TARGET DETECTION IN MIMO RADAR IN THE PRESENCE OF DOPPLER USING COMPLEMENTARY SEQUENCES

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ABSTRACT

In this paper, we present a method for detecting a point target using multiple antennas when the relative motion between the receivers and the target induces a non-negligible Doppler shift. As a key illustrative example, we consider a 4×4 system employing a unitary matrix waveform set, e.g., formed from Golay complementary sequences. When a non-negligible Doppler shift is induced by the target motion, the waveform matrix formed from the complementary sequences is no longer unitary, resulting in significantly degraded target range estimates. To solve this problem, we adopt a subspace based approach exploiting the observation that the receive matrix formed from matched filtering of the reflected waveforms has a (non-trivial) null-space. Through processing of the waveforms with the appropriate vector from the null-space, we can significantly improve the detection performance. We provide simulation results to confirm the theoretical analysis.

Index Terms— Golay complementary sequences, subspace signal processing, MIMO Radar

1. INTRODUCTION

In [1], Howard et al. proposed a new 2×2 multi-channel radar scheme employing polarization diversity for obtaining multiple independent views of the target. In this scheme, Golay pairs [2] of phase coded waveforms are used to provide synchronization while Alamouti coding [3] is used to coordinate transmission of these waveforms on the horizontal and vertical polarizations. The combination of Golay complementary sequences and Alamouti coding makes it possible to do unambiguous radar polarimetry on a pulse-by-pulse basis, which reduces the signal processing complexity as compared to distributed aperture radar. In [4], the 2×2 case was extended to multiple antennas, and more general waveforms families were developed that allowed for perfect separation in the case of negligible Doppler. In particular, scheduling for Golay pairs was described for a 4×4 system and it was demonstrated that Golay pairs achieve both *perfect separation* and *perfect reconstruction* (see [4]). However, in the presence of Doppler, Golay pairs are known to perform poorly, which is why they have not found widespread use in radar, since accurate target ranging with these sequences is impossible in the presence of Doppler when conventional processing techniques are employed. In [5], PTM sequences were used to make the Golay sequence transmissions resilient against Doppler shifts. The method achieves good results for small Doppler shifts, but the number of PRIs needed per transmission of the coded Golay sequence matrix is large, thereby requiring the ”radar channel” to stay constant over a relatively long time interval. In this paper, we describe a Doppler compensation scheme that exploits the subspace structure of the received waveform matrix. We show that the received waveform matrix can be processed in a way that imparts a specific structure on the subspace that it occupies, and the null-space of this matrix can be used to minimize the effects of Doppler. We develop a processing filter using the null-space of this matrix to alleviate the effects of Doppler in target ranging, and demonstrate that the method works over a wide range of target SNRs. We also show that the scheme works for multiple targets if their separation and velocities follow certain conditions.

2. GOLAY COMPLEMENTARY SEQUENCES AND TARGET DETECTION

In this section, we describe how Golay codes enable high-resolution detection of targets in the absence of Doppler shift. We make the assumptions that the target round-trip delay of d chip intervals doesn't change appreciably during the transmission of N pulses, where $N = 4$ in our case. Without loss of generality, we assume $d = 0$ for ease of explanation.

2.1. Golay Complementary Sequences

A pair of sequences $s_1(n)$ and $s_2(n)$ of length N_c satisfy the Golay property if the sum of their autocorrelation functions

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satisfy

$$R_{s_1s_1}(l) + R_{s_2s_2}(l) = \begin{cases} 2N_c & if \quad l = 0 \\ 0 & if \quad l \neq 0 \end{cases} \tag{1}$$

for $l = -N_c - 1, ..., N_c - 1$. If we take the DFT of the above equation, we get

$$|S_1(k)|^2 + |S_2(k)|^2 = 2N_c \tag{2}$$

In [4], we showed that if $s_1(n)$ and $s_2(n)$ are Golay complementary, then so are $s_1^*(-n)$ and $s_2^*(-n)$. Using this fact, we can develop a 4-waveform family using Golay complementary sequences by defining

$$s_3(n) = s_1^*(-n) \tag{3}$$

and

$$s_4(n) = s_2^*(-n) \tag{4}$$

We now describe how this 4-waveform family can be used for target detection in a 4×4 system.

2.2. Target Detection in the Absence of Doppler

In the case of negligible Doppler, the received signal over 4 PRIs is given by

$$\mathbf{R}(n) = \mathbf{H}^T\mathbf{S}(n) + \mathbf{N}(n) \tag{5}$$

where $\mathbf{S}(n)$ is the transmitted waveform matrix given by [4].

$$\mathbf{S}(n) = \begin{bmatrix} s_1(n) & s_2^*(-n) & s_3(n) & s_4^*(-n) \\ -s_2(n) & s_1^*(-n) & -s_4(n) & s_3^*(-n) \\ -s_3(n) & s_4^*(-n) & s_1(n) & -s_2^*(-n) \\ -s_4(n) & -s_3^*(-n) & s_2(n) & s_1^*(-n) \end{bmatrix} \tag{6}$$

$\mathbf{H}$ is 4×4 channel matrix which contains the various round-trip path gains from each transmit antenna to each receive antenna, and has i.i.d entries of the form.

$$[\mathbf{H}]_{ij} = h_{ij} \tag{7}$$

where $h_{ij} \sim CN(0, \sigma^2)$ is the channel co-efficient from transmit antenna i to receive antenna j, and $\mathbf{N}(n)$ is the noise matrix. To detect the presence of the target in the delay resolution bin n, We process the received waveform matrix as

$$\mathbf{R}(n) * \mathbf{S}^H(-n) = \mathbf{H}^T\mathbf{S}(n) * \mathbf{S}^H(-n) + \mathbf{N}'(n) \tag{8}$$

where $*$ is the pair-wise convolution of two matrices that follows the same order as matrix multiplication. It can be easily shown [4] that

$$\mathbf{S}(n) * \mathbf{S}^H(-n) = \alpha\mathbf{I}\delta(n) \tag{9}$$

From this, it follows that

$$\mathbf{R}(n) * \mathbf{S}^H(-n) = \alpha\mathbf{H}\delta(n) + \mathbf{N}'(n) \tag{10}$$

In order to detect the presence of a target in the delay resolution bin n, consider the test stastic

$$z(n) = \left\|\mathbf{R}(n) * \mathbf{S}^H(n)\right\|_F^2 \tag{11}$$

where the subscript F stands for Frobenius norm. A plot of $z(n)$ for target SNRs of 5dB and 10dB, respectively are shown in Figure 1. As we can see from the figure, the unitary waveform matrix signal design greatly facilitates high-resolution time-localization of a target when the Doppler shift is negligible.

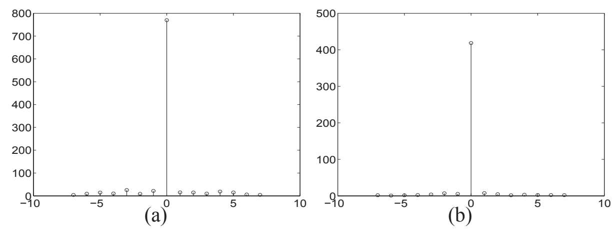

Fig. 1. Plot of $z(n)$ without Doppler for (a) SNR = 5dB (b) SNR = 10dB

3. DOPPLER COMPENSATION

In this section, we develop a signal model that incorporates the effects of Doppler. We assume that the target is moving at a constant speed, which means that between two successive PRIs, the differential Doppler phase shift is constant.

3.1. Effects of Doppler

In the presence of Doppler, the received signal may be expressed as

$$\mathbf{R}(n) = \mathbf{H}^T\mathbf{S}(n)\mathbf{D} + \mathbf{N}(n) \tag{12}$$

The Doppler shift matrix $\mathbf{D}$ is given by

$$\mathbf{D} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{j\upsilon} & 0 & 0 \\ 0 & 0 & e^{j2\upsilon} & 0 \\ 0 & 0 & 0 & e^{j3\upsilon} \end{bmatrix} \tag{13}$$

where υ is the Doppler-induced differential phase shift between two successive PRIs. As in the case of negligible Doppler, we process the received waveform matrix as

$$\mathbf{R}(n) * \mathbf{S}^H(-n) = \mathbf{H}^T\mathbf{S}(n)\mathbf{D} * \mathbf{S}^H(-n) + \mathbf{N}'(n) \tag{14}$$

In the presence of a non-negligible Doppler phase shift, the condition in equation (9) is not satisfied in general, i.e.,

$$\mathbf{S}(n)\mathbf{D} * \mathbf{S}^H(-n) \neq \alpha\mathbf{I}\delta(n) \tag{15}$$

and unambiguous range resolution becomes significantly more difficult. To illustrate this graphically, a plot of $z(n)$ for

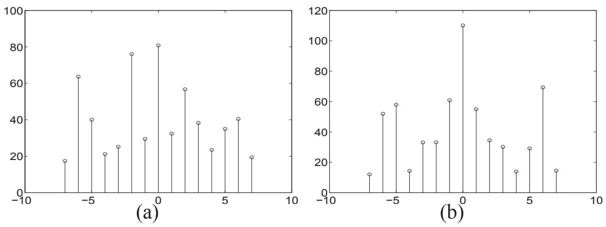

Fig. 2. Plot of $z(n)$ with Doppler ($\upsilon = \pi/3$) for (a) SNR = 5dB (b) SNR = 10dB

the same target SNRs of 5dB and 10dB are plotted in Figure 2 for the case of $\upsilon = \pi/3$. For this particular set of round-trip channel gains, the presence of Doppler makes it impossible to detect the target.

3.2. Doppler Processing

Towards combatting this problem, consider the matrix

$$\hat{\mathbf{R}}(n) = \mathbf{R}(n) * \mathbf{S}^H(-n) \tag{16}$$

Each term of this matrix is a sum of four individual convolution sequences. Define the vector $\mathbf{w}$ as

$$\mathbf{w} = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}^T \tag{17}$$

Now consider the 4×4 matrix $\mathbf{Y}_i$, given by

$$\mathbf{Y}_i(n) = \left(\mathbf{w}^T \otimes \mathbf{r}_i^T\right) \odot \mathbf{S}^H(-n) \tag{18}$$

where $\otimes$ and $\odot$ denote the Kronecker product and the Hadamard product, respectively, and $\mathbf{r}_i$ is the i^{th} row of $\mathbf{R}(n)$. Note that column j in $\mathbf{Y}_i(n)$ contains the individual convolution sequences that are summed up to yield the ij^{th} term in $\hat{\mathbf{R}}(n)$. In the case of no Doppler, and ignoring the noise, it is easy to verify that

$$\mathbf{w}^H \mathbf{Y}_i(n) = \gamma \mathbf{h}_i \delta(n) \tag{19}$$

where γ is just a scaling constant, and

$$\mathbf{h}_i = \begin{bmatrix} h_{1i} & h_{2i} & h_{3i} & h_{4i} \end{bmatrix} \tag{20}$$

The index i is associated with one of the receive antennas.

When Doppler is present, it is likewise easy to verify that

$$\mathbf{w}_D^H \mathbf{Y}_i(n) = \gamma \mathbf{h}_1 \delta(n) \tag{21}$$

where

$$\mathbf{w}_D = \begin{bmatrix} 1 & e^{j\upsilon} & e^{2j\upsilon} & e^{3j\upsilon} \end{bmatrix}^T \tag{22}$$

and this holds for all i. This means that for $n \neq 0$, the matrices $\mathbf{Y}_i$ are singular, and the vector producing the desired output lies in the null-space of these matrices. Also, because of the same waveform inputs, the matrices $\mathbf{Y}_i$ share the same null-space. Thus, we can form a concatenated matrix as

$$\mathbf{Y}_C(n) = \begin{bmatrix} \mathbf{Y}_1(n) & \mathbf{Y}_2(n) & \mathbf{Y}_3(n) & \mathbf{Y}_4(n) \end{bmatrix} \tag{23}$$

It is easy to verify that

$$\mathbf{w}_D^H \mathbf{Y}_C(n) = \gamma \mathbf{h} \delta(n) \tag{24}$$

where $\mathbf{h} = [\mathbf{h}_1\ \mathbf{h}_2\ \mathbf{h}_3\ \mathbf{h}_4]$. Now, since we don't know the true-target delay for which $\mathbf{Y}_C(n)$ is non-singular, we cannot simply find the null-space vector at every n. That is, an approach is needed to circumvent the fact $\mathbf{w}_D$ is not in the null-space of $\mathbf{Y}_C(n)$ at the true target delay. Again, WLOG the true target delay is assumed to be $n = 0$. The concatenated matrix $\mathbf{Y}_C(n)$ is formed to exploit the fact that the different matrices share the same null-space vector.

To counter the effect of $\mathbf{Y}_C(0)$ on the null-space, instead of working with a single chip interval, we take a length $2q+1$ lag window and form the matrix

$$\mathbf{X}_C(n) = \begin{bmatrix} \mathbf{Y}_C(n-q) & \ldots & \mathbf{Y}_C(n) & \ldots & \mathbf{Y}_C(n+q) \end{bmatrix} \tag{25}$$

The SVD of $\mathbf{X}_C$ can be written as

$$\mathbf{X}_C(n) = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^H \tag{26}$$

Since $\mathbf{X}_C(n)$ and $\mathbf{X}_C(n)\mathbf{X}_C^H(n)$ share the same singular vectors, we will work with the latter. The idea is to subtract out $\mathbf{Y}_C(0)$ in order to obtain the correct null space of $\mathbf{R}_{X_C X_C}(n)$. To do this, we compute the SVD of

$$\mathbf{R}_{X_C X_C}(n) - \mathbf{Y}_C(k)\mathbf{Y}_C^H(k) \tag{27}$$

for $n - q \leq k \leq n + q$ and store the singular vector associated with the smallest eigenvalue. Note that out of the $2q+1$ singular vectors that we store for each n, there is at most one singular vector that corresponds to $\mathbf{Y}_C(0)$ and this happens whenever $0 \in \{n-q, ..., n+q\}$. Again, WLOG the true target delay here is $n = 0$.

The inclusion of $\mathbf{Y}_C(0)$ alters the null-space structure. In order to find which matrix to subtract, since we don't know the true target delay, for each of the $2q+1$ singular vectors, we compute its inner product with the other $2q$ singular vectors, and choose that singular vector that yields the smallest inner product (magnitude) with the rest of the vectors.

This process is mathematically described as follows. Let $\mathbf{U}_{min}(n)$ be the matrix with the $2q+1$ singular vectors as columns. The inner product (Grammian) matrix is formed as

$$\mathbf{M}(n) = \mathbf{U}_{\text{min}}^H(n)\mathbf{U}_{\text{min}}(n) \tag{28}$$

We can write $\mathbf{M}(n)$ as

$$\mathbf{M}(n) = \begin{bmatrix} \mathbf{m}_{n-q} & \ldots & \mathbf{m}_{n+q} \end{bmatrix}^T \tag{29}$$

The index of the singular vector of interest is obtained as

$$k_{opt} = \arg\min_k \|\mathbf{m}_k\| \tag{30}$$

To check the presence of target in the delay bin n, we process the vector $\mathbf{Y}_C(n)$ as

$$z(n) = \left\| \mathbf{u}_{k_{opt}}^H(n)\mathbf{Y}_C(n) \right\|_F^2 \quad (31)$$

Ideally, the magnitude of $z(n)$ should exhibit a sharp peak at the true target delay and be near zero for all other values of n.

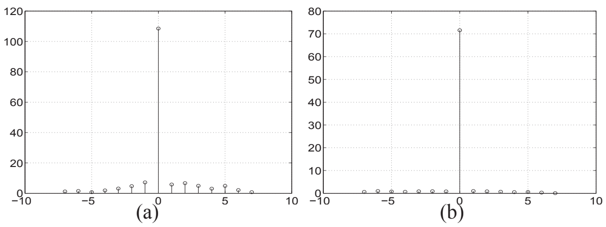

Fig. 3. Plot of $z(n)$ for a single target with Doppler shift of $\pi/3$ for (a) $SNR = 5dB$ and (b) $SNR = 10dB$

4. SIMULATION RESULTS

The simulation results show that, if the separation in chip intervals is greater than N_c or if the targets are moving with approximately the same speed with a separation greater than window length $2q + 1$, we are able to resolve targets moving at different speeds using our technique. We simulated a 4×4 system using Golay complementary sequences of length $N_c = 10$. The channel entries are i.i.d complex Gaussian with unit variance. The simulations results are plotted in Figure 3, 4 and 5 and we use a window length of $2q + 1 = 5$. Figure 3 shows a plot $z(n)$ for a single target with SNRs of 5dB and 10dB, respectively, with a Doppler shift of $\pi/3$. The system performance for the case when two targets introducing the same Doppler shift of $\pi/3$ and separated by more than window length is plotted in Figure 4. Figure 5 shows the contrast in system performance with the two targets moving at different velocities, introducing Doppler shifts of $\pi/3$ and $2\pi/3$ respectively while separated in time by more than the code length $N_c = 10$.

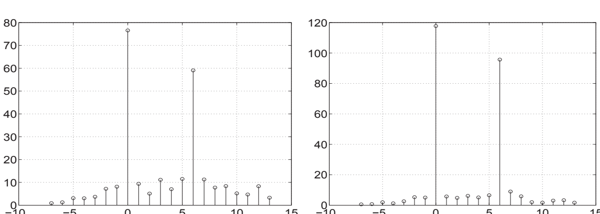

Fig. 4. Plot of $z(n)$ for two target with the same Doppler shift of $\pi/3$ for (a) $SNR = 5dB$ and (b) $SNR = 10dB$

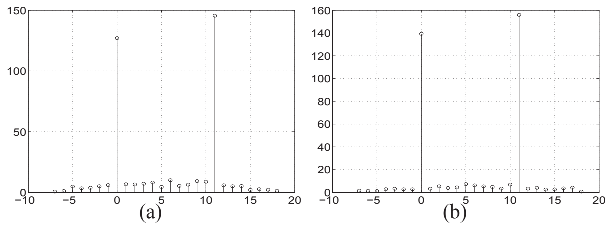

Fig. 5. Plot of $z(n)$ for two targets with respective Doppler shifts of $\pi/3$ and $2\pi/3$ for (a) $SNR = 5dB$ and (b) $SNR = 10dB$

5. CONCLUSIONS AND FUTURE WORK

We have developed a technique for doing accurate target ranging in the presence of Doppler using Golay complementary sequences. The technique is based on finding the null-space of the waveform matrix after matched filtering at the receiver, and then using an appropriate vector from the null-space to process the matched filtered received waveforms over multiple PRIs. Simulation results were presented that show how the proposed technique diminishes the effects of Doppler while still facilitating high-resolution, accurate target ranging over a range of target SNRs. Future work includes a theoretical analysis of the detection performance, and extending this technique for multiple targets in the same range cell with different radial velocities. The current technique can handle the latter as long as the targets are separated in time by at least the code length N_c.

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