Channel Estimation for MIMO-OFDM using Complementary Codes

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***Abstract*— We present a pilot-assisted method for estimating the frequency selective channel in a MIMO-OFDM (Multiple Input Multiple Output - Orthogonal Frequency Division Multiplexing) system. The pilot sequence is designed using the DFT (Discrete Fourier Transform) of the Golay complementary sequences. Novel exploitation of the perfect autocorrelation property of the Golay codes, in conjunction with OSTBC (Orthogonal Space-Time Block Code) based pilot waveform scheduling across multiple OFDM frames, facilitates simple separation of the channel mixtures at the receive antennas. The DFT length used to transform the complementary sequence into the frequency domain is shown to be a key critical parameter for correctly estimating the channel. NMSE (Normalized Mean Squared Error) between the actual and the estimated channel is used to characterize the estimation performance.**

***Index Terms*— MIMO, OFDM, OSTBC, DFT, Golay complementary sequences**

I. Introduction

MIMO-OFDM (Multiple Input Multiple Output - Orthogonal Frequency Division Multiplexing) systems provide performance gains because they combine the diversity and multiplexing gains of MIMO [1], [2] with the resilience of OFDM[3] against multi-path fading. In order to achieve these performance improvements, accurate CSI (Channel State Information) is required at the receiver. One way to get CSI at the receiver is to insert known pilot symbols in the transmitted signal that sample the multi-path channel. At the receiver, these samples are then used to obtain a representation of the entire channel through linear or higher order interpolation.

The difficulty in channel estimation for multi-antenna systems lies in the fact that at every receive antenna, we get a signal coming from different transmit antennas, and hence different channels. Therefore, we need to estimate the individual channels given mixtures of the channels. A technique for estimating the channel in a 2×1 transmit diversity system has been proposed in [4], which is an iterative technique and it requires the initialization of the channel estimates by sending the complementary codes sequences – no data – during the first two OFDM symbol periods followed by a successive interference cancelation procedure that kind of goes back and forth between data estimation and channel estimation. In this paper, we propose a simpler and computationally efficient method to obtain the sampled channel estimates using pilots without any need for initialization symbols or extra processing at the receiver. Our method uses OSTBC (Orthogonal Space-Time Block Coded) transformed Golay complementary sequences [5] to design pilots in the frequency domain. A lower bound on the number of pilots needed to estimate the frequency selective channel is derived and it has been shown that this channel estimation strategy can be used with transmit and receive antenna arrays of arbitrary sizes.

II. Pilot Design and Channel Estimation

We consider a 2×2 MIMO-OFDM system with N sub-carriers of which N_p are pilot sub-carriers. Each pair of transmit-receive antennas encounters a length L multi-path channel. We will assume that the channel is slowly fading and is quasi-static over $n \geq 2$ consecutive symbols. The system cyclically extends every OFDM symbols before transmission by appending the trailing N_g samples to the beginning of the symbol. We assume $N_g \geq L$ so that all the sub-carriers remain mutually orthogonal at the receiver [3]. The N_p point pilot sequence is designed in the frequency domain. We present a development without considering the effects of receiver noise and will later characterize the noisy estimation performance using the NMSE (Normalized Mean Squared Error) criterion.

A. Golay Complementary Sequences

A pair of sequences $s_1[n]$ and $s_2[n]$ of length N_c satisfy the Golay property if the sum of their autocorrelation functions satisfy

$$R_{s_1 s_1}[l] + R_{s_2 s_2}[l] = \begin{cases} 2N_c & if \quad l = 0 \\ 0 & if \quad l \neq 0 \end{cases} \tag{1}$$

for $l = -N_c - 1, ..., N_c - 1$. These sequences were first introduced by M. J. E. Golay in [5] and they can be constructed for length $2^N 10^K 26^M$ with $N, K, M \in \{\mathrm{N} \cup \{0\}\}$. If we take the DFT of the above equation, we get

$$|S_1[k]|^2 + |S_2[k]|^2 = 2N_c \tag{2}$$

This equation will play a fundamental role in our estimation scheme, as we now see.

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B. Frequency Domain Pilot Design

The pilot sequence is designed in the frequency domain using the DFT of complementary sequences. The N_p point DFT of the complementary sequence is given by

$$\tilde{E}_i[k] = DFT_{N_p}\{e_i[n]\} \tag{3}$$

We use $\tilde{E}[k]$ to emphasize the fact that this represents the pilot sequence with $k = 0, 1, ..., N_p$ and is the sequence carried by the pilot sub-carriers, as shown in Figure 1. The advantage of designing the pilot sequence in the frequency domain lies in the fact that the sequence is carried by orthogonal sub-carriers. The relative magnitudes of these sub-carriers change because each sub-carrier, upon passing through the channel, is multiplied by the corresponding value of the channel frequency response at that frequency. However, with the assumption that the guard interval N_g is longer than the maximum length of the sampled channel impulse response L, the sub-carriers still remain orthogonal at the receiver, and we can recover the pilot sequence without any interference from the data.

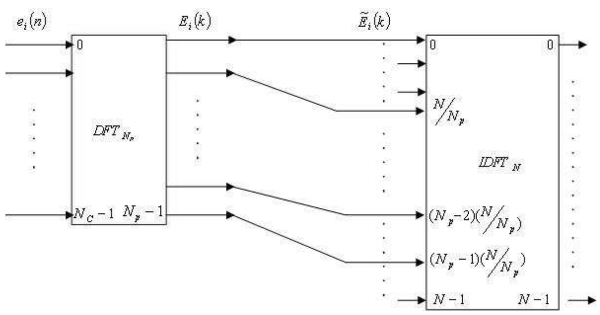

Fig. 1. Pilot Sequence Generation and Mapping

If $H_{ij}[k]$ represents the N-point DFT of the sampled channel impulse response between i^{th} transmit antenna and the j^{th} receive antenna, we define the sampled frequency response at the pilot frequencies as

$$\tilde{H}_{ij}[k] = H_{ij}[kN/N_p] \qquad k = 0, 1, ..., N_p - 1 \tag{4}$$

The received sequence, after removing the cyclic prefix and taking the DFT, can be written as

$$\mathbf{R}[k] = \left[\begin{array}{cc} \mathbf{Y}_1[k] & \mathbf{Y}_2[k] \end{array} \right] \tag{5}$$

where

$$\begin{aligned} \mathbf{Y}_1[k] &= \left[\begin{array}{c} Y_{11}[k] \\ Y_{21}[k] \end{array} \right] \\ &= \left[\begin{array}{cc} H_{11}[k] & H_{21}[k] \\ H_{12}[k] & H_{22}[k] \end{array} \right] \left[\begin{array}{c} E_1[k] \\ E_2[k] \end{array} \right] \end{aligned} \tag{6}$$

and

$$\begin{aligned} \mathbf{Y}_2[k] &= \left[\begin{array}{c} Y_{12}[k] \\ Y_{22}[k] \end{array} \right] \\ &= \left[\begin{array}{cc} H_{11}[k] & H_{21}[k] \\ H_{12}[k] & H_{22}[k] \end{array} \right] \left[\begin{array}{c} -E_2^*[k] \\ E_1^*[k] \end{array} \right] \end{aligned} \tag{7}$$

We know that the data sub-carriers do not play any part in the channel estimation because they are orthogonal to the pilot sub-carriers. Therefore, we only consider the sub-carrier indices pertaining to the pilot sub-carriers, i.e.

$$\tilde{Y}_i[k] = Y_i[kN/N_p] \qquad \tilde{k} = 0, 1, ..., N_p - 1 \tag{8}$$

With this information, the received matrix can also be written as

$$\begin{aligned} \tilde{\mathbf{R}}[k] &= \left[\begin{array}{cc} \tilde{\mathbf{Y}}_1[k] & \tilde{\mathbf{Y}}_2[k] \end{array} \right] \\ &= \tilde{\mathbf{H}}[k]\tilde{\mathbf{E}}[k] \end{aligned} \tag{9}$$

where

$$\tilde{\mathbf{E}}[k] = \left[\begin{array}{cc} \tilde{E}_1[k] & -\tilde{E}_2^*[k] \\ \tilde{E}_2[k] & \tilde{E}_1^*[k] \end{array} \right] \tag{10}$$

In order to separate the different channels at the receiver, we need to process the receiver waveform with a matrix $\mathbf{A}$ such that

$$\tilde{\mathbf{R}}[k]\mathbf{A} = \alpha\tilde{\mathbf{H}}[k] \tag{11}$$

This implies that we need

$$\tilde{\mathbf{E}}[k]\mathbf{A} = \alpha\mathbf{I} \tag{12}$$

This can be achieved by exploiting (2), from which we can directly infer that $\mathbf{A} = \tilde{\mathbf{E}}^H[k]$ gives the desired result, since

$$\begin{aligned} \tilde{\mathbf{E}}[k]\tilde{\mathbf{E}}^H[k] &= \left[\begin{array}{cc} \tilde{E}_1[k]\tilde{E}_1^*[k] + \tilde{E}_2^*[k]\tilde{E}_2[k] & \tilde{E}_1[k]\tilde{E}_2^*[k] - \tilde{E}_2^*[k]\tilde{E}_1[k] \\ \tilde{E}_2[k]\tilde{E}_1^*[k] - \tilde{E}_1^*[k]\tilde{E}_2[k] & \tilde{E}_2[k]\tilde{E}_2^*[k] + \tilde{E}_1^*[k]\tilde{E}_1[k] \end{array} \right] \\ &= \alpha\mathbf{I} \end{aligned} \tag{13}$$

Therefore, we have that

$$\tilde{\mathbf{R}}[k]\tilde{\mathbf{E}}^H[k] = \alpha \left[\begin{array}{cc} \tilde{H}_{11}[k] & \tilde{H}_{21}[k] \\ \tilde{H}_{12}[k] & \tilde{H}_{22}[k] \end{array} \right] \tag{14}$$

As we can see from these equations, the complementary sequences allow us to estimate all the different channels at the pilot sub-carrier frequencies without incurring any extra computational overhead. The noisy channel estimates can be represented as

$$\hat{\tilde{H}}_{ij}[k] = \tilde{H}_{ij}[k] + \tilde{n}[k] \tag{15}$$

Where $\tilde{n}[k]$ represents the noise sequence at the sub-carrier frequencies. In order to obtain the actual channel impulse response $\tilde{h}_{ij}[n]$ via IDFT of the estimated sampled channel frequency response $\hat{\hat{H}}_{ij}[k]$,i.e.

$$\hat{\hat{h}}_{ij}[n] = IDFT_{N_p}\left\{\hat{H}_{ij}[k]\right\} \quad (16)$$

certain conditions need to be met, as we now explain.

C. DFT Length

We saw in (9) that the channel estimation involves a product of three terms: $S_i[\tilde{k}]$, $H_{ij}[\tilde{k}]$, and $S_j^*[\tilde{k}]$. This operation represents a three-fold circular convolution in the time domain, i.e.

$$\tilde{S}_i[k]\tilde{H}_{ij}[k]\tilde{S}_j^*[k] = DFT\left\{s_i[n] \circledast h_{ij}[n] \circledast s_j^*[-n]\right\} \quad (17)$$

where $s_i[n]$ and $s_j^*[n]$ are length N_c sequences and $h_{ij}[n]$ has length L. We know that if two sequences of length M and Q are linearly convolved, the resulting sequence is of length $M+Q-1$. In order to get the same sequence through DFT processing, we need to take the $M+Q-1$ point DFT of both the sequences (by zero padding the two sequences to make them of length $M+Q-1$), multiply the DFTs together, and take the IDFT to get the $M+Q-1$ point linear convolution. This idea can be applied to our three-fold convolution by choosing the initial DFT length to be at least $2N_c+L-2$. However, since convolution satisfies both commutativity and associativity, we can first focus our attention on the convolution of the complementary sequences. The sum of the $2N_c-1$ point linear convolution of the complementary sequences is a Dirac delta function delayed by N_c-1, i.e. $\delta[n-N_c-1]$. The convolution of channel with this function is

$$h_{ij}[n] * \delta[n-N_c-1] = h_{ij}[n-N_c-1] \quad (18)$$

From this, we see that the resulting three-fold convolution is the sampled channel impulse response delayed by N_c-1. Note that this resulting $2N_c+L-2$ point sequence has N_c-1 zeros before and after the L point channel sequence. This tells us that if we were to reduce the initial DFT length to N_c+L-1, this would have the effect of aliasing the trailing N_c-1 zeros in the three-fold linear convolution with the leading N_c-1 zeros. What we get is an N_c+L-1 point sequence which represents the actual sampled channel response delayed by N_c-1. We can use the fact that this N_c+L-1 sequence still has N_c-1 zeros to further reduce the initial DFT length by observing that if we reduce the initial DFT length to N_c, the resulting three fold convolution would be the initial N_c-1 zeros being aliased with the values of the sampled channel response, which can also be thought of as a circular shift of the channel response. Since we know N_c and the maximum channel length L, we know the amount of circular shift present in the estimated channel, and we can circularly shift it back to get the actual channel response. Therefore, to estimate the channel correctly, we can establish a relationship between the number of pilots N_p, which also represents the initial DFT length used to transform the complementary sequences into the frequency domain, the length of the complementary sequences N_c, and maximum length of the sampled channel impulse response L, given by

$$N_p \geq N_c \geq L \quad (19)$$

Note that pilots represent an overhead in a communication system, and therefore, we need to minimize the number of pilots needed to achieve a desired performance level and the minimum number of pilots required by our scheme is L provided that we have $N_c = L$.

III. Simulation Results

We consider an OFDM system with $N = 256$ sub-carriers and $N_p = 16$ equi-spaced pilot sub-carriers. The channel is a unit variance Rayleigh fading channel with uniform power delay profile and $L = 5$ taps. We use real valued complementary sequence of length $N_c = 10$, given by

$$\begin{aligned} s_1[n] &= \{1, 1, -1, 1, -1, 1, -1, -1, 1, 1\} \\ s_2[n] &= \{1, 1, -1, 1, 1, 1, 1, 1, -1, -1\} \end{aligned} \quad (20)$$

Our performance measure is the $NMSE$ between the actual channel impulse response and the estimated channel response, i.e.

$$J_{NMSE}(SNR_{ave}) = \frac{\sum_{l=0}^{L-1}\left|h_{ij}[l] - \hat{h}_{ij}[l]\right|^2}{\sum_{l=0}^{L-1}|h_{ij}[l]|^2} \quad (21)$$

SNR_{ave} is the received SNR averaged over the two antennas and symbol periods given by

$$SNR_{ave} = \frac{1}{4}\sum_{i=1}^{2}\sum_{j=1}^{2} SNR_{ij} \quad (22)$$

Where SNR_{ij} is the SNR at the i^{th} receive antenna in the j^{th} symbol interval. We showed in the previous section that we can get the exact channel estimates in the case where there is no noise in the system. Therefore, the performance is limited only by the receiver noise. Figure 2 shows the $NMSE$ plotted against SNR_{ave}. The $NMSE$ decreases monotonically as the receiver SNR increases, thereby validating our claim that the estimation performance is limited only by the receiver noise. In Figure 3, we show the estimated and the actual channel impulse response for the channel from the transmit antenna 1 to receive antenna 1 in the case where there is no noise and we have $N_p \geq N_c + L$. The impulse responses are exactly the same, except that the estimated channel response is delayed by $N_c - 1$ samples. This happens because the autocorrelation on the complementary sequences is a delta function with a delay of $N_c - 1$, and since the estimation

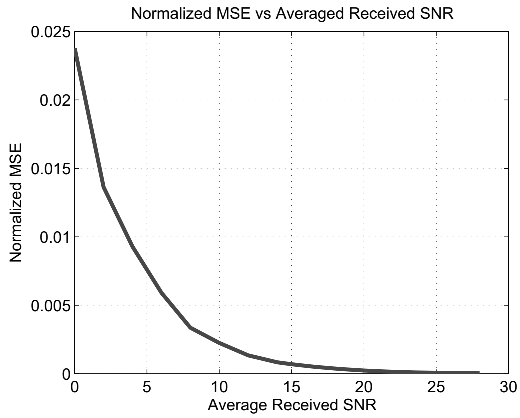

Fig. 2. Plot of NMSE against the average SNR

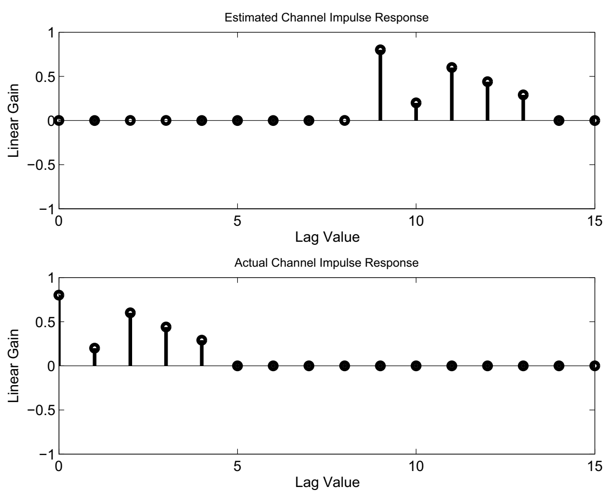

Fig. 3. Estimated and Actual CIR

of the channel impulse response involves the convolution of the actual channel response with the autocorrelation function of the complementary sequences, we observe a delay of $N_c - 1$ samples. In Figure 4, the estimated and the actual channel impulse responses are shown for the case where $N_p < N_c + L - 1$. Specifically, we have $N_p = 16$, $N_c = 10$, and $L = 10$ and there is no noise. We can see from this figure that the estimated channel impulse response is a time-domain aliased version of the actual channel impulse response, where the time-domain aliasing manifests itself as a circular shift of the channel impulse response. However, since we have $N_p \geq N_c$, if we left shift the sequence by $N_c + L - 1 - N_p = 3$, we have the same situation as the one shown in Figure 2, where the estimated channel is just a delayed version of the actual channel.

IV. Conclusions

We have introduced a new channel estimation technique that uses transformed Golay complementary sequences

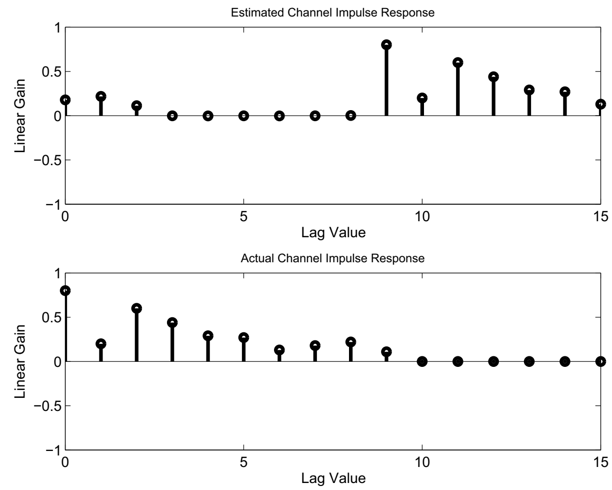

Fig. 4. Estimated and Actual CIR for $N_p < N_c + L - 1$

based pilot waveforms. The OSTBC based scheduling of these waveforms facilitates simple separation of the channel mixtures in a MIMO environment when certain constraints on the DFT size used to transform the complementary sequences into the frequency domain are met. In systems where the maximum sampled channel length does not exceed the cyclic prefix, it has been shown that the performance of this scheme is limited only by the receiver noise if we assume perfect timing and frequency synchronization. Simulation results have been provided to confirm the analytical results.

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