Target Detection in Clutter Using Adaptive OFDM Radar

Satyabrata Sen, *Student Member, IEEE*, and Arye Nehorai, *Fellow, IEEE*

***Abstract*—We address the problem of detecting a target moving in clutter environment using an orthogonal frequency division multiplexing (OFDM) radar. The broadband OFDM signal provides frequency diversity to improve the performance of the system. First, we develop a parametric model that accounts for the measurements at multiple frequencies including the Doppler shift. Then, we present a statistical detection test and evaluate its performance characteristics. Based on this, we propose an algorithm to adaptively design the parameters for the next transmitting waveform. Numerical examples illustrate our analytical results, demonstrating the achieved performance improvement due to the OFDM signaling method and adaptive waveform design.**

***Index Terms*—Asymptotic detection performance, generalized likelihood ratio test, optimal waveform design, OFDM radar.**

I. Introduction

IN THIS paper, we present a multifrequency radar that employs orthogonal frequency division multiplexing (OFDM) signals [1] to discriminate a moving target from undesired backscatter. In addition, we propose an algorithm to optimally design the parameters of the next transmitting waveform in order to achieve further improvement in detection performance. The motivation for employing multiple frequencies is that the different scattering centers of a target resonate differently at each frequency. The frequency diversity provides additional information that enhances the detection of targets from background clutter.

The advantage of using multiple frequencies has been well established in various radar applications, such as remote sensing of clouds and precipitation [2], detection of landmines [3], interpretation of an urban scene [4], etc. The use of multifrequency signals can be implemented either sequentially on a pulse-to-pulse basis or simultaneously in time. One of the ways to accomplish simultaneous use of several subcarrier is the OFDM signaling scheme which employs multiple subcarrier orthogonal signals in the time domain [6], [7]. Although OFDM has been elaborately studied and commercialized in the digital communication field [8], it is not so widely studied by the radar community apart from a few recent efforts [9], [10].

This paper is organized as follows. In Section II, we develop a parametric measurement model that accounts for a moving target in clutter due to multifrequency OFDM signaling. We also incorporate the realistic dependence of the clutter reflections on the transmitted signal into our model, following [11]. We assume that the measurement noise and the co-channel interference is Gaussian, temporally white but correlated between different subchannels with unknown positive definite covariance matrix. Then, in Section III, we formulate the detection problem as a hypothesis test to decide about the presence of a moving target. We partition the measured data in different range cells and apply the detection algorithm to the data corresponding to each range cell. Due to the lack of knowledge of some parameters in the model, we employ the generalized likelihood ratio (GLR) test. In Section IV, we propose an algorithm for adaptively computing the parameters of the next transmitting waveform based on the analysis of detection performance of our test. To illustrate the potential of our method, we present numerical examples in Section V. Our results demonstrate the achieved performance improvement due to the use of the OFDM signal. We also illustrate the benefit of using adaptive waveform design. Concluding remarks and highlights of a few possible future extensions are given in Section VI.

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The authors are with the Department of Electrical and Systems Engineering, Washington University, St. Louis, MO 63130 USA (e-mail: ssen3@ese.wustl.edu; nehorai@ese.wustl.edu).

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II. Problem Description and Modeling

In this section, we first develop a parametric measurement model in multiple orthogonal subchannels for a moving target in clutter. Then, we discuss our statistical assumption on the clutter and noise processes.

A. Measurement Model

We consider a radar employing OFDM signaling system with L active subcarriers, a bandwidth of B Hz, and pulse duration of T seconds. Let $a_l, l = 1, 2, \ldots, L$, represent the complex weights transmitted over the l-th subcarrier. The radar aims to detect a target that is assumed to be in far-field and is moving with a constant velocity $\vec{\boldsymbol{v}}$ relative to the radar. We further assume that for every range cell the radar knows the corresponding direction-of-arrival (DOA) vector $\vec{\boldsymbol{u}}$. Then, corresponding to a specific range cell containing the target, the complex envelope of the demodulated signal at the output of the l-th subchannel is given by

$$y_l(t) = a_l x_l^{\mathrm{t}} e^{j\omega_{l_{\mathrm{D}}} t} + a_l x_l^{\mathrm{c}} + e_l(t), \quad \text{for } l = 1, 2, \ldots, L,\ t = 1, 2, \ldots, N \quad (1)$$

where x_l^{t} and x_l^{c} represent the complex reflection coefficients of the target and clutter, respectively, in the l-th subchannel frequency, and $\omega_{l_{\mathrm{D}}} = 2\pi\beta f_l$ contains the Doppler information of the target in terms of $\beta = \langle \vec{\boldsymbol{v}}, \vec{\boldsymbol{u}} \rangle / c$ and $f_l = f_c + l\Delta f$ is the carrier frequency of the l-th subchannel. Here $\langle , \rangle$ denotes the inner-product operator over the real vector space. In addition, $\Delta f = B/L = 1/T$ denotes the subcarrier spacing, c is the speed of propagation, and N is the number of temporal measurements within a given coherent processing interval (CPI). The term $e_l(t)$ accounts for the measurement noise and co-channel interference (CCI) at the l-th subchannel. Note that in the second term of (1) there is no Doppler exponential factor because the clutter is considered to be static. Since the clutter effect depends on the transmitted signal (through the weights a_l's), this model can be classified as a signal-dependent clutter problem [11].

Stacking the outputs of all the L subchannels into a column vector, we get

$$\boldsymbol{y}(t) = \boldsymbol{A}\boldsymbol{X}^{\mathrm{t}}\boldsymbol{\phi}^{\mathrm{t}}(t,\boldsymbol{\eta}) + \boldsymbol{A}\boldsymbol{X}^{\mathrm{c}}\boldsymbol{\phi}^{\mathrm{c}}(t) + \boldsymbol{e}(t), \text{for } t = 1,2,\ldots,N \tag{2}$$

where

- $\boldsymbol{y}(t) = [y_1(t), y_2(t), \ldots, y_L(t)]^T$—here, "^{T}" denotes the transpose operator;
- $\boldsymbol{A} = \mathrm{diag}\{a_1, a_2, \ldots, a_L\}$ is an $L \times L$ complex diagonal matrix containing the transmitted weights;
- $\boldsymbol{X}^{\mathrm{t}} = \mathrm{diag}\{x_1^{\mathrm{t}}, x_2^{\mathrm{t}}, \ldots, x_L^{\mathrm{t}}\}$ is an $L \times L$ complex diagonal matrix representing the scattering coefficients of the target;
- $\boldsymbol{X}^{\mathrm{c}} = \mathrm{diag}\{x_1^{\mathrm{c}}, x_2^{\mathrm{c}}, \ldots, x_L^{\mathrm{c}}\}$ is an $L \times L$ complex diagonal matrix containing the clutter scattering coefficients;
- $\boldsymbol{\phi}^{\mathrm{t}}(t,\boldsymbol{\eta}) = [e^{j\omega_{1\mathrm{D}}t}, e^{j\omega_{2\mathrm{D}}t}, \ldots, e^{j\omega_{L\mathrm{D}}t}]^T$ is an $L \times 1$ vector containing the Doppler information of the target over all L subchannels;
- $\boldsymbol{\eta}$ is a column vector containing the unknown target-velocity components (for example, $\boldsymbol{\eta} = [v_x\, v_y]^T$ in a 2-D scenario);
- $\boldsymbol{\phi}^{\mathrm{c}}(t) = [1, 1, \ldots, 1]^T$ is an $L \times 1$ vector with all entries equal to 1;
- $\boldsymbol{e}(t) = [e_1(t), e_2(t), \ldots, e_L(t)]^T$ is an $L \times 1$ vector of measurement noise and co-channel interference.

Concatenating all the temporal data, defined in (2), into an $L \times N$ matrix, we obtain the measurement model as follows:

$$\boldsymbol{Y} = \boldsymbol{A}\boldsymbol{X}^{\mathrm{t}}\boldsymbol{\Phi}^{\mathrm{t}}(\boldsymbol{\eta}) + \boldsymbol{A}\boldsymbol{X}^{\mathrm{c}}\boldsymbol{\Phi}^{\mathrm{c}} + \boldsymbol{E} \tag{3}$$

where

- $\boldsymbol{Y} = [\boldsymbol{y}(t_1)\boldsymbol{y}(t_2)\cdots\boldsymbol{y}(t_N)]$;
- $\boldsymbol{\Phi}^{\mathrm{t}}(\boldsymbol{\eta}) = [\boldsymbol{\phi}^{\mathrm{t}}(t_1,\boldsymbol{\eta})\boldsymbol{\phi}^{\mathrm{t}}(t_2,\boldsymbol{\eta})\cdots\boldsymbol{\phi}^{\mathrm{t}}(t_N,\boldsymbol{\eta})]$ is an $L \times N$ matrix containing the Doppler information of the target through the parameter $\boldsymbol{\eta}$;
- $\boldsymbol{\Phi}^{\mathrm{c}} = [\boldsymbol{\phi}^{\mathrm{c}}(t_1)\boldsymbol{\phi}^{\mathrm{c}}(t_2)\cdots\boldsymbol{\phi}^{\mathrm{c}}(t_N)]$ is an $L \times N$ matrix with all entries equal to 1;
- $\boldsymbol{E} = [\boldsymbol{e}(t_1)\boldsymbol{e}(t_2)\cdots\boldsymbol{e}(t_N)]$ is an $L \times N$ matrix comprising measurement noise and interference.

B. Statistical Model

We start by assuming that the target response is deterministic but unknown in nature. On the other hand, the clutter in the range cell under test can be considered as a large collection of point scatterers producing incoherent reflections of the radar signal. Hence, the clutter response $[x_1^{\mathrm{c}}, x_2^{\mathrm{c}}, \ldots, x_L^{\mathrm{c}}]^T$ can be considered to be a zero-mean complex Gaussian random vector with unknown positive definite covariance matrix $\boldsymbol{\Sigma}_{\mathrm{c}}$. Therefore, we have

$$\boldsymbol{X}^{\mathrm{c}}\boldsymbol{\phi}^{\mathrm{c}}(t) \sim \mathbb{C}\mathcal{N}_L(\boldsymbol{0}, \boldsymbol{\Sigma}_{\mathrm{c}}). \tag{4}$$

From pulse to pulse, we assume that the clutter responses are temporarily independent, i.e., the columns of $\boldsymbol{X}^{\mathrm{c}}\boldsymbol{\Phi}^{\mathrm{c}}$ are independently and identically distributed according to (4). This implies that $\boldsymbol{A}\boldsymbol{X}^{\mathrm{c}}\boldsymbol{\Phi}^{\mathrm{c}} \sim \mathbb{C}\mathcal{N}_{L,N}(\boldsymbol{0}, \boldsymbol{I}_N \otimes \boldsymbol{A}\boldsymbol{\Sigma}_{\mathrm{c}}\boldsymbol{A}^H)$, where $\boldsymbol{I}_N$ is the identity matrix of dimension N, $\otimes$ represents the Kronecker product, and "^{H}" is the Hermitian operator.

The noise process is assumed to be temporally white zero-mean complex Gaussian process with covariance matrix $\sigma^2\boldsymbol{I}_L$, i.e., $\boldsymbol{E} \sim \mathbb{C}\mathcal{N}_{L,N}(\boldsymbol{0}, \boldsymbol{I}_N \otimes \sigma^2\boldsymbol{I}_L)$. For simplicity, we further assume that σ^2 is known. In particular, when the measurement noise is predominant over the co-channel interference, we can obtain some knowledge about σ^2 from the measured data when no signal has been transmitted. In addition, we assume that the clutter reflections and the measurement noise are statistically independent. Therefore, we can write

$$\tilde{\boldsymbol{E}} = \boldsymbol{A}\boldsymbol{X}^{\mathrm{c}}\boldsymbol{\Phi}^{\mathrm{c}} + \boldsymbol{E} \sim \mathbb{C}\mathcal{N}_{L,N}(\boldsymbol{0}, \boldsymbol{I}_N \otimes \tilde{\boldsymbol{\Sigma}}) \tag{5}$$

where $\tilde{\boldsymbol{\Sigma}} = \boldsymbol{A}\boldsymbol{\Sigma}_{\mathrm{c}}\boldsymbol{A}^H + \sigma^2\boldsymbol{I}_L$. Under this scenario, when the parameter $\boldsymbol{\eta}$ is known, (3) is known as the generalized multivariate analysis of variance (GMANOVA) [12] that has been studied extensively in statistics and applied to diverse applications in signal processing [14].

III. Detection Test

In this section, we develop a statistical detection test for the model presented in Section II. Our goal is to decide whether a target is present or not in the range cell under consideration.

We construct the decision problem to choose between two possible hypotheses: the null hypothesis $\mathcal{H}_0$ (target-free hypothesis) or the alternate hypothesis $\mathcal{H}_1$ (target-present hypothesis). This can be expressed as

$$\begin{cases} \mathcal{H}_0: & \boldsymbol{X}^{\mathrm{t}} = \boldsymbol{0}, \quad \tilde{\boldsymbol{\Sigma}} \text{ unknown} \\ \mathcal{H}_1: & \boldsymbol{X}^{\mathrm{t}} \neq \boldsymbol{0}, \quad \boldsymbol{\eta}, \tilde{\boldsymbol{\Sigma}} \text{ unknown.} \end{cases} \tag{6}$$

It is well known that the optimal detector for this problem is the Neyman-Pearson detector [15] that maximizes the probability of detection (P_{D}) for a certain probability of false alarm (P_{FA}). However, because we lack the knowledge of $\boldsymbol{\eta}$ and $\tilde{\boldsymbol{\Sigma}}$, we use the generalized likelihood ratio (GLR) test in which the unknown parameters are replaced with their maximum likelihood estimates (MLE). Although the GLR test is not optimal, in practice it appears to work quite well.

When the parameter $\boldsymbol{\eta}$ is known in (3), the GLR test compares the ratio of the likelihood functions under the two hypotheses with a threshold as follows ([15, Ch. 6.4.2]):

$$\mathrm{GLR}(\boldsymbol{\eta}) = \frac{f_{\mathcal{H}_1}(\boldsymbol{Y}; \boldsymbol{\eta}, \hat{\boldsymbol{X}}, \hat{\tilde{\boldsymbol{\Sigma}}}_1)}{f_{\mathcal{H}_0}(\boldsymbol{Y}; \hat{\tilde{\boldsymbol{\Sigma}}}_0)} \underset{}{\overset{\mathcal{H}_1}{>}} \gamma \tag{7}$$

where $f_{\mathcal{H}_0}$ and $f_{\mathcal{H}_1}$ are the likelihood functions under $\mathcal{H}_0$ and $\mathcal{H}_1$, $\hat{\tilde{\boldsymbol{\Sigma}}}_0$ and $\hat{\tilde{\boldsymbol{\Sigma}}}_1$ are the MLEs of $\tilde{\boldsymbol{\Sigma}}$ under $\mathcal{H}_0$ and $\mathcal{H}_1$, $\hat{\boldsymbol{X}}$ is the MLE of $\boldsymbol{X}$ under $\mathcal{H}_1$, and γ is the detection threshold. After some algebraic manipulations, it can be easily shown that the test statistics of this problem is [14]

$$\begin{aligned} \mathrm{GLR}(\boldsymbol{\eta}) &= \left|\hat{\tilde{\boldsymbol{\Sigma}}}_0\right| \Big/ \left|\hat{\tilde{\boldsymbol{\Sigma}}}_1\right| \\ &= \frac{|(1/N)\boldsymbol{Y}\boldsymbol{Y}^H|}{|(1/N)(\boldsymbol{Y} - \boldsymbol{A}\hat{\boldsymbol{X}}\boldsymbol{\Phi}(\boldsymbol{\eta}))(\boldsymbol{Y} - \boldsymbol{A}\hat{\boldsymbol{X}}\boldsymbol{\Phi}(\boldsymbol{\eta}))^H|} \end{aligned} \tag{8}$$

where $\hat{\boldsymbol{X}}(\boldsymbol{\eta}) = \mathrm{diag}(\boldsymbol{A}^{-1}\boldsymbol{Y}\boldsymbol{\Phi}(\boldsymbol{\eta})^H(\boldsymbol{\Phi}(\boldsymbol{\eta})\boldsymbol{\Phi}(\boldsymbol{\eta})^H)^-)$. Here, $|\cdot|$ denotes the determinant operator and $(\cdot)^-$ represents the generalized inverse of a matrix. In case of unknown $\boldsymbol{\eta}$, the GLR test compares $\max_{\boldsymbol{\eta}}\mathrm{GLR}(\boldsymbol{\eta}) = \mathrm{GLR}(\hat{\boldsymbol{\eta}})$ with a threshold.

IV. Adaptive Waveform Design

In this section, we develop an adaptive waveform design technique to improve the target-detection performance. To derive a mathematical formulation for the optimal waveform selection, we need to create a utility function and then determine the parameters of the next transmitting waveform by maximizing this utility function. To construct such a utility function we first consider the asymptotic performance characteristics of the GLR test considering the general case when $\boldsymbol{X}$ is a full matrix.

A. Detector Performance

The exact analytical derivation of the performance characteristics for our test statistic (8) for a finite value of N is very hard to find. Hence, we employ the asymptotic approximations under the most general scenario, i.e., when the matrix $\boldsymbol{X}$ is a full matrix. Then, following an analogous discussion on real Gaussian variables from ([16, Ch. 8]), we find that as $N \rightarrow \infty$, under $\mathcal{H}_0, N \ln \mathrm{GLR}(\boldsymbol{\eta})$ has a complex chi-square distribution with rL complex degrees of freedom, denoted as

$$N \ln \mathrm{GLR}(\boldsymbol{\eta}) \sim \mathbb{C}\chi^2_{rL} \quad (9)$$

where $r = \mathrm{rank}(\boldsymbol{\Phi}(\boldsymbol{\eta}))$. Note that the chi-square distribution does not depend on the unknown covariance matrix $\tilde{\boldsymbol{\Sigma}}$. Thus, when $\boldsymbol{\eta}$ is known, asymptotically (9) corresponds to a constant false-alarm rate (CFAR) test. Under $\mathcal{H}_1$, the limiting distribution of $N \ln \mathrm{GLR}(\boldsymbol{\eta})$ is a complex noncentral chi-square distribution with rL complex degrees of freedom, denoted as

$$N \ln \mathrm{GLR}(\boldsymbol{\eta}) \sim \mathbb{C}\chi^2_{rL}(\lambda) \quad (10)$$

where $\lambda = \sum_{l=1}^{L} \delta_l$ is the noncentrality parameter and $\delta_1, \delta_2, \ldots, \delta_L$ are the roots of $|\boldsymbol{M}\boldsymbol{M}^H - \delta\tilde{\boldsymbol{\Sigma}}| = 0$ and $\boldsymbol{M} = \boldsymbol{A}\boldsymbol{X}\boldsymbol{\Phi}(\boldsymbol{\eta})$. Obviously another way to represent the same noncentrality parameter is $\lambda = \mathrm{tr}(\tilde{\boldsymbol{\Sigma}}^{-1}\boldsymbol{M}\boldsymbol{M}^H)$. We may call the matrix $\tilde{\boldsymbol{\Sigma}}^{-1}\boldsymbol{M}\boldsymbol{M}^H$ as the "signal-to-noise ratio matrix," and hence its trace can be considered as a sum of squared Mahalanobis distances [17].

B. Waveform Design

In this subsection, we develop an adaptive waveform design method to improve the target-detection performance in the presence of background clutter. From the discussion of the previous subsection, we note that when $\boldsymbol{X}$ is a full matrix the GLR test is asymptotically CFAR when $\boldsymbol{\eta}$ is known and the detection performance depends on the system parameters through the noncentrality parameter λ. Although in our problem $\boldsymbol{X}$ is diagonal, we maximize the same noncentrality expression with respect to matrix $\boldsymbol{A}$ in order to maximize the probability of detection. Thus, in our optimization approach we aim to achieve

$$\hat{\boldsymbol{A}} = \underset{\boldsymbol{A}}{\mathrm{argmax}} [\mathrm{tr}((\boldsymbol{A}\boldsymbol{\Sigma}_{\mathrm{c}}\boldsymbol{A}^H + \sigma^2\boldsymbol{I}_L)^{-1}\boldsymbol{A}\boldsymbol{X}\boldsymbol{\Phi}\boldsymbol{\Phi}^H\boldsymbol{X}^H\boldsymbol{A}^H) - \mu(\mathrm{tr}(\boldsymbol{A}\boldsymbol{A}^H) - E_{\boldsymbol{A}})]. \quad (11)$$

Here, we also incorporate a pre-defined energy-constraint on $\boldsymbol{A}$ given by $E_{\boldsymbol{A}}$ and μ denotes the Lagrange multiplier. Note that in a real application the true values of $\boldsymbol{\eta}$, $\boldsymbol{X}$, and $\boldsymbol{\Sigma}_{\mathrm{c}}$ are not known. Hence, their estimates $\hat{\boldsymbol{\eta}}$, $\hat{\boldsymbol{X}}$, and $\hat{\boldsymbol{\Sigma}}_{\mathrm{c}}$ have to be used to obtain the optimal $\boldsymbol{A}$ for the next transmitting pulse based on the current measurements.

V. Numerical Results

We present below results of several simulations to illustrate our analytical results. The following parameters were common to all of the simulations unless otherwise mentioned.

- We considered an OFDM radar operating with $L = 5$ active subcarrier and the subcarrier spacing of $\Delta f = 20$ MHz.
- The radar operates by partitioning the whole surveillance area into several range cells. We simulated the situation of a particular range cell centered at 2 km north and 5 m east with respect to the radar.
- We assumed that a target is within this range cell and is moving with velocity $\vec{v} = 10\hat{i} + 10\hat{j}$ m/s.
- The radar collects $N = 15$ temporal measurements and jointly analyzes them to detect the presence/absence of the target.
- We defined the target-to-clutter ratio (TCR) as

$$\frac{\left[(1/N)\sum_{n=1}^{N}(\boldsymbol{A}\boldsymbol{X}\boldsymbol{\phi}(t_n, \boldsymbol{\eta}_{\mathrm{TRUE}}))^H \boldsymbol{A}\boldsymbol{X}\boldsymbol{\phi}(t_n, \boldsymbol{\eta}_{\mathrm{TRUE}})\right]}{\mathrm{tr}(\boldsymbol{A}\boldsymbol{\Sigma}_{\mathrm{c}}\boldsymbol{A}^H)} \quad (12)$$

where $\boldsymbol{\eta}_{\mathrm{TRUE}} = [10, 10]^T$ and clutter-to-noise ratio as

$$\mathrm{CNR} = \mathrm{tr}(\boldsymbol{A}\boldsymbol{\Sigma}_{\mathrm{c}}\boldsymbol{A}^H)/L\sigma^2. \quad (13)$$

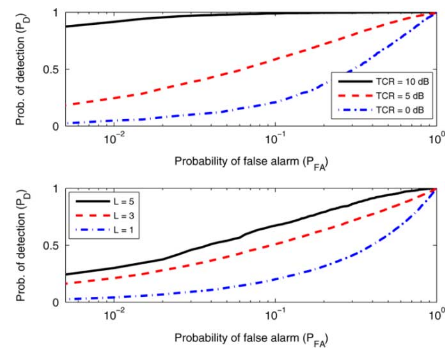

Fig. 1. (Top) Receiver operating characteristics for different target-to-clutter ratio values and (bottom) number of subchannels of OFDM signaling system.

A. Detector Performance

We performed Monte Carlo simulations based on 20,000 independent trials to characterize the performance of our proposed detector. The entries of $\boldsymbol{X}$ and $\boldsymbol{\Sigma}_{\mathrm{c}}$ were realized from the $\mathbb{C}\mathcal{N}(0, 1)$ distribution and then were scaled to satisfy the required TCR and CNR values. The matrix $\boldsymbol{A}$ was assumed to be the identity matrix. The performance of the detector is analyzed under the following scenarios:

- The upper subplot in Fig. 1 depicts the variations of probability of detection (P_{D}) as a function of probability of false alarm (P_{FA}) at three different TCR values, keeping the CNR fixed at 10 dB. As expected, the detection performance improves as TCR is increased.
- To show the advantage of using multifrequency OFDM signaling system, we simulated with different values of L by keeping the TCR fixed at 5 dB and CNR at 10 dB. The result is presented in the lower subplot of Fig. 1. It is evident that the frequency diversity improves the target-detection performance. For example, at $P_{\mathrm{FA}} = 10^{-2}$, the probability of detection (P_{D}) is 0.3 for five subcarrier OFDM system in comparison to 0.05 for a single-frequency operation.

B. Adaptive Waveform Design

In Section IV-B, we proposed an adaptive waveform design technique to improve the target detection performance. In order to study this improvement we devised a simple problem. We assumed a system in which we transmit $a_l = 1$ for $l = 1, 2, \ldots, L$ during the first radar dwell, and then based on the corresponding measurements we computed and transmitted

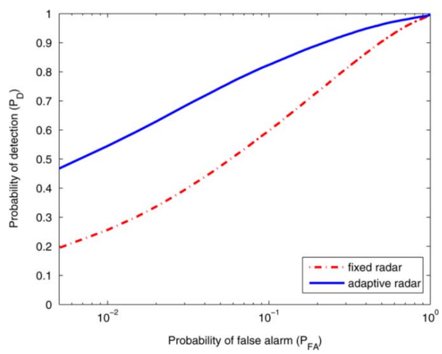

Fig. 2. Improvement on detection probability as a function of probability of false alarm due to the use of adaptive waveform design.

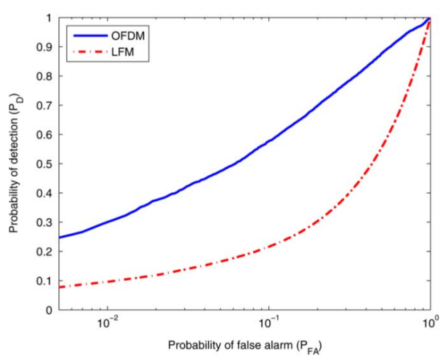

Fig. 3. Improvement, relative to a linear FM signal, in detection probability as a function of probability of false alarm due to the OFDM signal.

the optimized values of a_l's in the next radar dwell. We compared it with a system in which both the radar dwells transmit $a_l = 1$ for $l = 1, 2, \ldots, L$. We fixed the TCR at 5 dB and CNR at 10 dB for this simulation. Fig. 2 shows the average detection performance of 50 cases where parameters $\boldsymbol{X}$ and $\boldsymbol{\Sigma}$ were randomly selected. The figure shows the comparative study on the average performance of the conventional system (both the dwells transmit $a_l = 1 \; \forall l$) with that of the adaptive system in which the transmit weights (a_l) of the second dwell were optimally computed in accordance to (11). We observe that the detection performance of the adaptive system is considerably improved. For example, at $P_{\rm FA} = 10^{-2}$, the adaptive waveform shows an improvement from 0.25 to 0.55 in detection probability $(P_{\rm D})$ with respect to the nonadaptive system.

C. Comparison With Linear FM Signal

We compared the performance of our proposed method with that of a system employing the standard linear frequency modulated (LFM) signal. The complex envelope of the transmitted LFM signal is given by $s(t) = a_0 e^{j\pi k t^2}$, where a_0 is a normalizing constant and $k = B/T$ denotes the rate of change of instantaneous frequency. For a fair comparison, we considered the same bandwidth of $B = L\Delta f$ Hz and pulse duration of $T = 1/\Delta f$ seconds as were for OFDM transmission. We also chose a_0 in such a way that both the LFM and OFDM systems employ the same transmission energy. Fig. 3 shows the ROC curves based on 20,000 Monte Carlo trials. In this simulation, we kept the TCR at 5 dB and CNR at 10 dB. We observe that the use of OFDM signal provides better detection probability in comparison with that of LFM signal.

VI. Conclusions

We addressed the problem of detecting a moving target in clutter environment by employing an OFDM radar. We first developed the parametric model that accounts for the measurements over multiple orthogonal frequencies. Then, we formulated the detection problem as a hypothesis test to decide about the presence of a moving target. We analyzed the performance of this detector both numerically and analytically. Our numerical results demonstrate the achieved performance improvement through the use of OFDM signal in comparison with a single frequency operation. Furthermore, based on the asymptotic performance analysis, we proposed an algorithm to adaptively design the parameters for the next transmitting waveform. We presented a numerical simulation to show the performance improvement that could be obtained due to such adaptive waveform design.

In our future work, we will extend our algorithm to target detection and tracking problems in more complex and specific scenarios. We will explore other performance criteria, e.g., ambiguity function, mutual information, etc., to optimally design the transmit waveform to improve the accuracy. We will also validate the performance of our proposed detector with real data.

References

[1] A. Pandharipande, "Principles of OFDM," *IEEE Potentials*, vol. 21, no. 2, pp. 16–19, Apr. 2002.

[2] N. Majurec, S. M. Sekelsky, S. J. Frasier, and S. A. Rutledge, "The advanced multi-frequency radar (AMFR) for remote sensing of clouds and precipitation," in *Proc. 32nd Conf. Radar Meteorology*, Albuquerque, NM, 2004.

[3] P. V. Genderen, P. Hakkaart, J. V. Heijenoort, and G. P. Hermans, "A multi frequency radar for detecting landmines: Design aspects and electrical performance," in *Proc. 31st European Microwave Conf.*, 2001, vol. 2, pp. 249–252.

[4] M. L. Bryan, "Interpretation of an urban scene using multi-channel radar imagery," *Remote Sens. Environ.*, vol. 4, pp. 49–66, 1975.

[5] Z.-G. Xia, "Applications of multi-frequency, multi-polarization and multi-incident angle SAR systems in urban land use and land cover mapping," in *Proc. Int. Geoscience and Remote Sensing Symp.*, Lincoln, NE, May 27–31, 1996, vol. 4, pp. 2310–2314.

[6] S. B. Weinstein and P. M. Ebert, "Data transmission by frequency-division multiplexing using the discrete Fourier transform," *IEEE Trans. Commun. Technol.*, vol. 19, no. 5, pp. 476–488, Oct. 1971.

[7] A. F. Molisch, *Wideband Wireless Digital Communications.*. Upper Saddle River, NJ: Prentice-Hall, 2001.

[8] B. L. Floch, R. Halbert-Lassalle, and D. Castelain, "Digital sound broadcasting to mobile receivers," *IEEE Trans. Consumer Electron.*, vol. 35, no. 3, pp. 493–503, Aug. 1989.

[9] G. E. A. Franken, H. Nikookar, and P. V. Genderen, "Doppler tolerance of OFDM-coded radar signals," in *Proc. 3rd European Radar Conf.*, Manchester, U.K., Sep. 13–15, 2006, pp. 108–111.

[10] D. S. Garmatyuk, "Simulated imaging performance of UWB SAR based on OFDM," in *Proc. IEEE 2006 Int. Conf. Ultra-Wideband*, Sep. 2006, pp. 237–242.

[11] M. Hurtado and A. Nehorai, "Polarimetric detection of targets in heavy inhomogeneous clutter," *IEEE Trans. Signal Process.*, vol. 56, no. 4, pp. 1349–1361, Apr. 2008.

[12] M. S. Srivastava and C. G. Khatri, *An Introduction to Multivariate Statistics.*. New York: North-Holland, 1979.

[13] R. F. Potthoff and S. N. Roy, "A generalized multivariate analysis of variance model useful especially for growth curve problems," *Biometrika*, vol. 51, no. 3/4, pp. 313–326, Dec. 1964.

[14] A. Dogandzic and A. Nehorai, "Generalized multivariate analysis of variance: A unified framework for signal processing in correlated noise," *IEEE Signal Process. Mag.*, vol. 20, no. 5, pp. 39–54, Sep. 2003.

[15] S. M. Kay, *Fundamentals of Statistical Signal Processing: Detection Theory*. Upper Saddle River, NJ: Prentice-Hall, 1998.

[16] T. W. Anderson, *An Introduction to Multivariate Statistical Analysis*, 3rd ed. Hoboken, NJ: Wiley, 2003.

[17] P. C. Mahalanobis, "On the generalized distance in statistics," in *Proc. Nat. Inst. Sci. India*, 1936, vol. 2, pp. 49–55.