}(2)\ \cdots\ \boldsymbol{e}(K)]_{NM\times K}$ is the additive noise.

2.2. Statistical Model

In (4), we assume that $\boldsymbol{A}$ is known (due to the coherent processing assumption), and $\boldsymbol{X}$ (target RCS values) is unknown deterministic. Moreover, in our calculations $\boldsymbol{e}(k)$, for $k = 1, \ldots, K$ are independent identically distributed (i.i.d.) zero-mean complex multivariate normal random variables with $\sigma_e^2 \mathrm{I}_{MN\times MN}$ as covariance for unknown σ_e^2. Define $\boldsymbol{\phi}(k) = \exp(\boldsymbol{\theta}(k))$ with $\boldsymbol{\theta}(k) = [\theta_{11}(k), \theta_{21}(k),\ \ldots,\ \theta_{NM}(k)]$ where $\theta_{nm}(k)$ is the phase error between the n^{th} receiver and m^{th} transmitter pair at the k^{th} pulse. Then, $\boldsymbol{\Theta} = [\boldsymbol{\theta}(1)\ \cdots\ \boldsymbol{\theta}(K)]$. We model $\theta_{nm}(k)$ as i.i.d. von-Mises distributed random variables following [4]

$$p(\theta_{nm}(k);\Delta) = \frac{\exp[\Delta\cos\theta_{nm}(k)]}{2\pi I_0(\Delta)} \qquad -\pi \le \theta_{nm}(k) \le \pi, \tag{6}$$

where $\Delta \ge 0$ is the (unknown) shape parameter and $I_0(\cdot)$ is the modified Bessel function of the first kind; Δ controls the spread of the density reducing the density to uniform distribution when $\Delta = 0$ [5]. Observe that conditioned on $\boldsymbol{\Theta}$ with known $\boldsymbol{A}$ and unknown $\boldsymbol{X}$ and σ_e^2, (5) is a GMANOVA model with the following distribution

$$\prod_{k=1}^{K} p(\boldsymbol{y}(k)|\boldsymbol{\theta}(k);\boldsymbol{X},\sigma_e^2) = \prod_{k=1}^{K} \frac{1}{|\pi\sigma_e^2 I|} \exp\left\{\frac{-1}{\sigma_e^2}\left[\boldsymbol{y}(k) - \boldsymbol{A}\boldsymbol{X}\boldsymbol{\phi}(k)\right]^H \left[\boldsymbol{y}(k) - \boldsymbol{A}\boldsymbol{X}\boldsymbol{\phi}(k)\right]\right\}, \tag{7}$$

3. DETECTION AND ESTIMATION ALGORITHM

We derive GLRT using the observed data likelihood function, with Y as observed and $\boldsymbol{\theta}(k)$, $k = 1, \ldots, K$ as unobserved data to decide about the presence of a target in the COI. Namely, we choose between two hypotheses in the following parametric test

$$\begin{cases} \mathcal{H}_0 & : \quad \boldsymbol{X} = 0, \sigma_e^2, \Delta \\ \mathcal{H}_1 & : \quad \boldsymbol{X} \neq 0, \sigma_e^2 \end{cases}, \tag{8}$$

with σ_e^2 and Δ as the nuisance parameters. We compute the GLRT and reject $\mathcal{H}_0$ (target-free case) in favor of $\mathcal{H}_1$ (target-present case) when

$$\text{GLRT} = \frac{p_1(Y; \hat{\boldsymbol{X}}, \hat{\sigma_{e}^2}_1, \hat{\Delta})}{p_0(Y; \hat{\sigma_{e}^2}_0)} > \eta, \tag{9}$$

where $p_i(\cdot)$ and $\hat{\sigma_{e}^2}_i$ are the observed data likelihood function and MLE of $\hat{\sigma_e^2}$ under $\mathcal{H}_i$, respectively for $i = 0, 1$; $\boldsymbol{X}$ and Δ are the MLEs of $\boldsymbol{X}$ and Δ under $\mathcal{H}_1$; η is the detection threshold.

We compute the MLEs of the unknown parameters using an EM algorithm with the hierarchical data model presented in (6) and (7). First, we write the complete data log-likelihood function in canonical exponential family form [6]

$$\begin{aligned} L_c(\boldsymbol{X}, \sigma_e^2, \Delta) &= \ln \textstyle\prod_{k=1}^K p(\boldsymbol{y}(k)|\boldsymbol{\theta}(k); \boldsymbol{X}, \sigma_e^2) p(\boldsymbol{\theta}(k); \Delta) \\ &= const + \tfrac{-K}{\sigma_e^2}\Big(\text{tr}[\boldsymbol{X}^H \boldsymbol{A}^H \boldsymbol{A}\boldsymbol{X}\, \boldsymbol{T}_3] \\ &\quad + \text{tr}[\boldsymbol{T}_2] - 2\text{Re}(\text{tr}[\boldsymbol{T}_1^H\, \boldsymbol{A}\boldsymbol{X}])\Big) \\ &\quad - NMK \ln I_0(\Delta) + \Delta T_4, \end{aligned} \tag{10}$$

where $\boldsymbol{T}_i$, $i = 1, \ldots, 4$, are the natural complete-data sufficient statistics (see (11) for the definitions) and "tr" stands for the trace. Since the complete-data likelihood function belongs to an exponential family, we simplify the EM algorithm [6].

E Step: We define the i^{th} iteration estimates of the set of the unknown parameters as $\Gamma^{(i)} = \{\hat{\boldsymbol{X}}^{(i)}, (\hat{\sigma_e}^2)^{(i)}, \hat{\Delta}^{(i)}\}$, and compute the conditional expectation ($\text{E}_{p(\boldsymbol{\Theta}|Y)}(\cdot)$ expectation w.r.to $p(\boldsymbol{\Theta}|Y)$) of the sufficient statistics under $\mathcal{H}_1$

$$\boldsymbol{T}_1^{(i)} = \text{E}_{p(\boldsymbol{\Theta}|Y)}\Big(\frac{1}{K}\sum_{k=1}^K \boldsymbol{y}(k)\boldsymbol{\phi}(k)^H; \Gamma^{(i)}\Big) \tag{11a}$$

$$\boldsymbol{T}_2^{(i)} = \frac{1}{K}\sum_{k=1}^K \boldsymbol{y}(k)\boldsymbol{y}(k)^H \tag{11b}$$

$$\boldsymbol{T}_3^{(i)} = \text{E}_{p(\boldsymbol{\Theta}|Y)}\Big(\frac{1}{K}\sum_{k=1}^K \boldsymbol{\phi}(k)\boldsymbol{\phi}(k)^H; \Gamma^{(i)}\Big) \tag{11c}$$

$$T_4^{(i)} = \text{E}_{p(\boldsymbol{\Theta}|Y)}\Big(\sum_{k=1}^K\sum_{n=1}^N\sum_{m=1}^M \cos(\theta_{nm}(k)); \Gamma^{(i)}\Big) \tag{11d}$$

M Step: We simply replace the natural complete-data sufficient statistics in the MLE expressions. We use the results of GMANOVA [8] for the MLEs of $\boldsymbol{X}$ and σ_e^2. We define $\boldsymbol{S}^{(i)} = \boldsymbol{T}_2^{(i)} - \boldsymbol{T}_1^{(i)}\left(\boldsymbol{T}_3^{(i)}\right)^{-1}\left(\boldsymbol{T}_1^{(i)}\right)^H$ and $P = NMK$ then compute

$$\begin{aligned} \hat{\boldsymbol{X}}^{(i+1)} = & \left[\boldsymbol{A}^H \left(\boldsymbol{S}^{(i)}\right)^{-1} \boldsymbol{A}\right]^{-1} \boldsymbol{A}^H \cdot \\ & \left(\boldsymbol{S}^{(i)}\right)^{-1} \boldsymbol{T}_1^{(i)} \left(\boldsymbol{T}_3^{(i)}\right)^{-1} \end{aligned} \tag{12a}$$

$$\begin{aligned} (\hat{\sigma}_e^2)^{(i+1)} = & \tfrac{1}{P}\Big(\text{tr}[\boldsymbol{T}_2^{(i)}] - 2\text{Re}(\text{tr}[(\boldsymbol{T}_1^H)^{(i)}\boldsymbol{A}\boldsymbol{X}^{(i+1)}]) \\ & + \text{tr}[(\boldsymbol{X}^H)^{(i+1)}\boldsymbol{A}^H\boldsymbol{A}\boldsymbol{X}^{(i+1)}]\Big) \end{aligned} \tag{12b}$$

Concentrating (10) w.r.to $\hat{\boldsymbol{X}}^{(i+1)}$ and $(\hat{\sigma}_e^2)^{(i+1)}$, we compute

$$\hat{\Delta}^{(i+1)} = \arg\max_{\Delta} -P \ln I_0(\Delta) + \Delta T_4^{(i)}. \tag{13}$$

Under $\mathcal{H}_0$, $p_0(Y; \hat{\sigma_e^2})$ is Gaussian distributed and hence the only unknown $\hat{\sigma}_e^2 = \text{tr}(\boldsymbol{T}_2)$.

Recalling the assumptions from Sec. 2.2, we compute the conditional distribution, $p(\boldsymbol{\Theta}|Y) = \prod_{k=1}^K p(\boldsymbol{\theta}(k)|\boldsymbol{y}(k))$. We employ $\int_{-\pi}^{\pi}\exp(a\cos\theta + b\sin\theta)\text{d}\theta = 2\pi I_0(\sqrt{a^2+b^2})$ for any complex a and b (see [4], [5] and [10]), and obtain

$$p(\boldsymbol{y}(k)) = \frac{\exp(\frac{-1}{\sigma_e^2}(\boldsymbol{y}(k)^H\boldsymbol{y}(k) + h)}{(\sigma_e^2\pi)^{NM}} \prod_{m=1}^M\prod_{n=1}^N \frac{I_0(c_{nm}(k))}{I_0(\Delta)}, \tag{14}$$

where $h = \boldsymbol{1}^T\boldsymbol{X}^H\boldsymbol{A}^H\boldsymbol{A}\boldsymbol{X}\boldsymbol{1}$ and $\boldsymbol{1}$ is an $NM \times 1$ vector of ones, $c_{nm}(k) = \sqrt{a_{nm}(k)^2 + b_{nm}(k)^2}$, $a_{nm}(k) = \frac{2}{\sigma_e^2}\text{Re}\{r_{nm}^*(k)\alpha_{nm}\beta_m e^{-j\psi_{nm}\sigma_{nm}}\} + \Delta$, $b_{nm}(k) = \frac{2}{\sigma_e^2}\text{Im}\{r_{nm}^*(k)\alpha_{nm}\beta_m e^{-j\psi_{nm}\sigma_{nm}}\}$. Here $r_{nm}^*(k)$ is the complex conjugate of the $(N(m-1)+n)^{th}$ entry of $\boldsymbol{y}(k)$, $\alpha_{nm}\beta_m e^{-j\psi_{nm}}$ and σ_{nm} are the $(N(m-1)+n)^{th}$ diagonal entries of $\boldsymbol{A}$ and $\boldsymbol{X}$ respectively (see (4) and (5)). $\text{Re}(\cdot)$ and $\text{Im}(\cdot)$ mean the real and imaginary parts, respectively.

Then, using (6), (7) and (14), we find

$$p(\boldsymbol{\theta}(k)|\boldsymbol{y}(k)) = \prod_{m=1}^M\prod_{n=1}^N \frac{\exp(d_{nm}(k))}{2\pi I_0(c_{nm}(k))}, \tag{15}$$

where $d_{nm}(k) = a_{nm}(k)\cos\theta_{nm}(k) + b_{nm}(k)\sin\theta_{nm}(k)$.

We compute the conditional mean of the natural complete-data sufficient statistics in (11) using (15) and applying $\text{E}_{p(\boldsymbol{\Theta}|Y)}(e^{\pm j\theta_{nm}(k)}) = \frac{a_{nm}(k)\pm jb_{nm}(k)}{c_{nm}(k)}\frac{I_1(c_{nm}(k))}{I_0(c_{nm}(k))}$ (see [10] for the proof.) We denote $\hat{\boldsymbol{X}}$, $\hat{\sigma}_e^2$ and $\hat{\Delta}$ as the estimates of $\boldsymbol{X}$ and σ_e^2 and Δ obtained upon the convergence of the EM algorithm under $\mathcal{H}_1$, and using (9), (12), (13) and (14), we obtain

$$(\text{GLRT})^{\frac{1}{P}} = \frac{\exp(\frac{-1}{NM\hat{\sigma}_e^2}(\text{tr}(\boldsymbol{T}_2)+h))}{exp(-1)}\frac{\text{tr}(\boldsymbol{T}_2)}{\hat{\sigma}_e^2}\frac{g^{\frac{1}{P}}}{I_0(\hat{\Delta})}, \tag{16}$$

where $g = \prod_{k=1}^K\prod_{m=1}^M\prod_{n=1}^N I_0(c_{nm}(k))$.

4. NUMERICAL EXAMPLES

We demonstrate the performance of the GLRT detector with numerical examples using Monte Carlo (MC) simulations. The results are obtained from 10^4 MC runs. We assume that M transmitters and N receivers (denoting MIMO $M \times N$) are located on the y-axis and x-axis, respectively. The target is 10km from each axes. The antenna gains (G_{tx} and G_{rx}) are 30dB; the signal frequency (f_c) is 1GHz. The angle between the transmitters $a_1 = ... = a_M = 10°$ and similarly between the receivers $b_1 = ... = b_N = 10°$ (see Fig. 1). We define

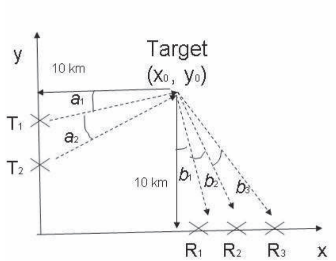

Fig. 1. MIMO antenna system with M transmitters and N receivers.

the signal-to-noise ratio (SNR) similar to [2],[10],

$$\text{SNR} = \sum_{k=1}^{K} (\boldsymbol{AX1})^H (\boldsymbol{AX1}) / (NMK\sigma_e^2) \qquad (17)$$

The RCS values, $\boldsymbol{X}$, are assigned as the realizations of zero mean complex Gaussian random variable with unit variance for the simulation purposes. Later, σ_e^2 is chosen to meet the desired SNR.

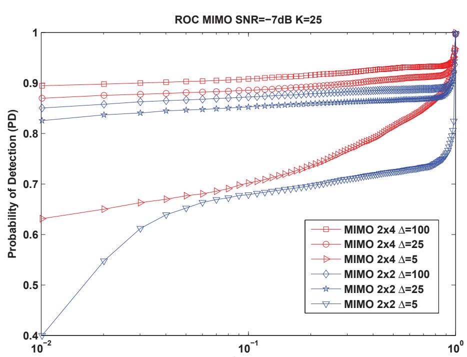

Fig. 2. Receiver operating characteristics of MIMO radar for different number of receivers and shape parameter values.

In Fig. 2, we plot the receiver operating characteristics (ROC) for different number of the receivers, N, and shape parameter (of the von-Misses distribution), Δ, values assuming $M = 2$, $K = 25$ pulses and SNR=-7 dB. As expected when N, increases, the performance of the MIMO system improves. We also observe that as Δ increases, the detection performance increases. This behavior is due to the characteristics of the von-Misses distribution. When $\Delta = 0$, it has the maximum entropy (maximum uncertainty, minimum performance), as Δ increases this entropy decreases giving rise to an increase in the detection performance [5].

5. CONCLUSION

We developed a GLRT target detector for a coherent MIMO radar system with widely separated antennas in the presence of phase synchronization error. Representing the phase error terms using von-Mises distribution, we introduced a measurement model under the GMANOVA framework, and applied the EM algorithm to estimate the unknown parameters. We developed the GLR test detector using these estimates. With different shape parameters, we modeled different uncertainties in the phase error distribution, and demonstrated their effect on the detection performance using Monte Carlo simulations. Future work will also include the adaptive power allocation and effect of the clutter.

6. REFERENCES

[1] J. Li and P. Stoica, *MIMO Radar Signal Processing*. Wiley-IEEE Press, Oct. 2008.

[2] M. Akcakaya, M. Hurtado, and A. Nehorai, "MIMO radar detection of targets in compound-Gaussian clutter,," in *Proc. 42nd Asilomar Conf. Signals, Syst. Comput.*, Pacific Groove, CA, USA, Oct. 2008.

[3] I. Papoutsis, C. Baker, and H. Griffiths, "Fundamental performance limitations of radar networks,," in *Proc. 1st EMRS DTC Technical Conf.*, Edinburgh, 2004.

[4] H. L. V. Trees, *Detection, Estimation, and Modulation Theory, Part I*, 1st ed. Wiley-Interscience, Sep. 2001.

[5] K. V. Mardia and P. E. Jupp, *Directional Statistics*, 2nd ed. Wiley, 1999.

[6] P. Bickel and K. Doksum, *Mathematical Statistics: Basic Ideas and Selected Topics*, 2nd ed. Upper Saddle River, NJ: Prentice Hall, 2000.

[7] S. M. Kay, *Fundamentals of Statistical Signal Processing: Detection Theory*. Upper Saddle River, NJ: Prentice Hall PTR, 1998.

[8] A. Dogandzic and A. Nehorai, "Generalized multivariate analysis of variance: A unified framework for signal processing in correlated noise," *IEEE Signal Process. Mag.*, vol. 20, pp. 39–54, Sep. 2003.

[9] Y. Abramovich and G. Frazer, "Bounds on the volume and height distributions for the MIMO radar ambiguity function," *IEEE Signal Processing Letters*, vol. 15, pp. 505–508, 2008.

[10] M. Akcakaya and A. Nehorai, "MIMO radar detection and adaptive design under a phase synchorization mismatch,," submitted to *IEEE Trans. Sign. Process.*