A NOVEL NONLINEAR TECHNIQUE FOR SIDELOBE SUPPRESSION IN RADAR

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Abstract

Sidelobes can be completely removed if the returns from two pulses based on Golay complementary sequences are combined after pulse compression. However these pulses cannot be sent simultaneously via frequency separation because of a phase term which is dependent on the unknown range. We propose to transmit one of the sequences at two offsets, above and below the carrier frequency. This facilitates recovery of the square of the autocorrelation function independently of signal phase. A modification to the Golay pair results in sequences having complementary squares of autocorrelation sequences. This enables complementary behaviour with frequency offset pulse pairs. The nonlinear squaring operation introduces small manageable cross–terms in place of sidelobes.

1 Introduction

Modern radars employ pulse compression techniques in order to achieve acceptable range resolution with limited energy transmitters. The received returns are match–filtered with the transmitted pulse shape, causing returns to be compressed in range. Ideally the filter output would be an impulse for each return. However this is impossible. Much research has thus focussed on finding ways to minimise the size of sidelobes in compressed returns.

One method of managing sidelobes is to modulate the radar pulse with a code having favourable autocorrelation properties. Binary codes (like the Barker code [1]) and polyphase codes based on frequency chirps (Frank codes and P codes [2, 5]) are frequently used. Another method of sidelobe reduction is to employ amplitude–weighting in the filter (eg [3]). However this results in the filter being mismatched with the transmitted signal, thereby becoming suboptimal. Another method involves post–processing the filter output with a delay–and–sum circuit [4, 7]. This has the effect of minimising peak sidelobe level at the expense of widening the main peak.

While an ideal filter output cannot be achieved with a single code sequence, it can be achieved through the use of two sequences whose autocorrelations sum to an impulse function. Such pairs of sequences were introduced by Marcel Golay and are referred to as a Golay complementary pair [6]. A radar pulse can be modulated with the first sequence of a Golay pair, and a subsequent pulse by the second sequence. Sidelobes will cancel out when the two pulse compressed returns are combined. The main drawback is that one is forced to wait for an entire pulse interval before obtaining the result in order for the two pulses to be transmitted and processed.

This study demonstrates that a Golay pair may not be transmitted simultaneously via frequency separation without destroying their complementarity. A modification to the Golay pair which results in sequences with complementary squares of autocorrelations is proposed. A method of multiplexing these sequences in frequency so that they might be transmitted simultaneously while preserving complementarity is described. This technique achieves complete suppression of sidelobes, thereby performing better than any single–sequence pulse compression regime, in a much shorter time than is necessary with time–separation of a Golay code pair.

2 Use of Golay codes as radar pulses

2.1 Review of complementary codes

Two discrete sequences $p_1(n)$ and $p_2(n)$ having length N are termed *complementary* if

$$R_{p_1}(k) + R_{p_2}(k) = 2N\delta(k) \qquad (1)$$

Let $p_1(n)$ and $p_2(n)$ be a pair of binary complementary Golay sequences of length N. The following properties hold.

$$\begin{aligned} R_{p_1}(k) + R_{p_2}(k) &= 2N,\ k = 0 &(2)\\ R_{p_1}(k) &= -R_{p_2}(k),\ k \neq 0 &(3)\\ R_{p_1}(k)^2 &= R_{p_2}(k)^2 &(4) \end{aligned}$$

where $R(k)$ denotes the autocorrelation sequence. For Golay sequences of length $N = 2^n$ constructed in a standard manner (eg via the PONS method [8]) then the following also holds.

$$R_{p_1}(2k) = R_{p_2}(2k) = 0 \ \forall k \neq 0 \qquad (5)$$

2.2 Signal model

If $p(n)$ is an arbitrary sequence of length N, define

$$s_p(t) = \sum_{n=0}^{N-1} p(n)\Omega\left(\frac{1}{T}t - n - \frac{1}{2}\right) \qquad (6)$$

where T is the chip length and $\Omega(t)$ is an arbitrary pulse–shaping function. This function should have most of its power in the interval $(-0.5, 0.5)$.

$$\int_{-0.5}^{0.5} \Omega(t)^2 \; dt \quad \approx \quad 1 \tag{7}$$

The continuous autocorrelation of $s_p(t)$ is thus

$$R_s(\tau) = \int_{-\infty}^{\infty} s(t)s^*(t-\tau)\,dt \tag{8}$$

$$= \int_{-\infty}^{\infty} \sum_{n=0}^{N-1}\sum_{m=0}^{M-1} p(n)p^*(m) \times \Omega\left(\frac{t}{T}-n-\frac{1}{2}\right)\Omega\left(\frac{t+\tau}{T}-m-\frac{1}{2}\right) dt \tag{9}$$

$$= \sum_{k=-\infty}^{\infty}\sum_{n=0}^{N-1} p(n)p(n-k)T\gamma(k,\tau) \tag{10}$$

where $\gamma(k,\tau)$ is the integral

$$\gamma(k,\tau) = \int_{-\infty}^{\infty} \Omega(t)\Omega(t+k-\frac{\tau}{T})\,dt \tag{11}$$

Note that the sum over m has changed to a sum over all possible lags k, where $k = n-m$. It follows that

$$R_s(\tau) \approx T \sum_{k=-\infty}^{\infty} \Pi\left(\frac{1}{2}(k-\frac{\tau}{T})\right)\gamma(k,\tau)R_p(k) \tag{12}$$

Where $\Pi(x)$ is the rectangle function

$$\Pi(x) = \begin{cases} 1 & \text{if } |x| < \frac{1}{2} \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

For any given value of τ there will be at most two terms for which $\Pi(\cdot) \neq 0$,

$$k_1(\tau) = \left\lceil \frac{\tau}{T} \right\rceil - 1 \quad \text{and} \quad k_2(\tau) = \left\lceil \frac{\tau}{T} \right\rceil \tag{14}$$

So eq. (12) could be rewritten as

$$R_s(\tau) = T\gamma(k_1(\tau),\tau)R_p(k_1(\tau)) + T\gamma(k_2(\tau),\tau)R_p(k_2(\tau)) \tag{15}$$

2.3 Offset frequency modulation

Earlier studies have assumed that Golay sequences can be transmitted separately and recovered via matched filtering [8]. However we demonstrate here that it is not possible to recover complementary sequences that have been separated in frequency.

Assume that $p_1(n)$ and $p_2(n)$ form a Golay complementary pair. Further assume that each is used to encode a radar pulse in the manner described above, yielding $s_1(t)$ and $s_2(t)$ respectively. The two signals may be multiplexed by offsetting the second signal in frequency by an amount ω_b which is at least equal to its bandwidth. This is added to the first signal, carrier modulated at ω_c and transmitted.

$$\tilde{s}(t) = \left(s_1(t) + s_2(t)e^{i\omega_b t}\right)e^{i\omega_c t} \tag{16}$$

This signal is returned from a reflector at delay d with an attenuation factor of a, producing the returned signal $\tilde{y}(t)$

$$\tilde{y}(t) = a\tilde{s}(t-d) \tag{17}$$

$$= as_1(t-d)e^{i\omega_c(t-d)} + as_2(t-d)e^{i(\omega_c+\omega_b)(t-d)} \tag{18}$$

The two components can be downmixed and lowpass filtered to produce the following received signals

$$y_1(t) = as_1(t-d)e^{-i\omega_c d} \tag{19}$$

$$y_2(t) = as_2(t-d)e^{-i(\omega_c+\omega_b)d} \tag{20}$$

The cross–correlations of these with the transmitted signals are

$$R_{s_1y_1}(\tau) = ae^{-i\omega_c d}R_{s_1}(\tau-d) \tag{21}$$

$$R_{s_2y_2}(\tau) = ae^{-i(\omega_c+\omega_b)d}R_{s_2}(\tau-d) \tag{22}$$

and their sum is

$$R_{s_1y_1}(\tau) + R_{s_2y_2}(\tau) = ae^{-i\omega_c d}\left(R_{s_1}(\tau-d) + R_{s_2}(\tau-d)e^{-i\omega_b d}\right) \tag{23}$$

It is evident that complementary behaviour will occur only when $e^{-i\omega_b \tau} = 1$. This will be the case only if

$$\tau = \frac{2\pi m}{\omega_b} \tag{24}$$

where m is some integer. Since a target may lie at any range, complementary behaviour cannot be guaranteed.

3 Achieving complementary behaviour

3.1 Equal and opposite frequency offsets

What prevents the successful recovery of a frequency–offset Golay pair is the phase term on the second signal which is a function of the offset frequency and the unknown range. Observe that when a complex number is multiplied by its conjugate then the result is real–valued; the phasors cancel. Further observe that if the second signal $s_2(n)$ had been frequency–offset by the same amount in the negative direction then the resulting demodulated signal would be the conjugate (with respect to the frequency–offset phase term only) of eq. (20).

$$\tilde{y}_{2b}(n) = as_2(t-d)e^{-i(\omega_c-\omega_b)\tau} \tag{25}$$

Taking the cross–correlations of y_2 and y_{2b} with s_2 and multiplying together, the offset phase terms cancel out.

$$R_{s_2y_2}(k) \times R_{s_2y_{2b}}(k) = a^2e^{-i2\omega_c d}R_{s_2}^2(\tau-d) \tag{26}$$

However the square of the autocorrelation has been obtained rather than the autocorrelation itself. One can thus recover the autocorrelation function up to the sign of the terms.

3.2 Modification of one member of a Golay pair

We desire a pair of sequences such that $R_p^2(k) + R_q^2(k) = C\delta(k)$. For this to occur it is necessary that at least one of the R^2 functions be negative at some values of k. This is possible only if the underlying sequence has imaginary components.

Let $p_1(n)$ and $p_2(n)$ be a Golay complementary pair. Define a sequence q_2 in terms of p_2 as

$$q_2(n) = p_2(n)e^{i\frac{\pi}{2}n} \tag{27}$$

Then

$$R_{q_2}(k) = R_{p_2}(k)e^{i\frac{\pi}{2}k} \tag{28}$$

and

$$R^2_{q_2}(k) = R^2_{p_2}(k)e^{i\pi k} \tag{29}$$

$$= \begin{cases} -R^2_{p_1}(k) & \text{if k is odd} \\ 0 & \text{if k is even, } k \neq 0 \\ R^2_{p_1}(k) & \text{if } k = 0 \end{cases} \tag{30}$$

and thus

$$R^2_{q_2}(k) + R^2_{p_1}(k) = 2N^2\delta(k) \tag{31}$$

thus p_1 and q_2 may be taken as the desired code pair.

3.3 Illustration

Figure 1 shows two complementary binary sequences and their autocorrelation functions. It is evident from inspection that the autocorrelations are complementary. The p_2 code is modified

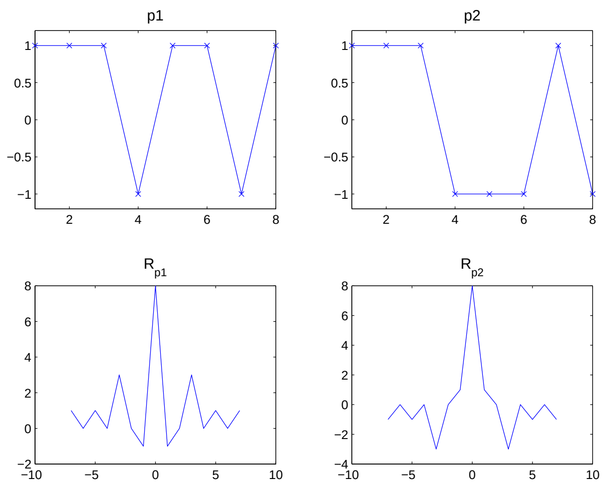

Figure 1: Complementary codes & their autocorrelations

by multiplication with a complex phasor sequence to produce code q_2, illustrated in figure 2. Here complex values are represented as two sequences, real parts (solid line and crosses) and imaginary parts (dashed line and circles). The autocorrelations of p_1 and q_2 are plotted in figure 3. Note that the autocorrelation of q_2 is imaginary at all locations except $k = 0$. This is due to the complex phasor term being imaginary at odd–valued lags. Since the autocorrelation of p_2 was zero at all even–valued lags except $k = 0$, there are no real–valued terms apart from at zero lag. Because of this the squares of the autocorrelation values at $k \neq 0$ are all negative. Being formed from the complementary Golay sequence p_2, their magnitudes are identical to those of the p_1 sequence. Therefore on summation they cancel completely, leaving only the zero–lag term.

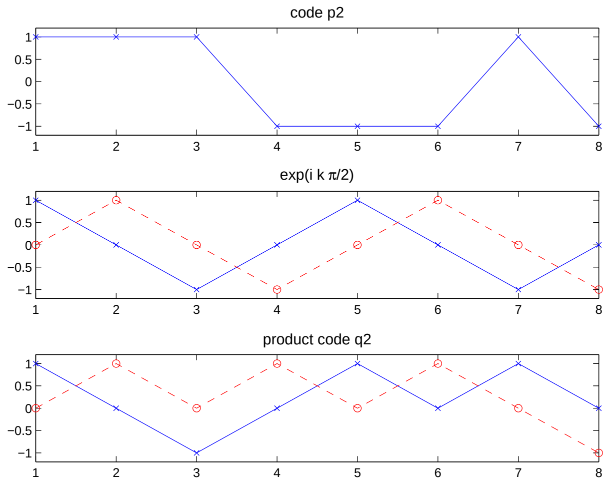

Figure 2: Construction of q_2 from p_2 and a complex sequence

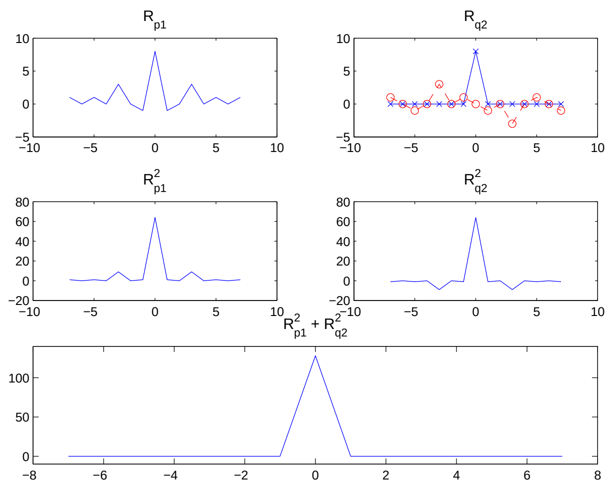

Figure 3: Complementarity of $R^2_{p_1}$ & $R^2_{p_2}$

3.4 Use of modified Golay pair as radar signals

Consider a modified Golay pair p_1 & q_2 which is used to modulate a pulse sequence. The first code in the pair is transmitted at carrier. The second code is transmitted twice, offset an equal amount both above and below carrier. The received signal is demodulated and separated into its components, giving

$$y_1(t) = ae^{-i\omega_c d}s_1(t-d) \tag{32}$$

$$y_{2a}(t) = ae^{-i(\omega_c+\omega_b)d}s_2(t-d) \tag{33}$$

$$y_{2b}(t) = ae^{-i(\omega_c-\omega_b)d}s_2(t-d) \tag{34}$$

The square of the cross–correlation of y_1 with s_1 is

$$R^2_{s_1y_1}(\tau) = a^2e^{-i2\omega_c d}R^2_{s_1}(\tau-d) \tag{35}$$

The product of correlations of the two offset signals with s_2 is

$$R_{s_2y_{2a}}(\tau) \times R_{s_2y_{2b}}(\tau) = a^2e^{-i2\omega_c d}R^2_{s_2}(\tau-d) \tag{36}$$

The sum of these is

$$\Upsilon(\tau) = R^2_{s_1y_1}(\tau) + R_{s_2y_{2a}}(\tau) \times R_{s_2y_{2b}}(\tau)$$

$$
\begin{aligned}
&= a^2 e^{-i2\omega_c d}\left(R^2_{s_1}(\tau-d)+R^2_{s_2}(\tau-d)\right) && (37)\\
&= T^2 a^2 e^{-i2\omega_c d}\left[\gamma^2(k_1)2N^2\delta(k_1)+\right.\\
&\quad \gamma^2(k_2)2N^2\delta(k_2)+2N\gamma(k_1)\gamma(k_2)R_p(1)\times\\
&\quad \left.((1+i)\delta(k_2)+(1-i)\delta(k_1))\right] && (38)
\end{aligned}
$$

where the τ argument is omitted from $\gamma(k)$, k_1 and k_2 for convenience. This is clearly zero except when $k_1=0$ or $k_2=0$.

In the following example a modified Golay pair is upsampled by a factor of 5. The continuous squared autocorrelation sequences are computed and plotted in figure 4. Also plotted is the sum of the squared autocorrelations. Observe that the sum

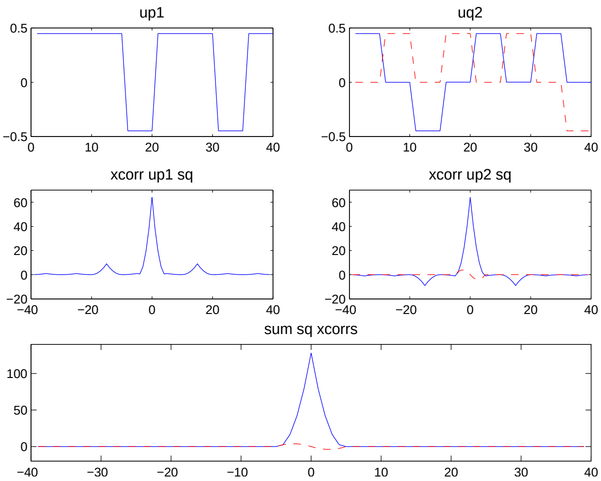

Figure 4: Complementarity of continuous signals

is zero at delays greater than one chip length from zero lag, and that the function has one lobe which is maximised at zero lag.

4 Cross terms caused by two returns

Now assume that there are two returns with delays d_1 and d_2, $|d_1-d_2|<2NT$. The separated return signals are

$$
\begin{aligned}
y_1(t) &= ae^{-i\omega_c d_1}s_1(t-d_1)+\tilde{a}e^{-i\omega_c d_2}s_1(t-d_2) && (39)\\
y_{2a}(t) &= ae^{-i(\omega_c+\omega_b)d_1}s_2(t-d_1)+\\
&\quad \tilde{a}e^{-i(\omega_c+\omega_b)d_2}s_2(t-d_2) && (40)\\
y_{2b}(t) &= ae^{-i(\omega_c-\omega_b)d_1}s_2(t-d_1)+\\
&\quad \tilde{a}e^{-i(\omega_c-\omega_b)d_2}s_2(t-d_2) && (41)
\end{aligned}
$$

The cross–correlations of these with the transmitted signal are

$$
\begin{aligned}
R_{s_1y_1} &= ae^{-i\omega_c d_1}R_{s_1}(\tau-d_1)+\tilde{a}e^{-i\omega_c d_2}R_{s_1}(\tau-d_2) && (42)\\
R_{s_2y_{2a}} &= ae^{-i(\omega_c+\omega_b)d_1}R_{s_2}(\tau-d_1)+\\
&\quad \tilde{a}e^{-i(\omega_c+\omega_b)d_2}R_{s_2}(\tau-d_2) && (43)\\
R_{s_2y_{2b}} &= ae^{-i(\omega_c-\omega_b)d_1}R_{s_2}(\tau-d_1)+\\
&\quad \tilde{a}e^{-i(\omega_c-\omega_b)d_2}R_{s_2}(\tau-d_2) && (44)
\end{aligned}
$$

Note that $R_{s_{2a}}=R_{s_{2b}}$ and is written above as R_{s_2}.

Computing the sum of squared cross–correlations,

$$
\begin{aligned}
\Upsilon(\tau) &= R^2_{s_1y_1}(\tau)+R_{s_2y_{2a}}(\tau)R_{s_2y_{2b}}(\tau) && (45)\\
&= a^2e^{-i2\omega_c d_1}\left(R^2_{s_1}(\tau-d_1)+R^2_{s_2}(\tau-d_1)\right)+\\
&\quad \tilde{a}^2e^{-i2\omega_c d_2}\left(R^2_{s_1}(\tau-d_2)+R^2_{s_2}(\tau-d_2)\right)+\\
&\quad 2a\tilde{a}e^{-i\omega_c(d_1+d_2)}\chi(\tau) && (46)
\end{aligned}
$$

where $\chi(\tau)$ is dependent upon cross terms,

$$
\begin{aligned}
\chi(\tau) &= R_{s_1}(\tau-d_1)R_{s_1}(\tau-d_2)+\\
&\quad R_{s_2}(\tau-d_1)R_{s_2}(\tau-d_2)\cos\omega_b(d_1-d_2) && (47)
\end{aligned}
$$

The first two terms of $\Upsilon(\tau)$ are due to the targets at delay d_1 and d_2. Each will cause a single main lobe as described by eq. (38). The third term is due to cross terms in the square operation. Where this is zero depends upon exact values of d_1, d_2 and ω_b. It can be shown that the cross terms disappear at chip intervals when the delay $\delta_d=d_2-d_1$ is a whole number of chips, provided that ω_b is chosen such that

$$2T\omega_c=(2k+1)\pi \quad (48)$$

for some integer k. When δ_d is not a whole number of chips, ω_b may be chosen in order to force the cosine in $\chi(\tau)$ to be ±1 for a given value of δ_d, enabling control of cross terms based on hypothetical distance.

$$\omega_b=\left[\arccos(\mathrm{sign}(\cos(\pi\delta_d(2T)^{-1})))+k2\pi\right]\delta_d^{-1} \quad (49)$$

4.1 Example

$\Upsilon(\tau)$ has been computed for two returns of equal power, using a modified Golay pair of 8 chips and using $\Pi(t)$ as the pulse–shaping function. Individual terms of Υ for a "poor case" combination of d_1, d_2 and ω_b are presented in figure 5. Observe that

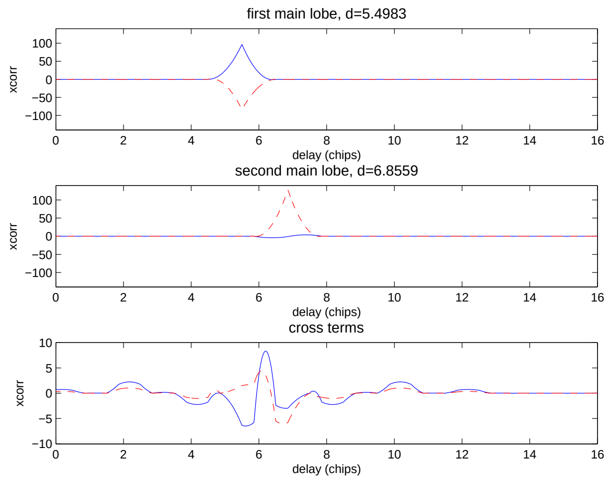

Figure 5: Individual terms of Υ, 2 returns of equal power

much of the energy in the cross terms occurs in the vicinity of either target lobe. Examining the magnitude of the sum of these

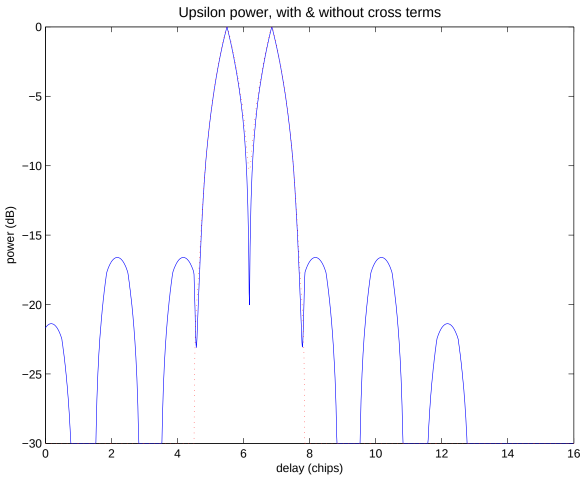

Figure 6: Magnitude of Υ, 2 returns of equal power

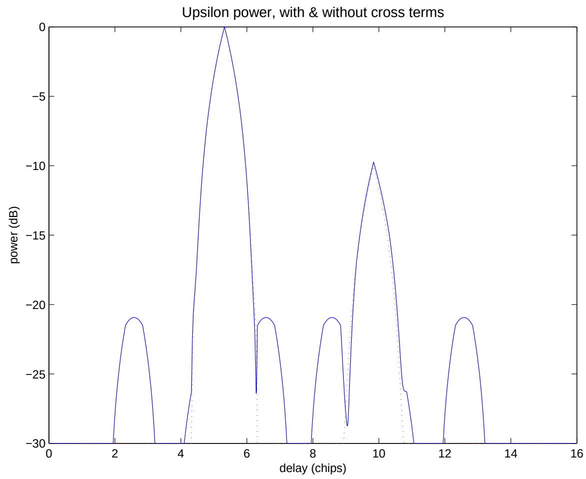

Figure 8: Magnitude of Υ, 2 returns of different power

terms (fig. 6) one sees that these cross terms cause small peaks away from the main lobes.

Another "poor case" scenario in which the secondary target has a tenth of the power of the first is presented in figure 7. The size

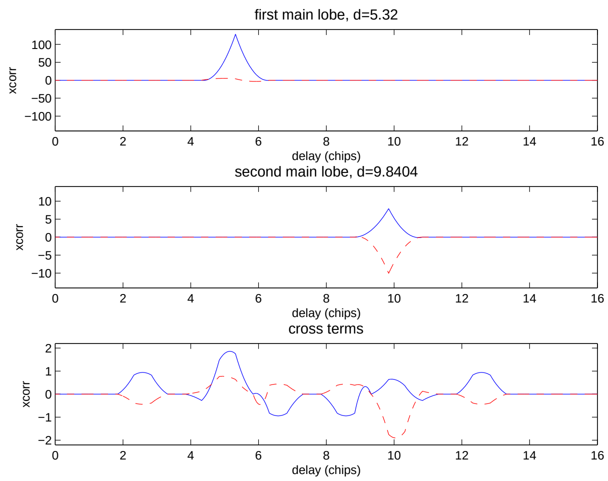

Figure 7: Individual terms of Υ, 2 returns of different power

of the cross terms is noticeably smaller than for the previous case, due to their dependence on the power of both returns. When combined, the cross terms do not conceal the presence of the second peak (fig. 8).

5 Conclusion

This study has presented a means of achieving complementary behaviour with two waveforms which are multiplexed in frequency and transmitted simultaneously. The method is based upon a Golay complementary pair, modified so that the squares of the autocorrelations exhibit complementarity. Offsetting the modified sequence above and below the carrier frequency allows the square of the matched filter output to be recovered irrespective of any range–dependent phase factor. The chief advantage of this method is the complete removal of sidelobes in pulse compression output in a fraction of the time it would take via time separation. Cross terms introduced when two returns are closely spaced are small and can be easily managed. Future work may examine how this nonlinear technique is affected by significant levels of sensor noise.

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