OPTIMAL BEAMPATTERN SYNTHESIS OF A POLARIZED ARRAY

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ABSTRACT

Utilizing the polarization information in waveforms about targets enables improving the performance of radar systems. We consider the optimal synthesis of the polarized beampattern of an array of antennas, each having two orthogonal electric-dipole elements. We control the amplitudes and phases of the electric fields emitted from these dipole antennas to synthesize the electromagnetic beam with desired power and polarization patterns. The problem is formulated in a convex form which is thus efficiently solvable. We compare the performance of two synthesizing methods: (i) separate design of the two co-aligned dipole antenna sub-arrays; and (ii) joint design of the cross-dipole antenna array. Our results indicate that these two separate designs have the equivalent capability of suppressing the sidelobe power density, whereas the joint design has the additional advantage of controlling the beam polarizations. The results are also demonstrated by numerical examples.

Index Terms— Waveform polarization, vector antenna, beamforming, beampattern synthesis, convex optimization

1. INTRODUCTION

In active sensing systems such as radars, polarimetric scattering information is useful for discriminating the targets' properties such as geometrical structure, shape, reflectivity, and orientation. Thus, exploiting the polarization information in the backscattered waveforms can significantly improve the sensing performance [1, 2]. Polarimetric radar systems which transmit waveforms with both horizontal and vertical orientations have been developed and adopted in various applications [3].

In this paper, we consider the beampattern synthesis of a polarized array. In traditional non-polarized arrays, radio signals from a set of small non-directional antennas are combined with different weights to achieve the beam directionality. The beam emitted by such arrays are of a scalar form. To obtain a polarized beam, we use an array of vector antennas, each having two co-aligned orthogonal electric-dipole elements. Such a vector sensor array can be equivalently viewed as two co-aligned dipole antenna sub-arrays in horizontal and vertical orientations. Our aim is to design complex weights for individual array elements to achieve a beam with both desired spatial power density and desired polarizations.

There has been a long history of research on the problem of beampattern synthesis [4, 5, 6]. The main difference between the present paper and previous work is that our paper jointly designs waveform polarization and spatial power pattern, whereas the existing literature focuses on the latter only. We consider the joint design of the cross-dipole antenna array, and compare its performance with the scalar arrays. By formulating the problem in an efficiently solvable convex form, we show that the vector array and scalar array have the equivalent capability of attaining power density, whereas the vector array has the additional capability of fully controlling the beam polarizations.

2. WAVEFORM POLARIZATION

Consider a harmonic plane wave $E = [E_1, E_2]$ traveling in a uniform medium in the 3-D space. The polarization is the locus of the electric field vector as a function of time. It describes the direction of wave oscillation in the plane perpendicular to the direction of propagation [7]. One efficient method of describing the waveform polarization is the so-called polarization ellipse [1].

2.1. Polarization ellipse

Write E in the complex form $\mathbf{E}(z;t) = [\xi_1, \xi_2]^T e^{j\omega t}$, where $[\xi_1, \xi_2]^T$ is the convex envelope of E. The following theorem gives a one-to-one relationship between the polarization of E and the ratio $\xi_2/\xi_1 = \gamma e^{j\delta_\phi}$.

Theorem 1 ([1]) *Every non-zero* $\boldsymbol{\xi} = [\xi_1,\ \xi_2]^T \in \mathbb{C}^2$ *has a unique presentation*

$$\boldsymbol{\xi} = re^{j\psi} \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\beta \\ j\sin\beta \end{bmatrix} \qquad (1)$$

where $r > 0$, $\phi \in (-\pi, \pi]$, $\alpha \in (-\frac{\pi}{2}, \frac{\pi}{2}]$, $\beta \in [-\frac{\pi}{4}, \frac{\pi}{4}]$.

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In (1), α is called the orientation angle and β is the ellipticity angle (see Fig. 1). For example, $\beta = 0$ and $\beta = \pm\pi/4$ gives linear and circular polarizations respectively. The inverse of (1) can be given as follows. First, $\alpha \in (-\pi/2, \pi/2]$ is uniquely determined from

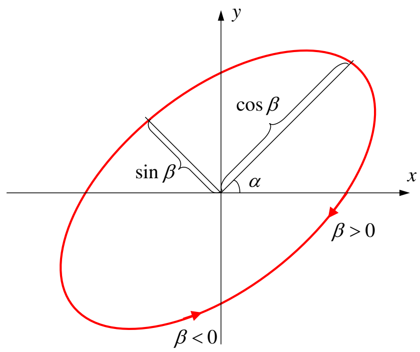

Fig. 1. Polarization ellipse.

$$\cos 2\alpha = \frac{1-\gamma^2}{\sqrt{1+\gamma^4+2\gamma^2\cos 2\delta_\phi}};$$
$$\sin 2\alpha = \frac{-2\gamma\cos\delta_\phi}{\sqrt{1+\gamma^4+2\gamma^2\cos 2\delta_\phi}}.$$

Moreover, $\beta \in [-\pi/4, \pi/4]$ can be solved from

$$\beta = \arcsin \frac{\sqrt{1-\frac{2\gamma}{1+\gamma^2}\sin\delta_\phi} - \sqrt{1+\frac{2\gamma}{1+\gamma^2}\sin\delta_\phi}}{2}.$$

Thus, the relationship between $\{\alpha, \beta\}$ and $\xi_2/\xi_1 = \gamma e^{j\delta_\phi}$ is one-to-one.

2.2. Polarization transformation

For any pair of angles $\alpha \in (-\pi/2, \pi/2]$, $\beta \in [-\pi/4, \pi/4]$, let us introduce notation

$$\mathcal{P}(\alpha,\beta) = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\beta \\ j\sin\beta \end{bmatrix}. \qquad (2)$$

It turns out that any polarization $\mathcal{P}(\alpha,\beta)$ can be mathematically transformed to $\mathcal{P}(0,0)$ by an *orthogonal transformation*. To show that, we introduce transformation $T(\alpha,\beta)$: $[x_1, x_2]^T \to [y_1, y_2]^T$ in which

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \cos\beta & -j\sin\beta \\ j\sin\beta & -\cos\beta \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

It is easy to verify that $T(\alpha,\beta)$ ia an orthogonal transformation, and in addition,

$$T(\alpha,\beta)\mathcal{P}(\alpha,\beta) = \begin{bmatrix} \cos\beta & -j\sin\beta \\ j\sin\beta & -\cos\beta \end{bmatrix} \begin{bmatrix} \cos\beta \\ j\sin\beta \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \mathcal{P}(0,0).$$

We will use such transformations in Section 3 to simply the design of a polarized beampattern.

3. POLARIZED BEAMPATTERN SYNTHESIS

In this section, we consider the problem of polarized beampattern synthesis using an array of co-aligned vector antennas. We use the tools from convex optimization to obtain optimal design under various criteria.

3.1. Electromagnetic vector antenna (EMVA) array

Suppose there are N transmitters in the array (see Fig. 2). The antennas are driven by a signal with same wavelength λ and convex envelope $s(t)$. We consider the following two types of arrays:

- *Scalar array.* Each transmitter has one electrical dipole with antenna current ξ_n.
- *Vector array.* Each transmitter has two orthogonal electrical dipoles which are co-aligned in the H-direction and V-direction. The antenna currents are denoted by $\boldsymbol{\xi}_n = [\xi_n^{(H)}, \ \xi_n^{(V)}]^T$.

Suppose the transmitters are located at N locations in $\mathbb{R}^3$ with coordinates $\mathbf{x}_n$: $n = 1, 2, \ldots, N$. The antenna array response at any spatial direction $\mathbf{r}$ is

$$\mathbf{a}(\mathbf{r}) = [e^{-j\psi_1(\mathbf{r})}, e^{-j\psi_2(\mathbf{r})}, \ldots, e^{-j\psi_N(\mathbf{r})}]^T,$$

where $\psi_n(\mathbf{r}) = k\mathbf{r} \cdot \mathbf{x}_n$ and $k = 2\pi/\lambda$ is the wave number.

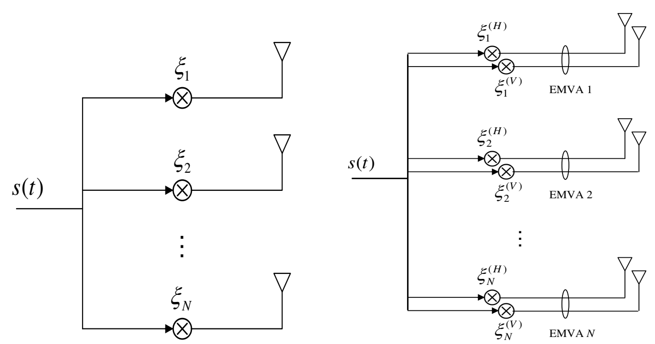

Fig. 2. Left: scalar array; Right: vector array.

The normalized electrical field emitted from the antenna array (by ignoring the common carrier and the baseband signal $s(t)$) can be written as $\mathbf{E}(\mathbf{r}) = \mathbf{a}(\mathbf{r}) \cdot \boldsymbol{\xi}$, where for the case of scalar array $\boldsymbol{\xi} = [\xi_1, \xi_2, \ldots, \xi_N]^T$, and for the case of vector array,

$$\boldsymbol{\xi} = [\boldsymbol{\xi}_1, \boldsymbol{\xi}_2, \ldots, \boldsymbol{\xi}_N]^T \quad \text{with} \quad \boldsymbol{\xi}_n = [\boldsymbol{\xi}_n^{(H)}, \boldsymbol{\xi}_n^{(V)}]^T.$$

Specifically for the vector array, the emitted electrical field has a vector form

$$\mathbf{E}(\mathbf{r}) = \mathbf{a}(\mathbf{r}) \cdot \boldsymbol{\xi} = \begin{bmatrix} \mathbf{a}(\mathbf{r}) \cdot \boldsymbol{\xi}^{(H)} \\ \mathbf{a}(\mathbf{r}) \cdot \boldsymbol{\xi}^{(V)} \end{bmatrix} = \begin{bmatrix} E^{(H)}(\mathbf{r}) \\ E^{(V)}(\mathbf{r}) \end{bmatrix}. \qquad (3)$$

The beampattern synthesis for the scalar array is to design weights $\boldsymbol{\xi} = [\xi_1, \xi_2, \ldots, \xi_N]^T$ to achieve certain spatial power pattern, whereas the polarized beampattern synthesis in a vector array is to design weights $\boldsymbol{\xi} = [\boldsymbol{\xi}_1, \boldsymbol{\xi}_2, \ldots, \boldsymbol{\xi}_N]^T$ such that

- the main beam (at a direction $\mathbf{r}_0$) has desired power P and polarization (μ, ν);
- the power pattern of sidelobes (in a region denoted by $\mathcal{S}_\mathbf{r}$) are suppressed.

Compared with the scalar array, the beampattern synthesis in a vector array enforces an additional polarization constraint on the beam. If desired, the polarization of the sidelobes can also be controlled.

3.2. Polarized beampattern synthesis

There are various criteria in suppressing the sidelobe power while maintaining the mainlobe power and polarization. For example, minimizing the total power of sidelobes gives the following problem

$$\min_{\boldsymbol{\xi}} \sum_{\mathbf{r}_s \in \mathcal{S}_\mathbf{r}} \left\{ |E^{(H)}(\mathbf{r}_s)|^2 + |E^{(V)}(\mathbf{r}_s)|^2 \right\} \tag{4}$$
$$\text{s.t.} \begin{bmatrix} E^{(H)}(\mathbf{r}_0) \\ E^{(V)}(\mathbf{r}_0) \end{bmatrix} = \sqrt{P} e^{j\phi} \begin{bmatrix} \cos\mu & \sin\mu \\ -\sin\mu & \cos\mu \end{bmatrix} \begin{bmatrix} \cos\nu \\ j\sin\nu \end{bmatrix}$$

where the relation between $\boldsymbol{\xi}$ and $\mathbf{E}$ is given in (3). An alternative criterion is to minimize the maximal sidelobe which instead leads to the following optimization problem

$$\min_{\boldsymbol{\xi}} \max_{\mathbf{r}_s \in \mathcal{S}_\mathbf{r}} \left\{ |E^{(H)}(\mathbf{r}_s)|^2 + |E^{(V)}(\mathbf{r}_s)|^2 \right\} \tag{5}$$
$$\text{s.t.} \begin{bmatrix} E^{(H)}(\mathbf{r}_0) \\ E^{(V)}(\mathbf{r}_0) \end{bmatrix} = \sqrt{P} e^{j\phi} \begin{bmatrix} \cos\mu & \sin\mu \\ -\sin\mu & \cos\mu \end{bmatrix} \begin{bmatrix} \cos\nu \\ j\sin\nu \end{bmatrix}$$

Besides the above two examples we will in addition consider other design criteria including null region (of completely suppressing the power in a certain range) and also polarization control of sidelobes.

3.2.1. *Sum sidelobe minimization*

We first consider the total sidelobe power minimization. Problem (4) is clearly equivalent to

$$\begin{aligned} \min_{\boldsymbol{\xi}} \quad & \sum_{\mathbf{r}_s \in \mathcal{S}_\mathbf{r}} |E^{(H)}(\mathbf{r}_s)|^2 + \sum_{\mathbf{r}_s \in \mathcal{S}_\mathbf{r}} |E^{(V)}(\mathbf{r}_s)|^2 \\ \text{s.t.} \quad & E^{(H)}(\mathbf{r}_0) = \sqrt{P} e^{j\phi} (\cos\mu\cos\nu + j\sin\mu\sin\nu) \\ & E^{(V)}(\mathbf{r}_0) = \sqrt{P} e^{j\phi} (-\sin\mu\cos\nu + j\cos\mu\sin\nu) \end{aligned}$$

In the above problem, both objective function and constraints are *separable*. Thus, it can be broken down to the following two scalar array problems:

(i) H-channel:

$$\begin{aligned} \min_{\boldsymbol{\xi}^{(H)}} \quad & \sum_{\mathbf{r}_s \in \mathcal{S}_\mathbf{r}} |E^{(H)}(\mathbf{r}_s)|^2 \\ \text{s.t.} \quad & E^{(H)}(\mathbf{r}_0) = \sqrt{P} e^{j\phi} (\cos\mu\cos\nu + j\sin\mu\sin\nu) \end{aligned}$$

(ii) V-channel:

$$\begin{aligned} \min_{\boldsymbol{\xi}^{(H)}} \quad & \sum_{\mathbf{r}_s \in \mathcal{S}_\mathbf{r}} |E^{(V)}(\mathbf{r}_s)|^2 \\ \text{s.t.} \quad & E^{(V)}(\mathbf{r}_0) = \sqrt{P} e^{j\phi} (-\sin\mu\cos\nu + j\cos\mu\sin\nu) \end{aligned}$$

It is easy to see that in the above problems, the factor $\sqrt{P} e^{j\phi}$ can be omitted after performing a scaling on corresponding parameters. We thus discard it in the remaining part of the paper (by assuming a $0dB$ main beam power gain).

The above analysis implies that designing the weights for the vector array can be achieved by designing the two subarrays separately without loss of optimality. Fig. 3 shows the computer simulation of a linear array of 15 elements that are separated by half wavelength. The main beam is at 10° with width 15°. The main beam of the vector array has polarization $(\mu, \nu) = (-39.03^o, 7.35^o)$. The formulated optimization problems are solved using the software package SeDuMi [8, 9]. As can be seen, both scalar and vector arrays achieve the same spatial power pattern, whereas the beam from vector array has the desired polarization property.

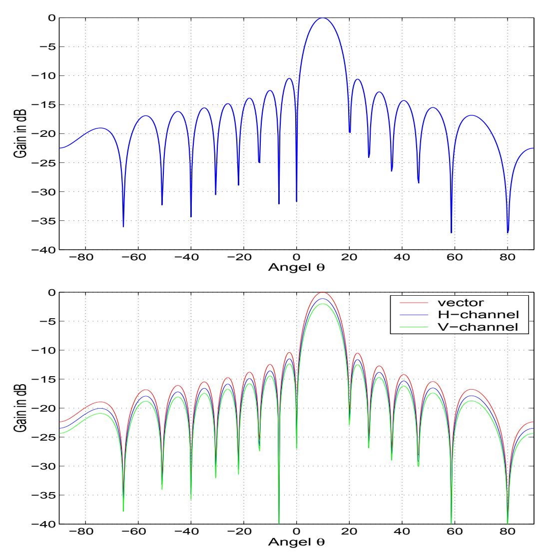

Fig. 3. Beampattern synthesis of a scalar array (top); vector array (bottom).

3.2.2. *Maximal sidelobe minimization*

We now consider the problem of minimizing the maximum sidelobe. After introducing an auxiliary variable τ, we can write (5) as

$$\begin{aligned} \min_{\boldsymbol{\xi};\ \tau} \quad & \tau \\ \text{s.t.} \quad & \begin{bmatrix} E^{(H)}(\mathbf{r}_0) \\ E^{(V)}(\mathbf{r}_0) \end{bmatrix} = \begin{bmatrix} \cos\mu & \sin\mu \\ -\sin\mu & \cos\mu \end{bmatrix} \begin{bmatrix} \cos\nu \\ j\sin\nu \end{bmatrix} \\ & ||E^{(H)}(\mathbf{r}_s)||^2 + ||E^{(V)}(\mathbf{r}_s)||^2 \le \tau^2; \quad \mathbf{r}_s \in \mathcal{S}_\mathbf{r} \\ & \begin{bmatrix} E^{(H)}(\mathbf{r}) \\ E^{(V)}(\mathbf{r}) \end{bmatrix} = \sum_{n=1}^{N} a_n(\mathbf{r}) \begin{bmatrix} \xi_n^{(H)}(\mathbf{r}) \\ \xi_n^{(V)}(\mathbf{r}) \end{bmatrix} \end{aligned} \tag{6}$$

We can not reduce the above problem to two scalar sub-array problems since the second constraint can not be decoupled between the H-component and V-component in the sense that

$$\max_{\mathbf{r}_s\in\mathcal{S}_\mathbf{r}}\left\{||E^{(H)}(\mathbf{r}_s)||^2+||E^{(V)}(\mathbf{r}_s)||^2\right\}$$
$$\neq \max_{\mathbf{r}_s\in\mathcal{S}_\mathbf{r}}||E^{(H)}(\mathbf{r}_s)||^2+\max_{\mathbf{r}_s\in\mathcal{S}_\mathbf{r}}||E^{(V)}(\mathbf{r}_s)||^2.$$

However, to simplify the above problem and relate it to its scalar counterpart, we can apply the orthogonal transformations introduced in Section 2.2. Recall that the orthogonal transformation $T(\mu,\nu)$ maps the $\mathcal{P}(\mu,\nu)$ to $\mathcal{P}(0,0)$. We can thus change the reference coordinate system and introduce

$$\begin{bmatrix}\xi_n^{(1)}\\ \xi_n^{(2)}\end{bmatrix}=T(\mu,\nu)\begin{bmatrix}\xi_n^{(H)}\\ \xi_n^{(V)}\end{bmatrix},\quad \begin{bmatrix}E^{(1)}\\ E^{(2)}\end{bmatrix}=T(\mu,\nu)\begin{bmatrix}E^{(H)}\\ E^{(V)}\end{bmatrix}.$$

Therefore, $||E^{(1)}(\mathbf{r}_s)||^2+||E^{(2)}(\mathbf{r}_s)||^2=||E^{(H)}(\mathbf{r}_s)||^2+||E^{(V)}(\mathbf{r}_s)||^2$ for all $\mathbf{r}_s$ since an orthogonal transformation does not change the norm of a vector. In terms of $(E^{(1)},E^{(2)})$, (6) becomes

$$\begin{aligned}\min_{\boldsymbol{\xi};\ \tau}\quad & \tau\\ \text{s.t.}\quad & \begin{bmatrix}E^{(1)}(\mathbf{r}_0)\\ E^{(2)}(\mathbf{r}_0)\end{bmatrix}=\begin{bmatrix}1\\ 0\end{bmatrix}\\ & ||E^{(1)}(\mathbf{r}_s)||^2+||E^{(2)}(\mathbf{r}_s)||^2\le\tau^2;\quad \mathbf{r}_s\in\mathcal{S}_\mathbf{r}\\ & E^{(1)}(\mathbf{r})=\sum_{n=1}^N a_n(\mathbf{r})\xi_n^{(1)};\ E^{(2)}(\mathbf{r})=\sum_{n=1}^N a_n(\mathbf{r})\xi_n^{(2)}\end{aligned}\tag{7}$$

In (7), the optimum is reach at the points when $E^{(2)}=0$ and $\xi_n^{(2)}=0$. We can thus remove the dimension at $\xi_n^{(2)}$ and solve for $\xi_n^{(1)}$ as a scalar array problem below

$$\begin{aligned}\min_{\boldsymbol{\xi}^{(1)};\ \tau}\quad & \tau\\ \text{s.t.}\quad & E^{(1)}(\mathbf{r}_0)=1\\ & ||E^{(1)}(\mathbf{r}_s)||^2\le\tau^2;\quad \mathbf{r}_s\in\mathcal{S}_\mathbf{r}\\ & E^{(1)}(\mathbf{r})=\sum_{n=1}^N a_n(\mathbf{r})\xi_n^{(1)}.\end{aligned}$$

Therefore, if $\boldsymbol{\xi}^{(1)^*}$ is the solution to the above scalar array problem, then $[\xi_n^{(1)^*},0]^T$ is the solution to (7), and

$$\begin{bmatrix}\xi_n^{(H)}\\ \xi_n^{(V)}\end{bmatrix}=T^{-1}(\alpha,\beta)\begin{bmatrix}\xi_n^{(1)^*}\\ 0\end{bmatrix}$$
$$=\begin{bmatrix}\cos\mu & \sin\mu\\ -\sin\mu & \cos\mu\end{bmatrix}\begin{bmatrix}\cos\nu & j\sin\nu\\ -j\sin\nu & -\cos\nu\end{bmatrix}\begin{bmatrix}\xi_n^{(1)^*}\\ 0\end{bmatrix}$$

is an optimal solution to (6).

The above results imply that in the case of maximal sidelobe minimization, the vector array can achieve the same power pattern as the scalar array, while preserving the advantage of controlling the beam polarization. Such conclusion is illustrated in Fig. 4 which shows a computer simulation of minimizing the maximum sidelobe where the array configuration is the same as in Fig. 3. In general, the above result can be extended to the case when the sidelobe control criteria is on its power only, *i.e.*, the design aims to minimizing a cost function in the form of $\mathcal{F}\left(||\mathbf{E}(\mathbf{r}_s)||,\ \mathbf{r}_s\in\mathcal{S}_\mathbf{r}\right)$, where $\mathcal{F}$ is an increasing function in its variables.

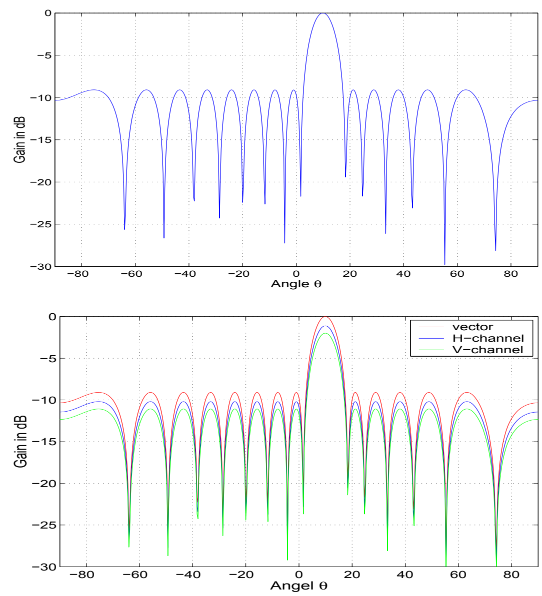

Fig. 4. Top: scalar array; Bottom: vector array.

3.2.3. *Polarized beampattern with null region*

We further consider the case of completely suppressing the beam at a null region denoted by $\mathcal{S}_n$. Due to the limited degree of freedom in the design (determined by the size of the array), completely nulling out the power in $\mathcal{S}_n$ is not possible (unless in the trivial case when it only consists of a few discrete points). In practice, a constraint of the form $P(\mathbf{r}_n)\le\epsilon$ is imposed for some $\epsilon\ll 1$ and for all $\mathbf{r}_n\in\mathcal{S}_n$. We thus obtain the following problem

$$\begin{aligned}\min_{\boldsymbol{\xi}}\quad & \tau\\ \text{s.t.}\quad & \begin{bmatrix}E^{(H)}(\mathbf{r}_0)\\ E^{(V)}(\mathbf{r}_0)\end{bmatrix}=\begin{bmatrix}\cos\mu & \sin\mu\\ -\sin\mu & \cos\mu\end{bmatrix}\begin{bmatrix}\cos\nu\\ j\sin\nu\end{bmatrix}\\ & ||E^{(H)}(\mathbf{r}_s)||^2+||E^{(V)}(\mathbf{r}_s)||^2\le\tau;\quad \mathbf{r}_n\in\mathcal{S}_\mathbf{r}\\ & ||E^{(H)}(\mathbf{r}_n)||^2+||E^{(V)}(\mathbf{r}_n)||^2\le\epsilon;\quad \mathbf{r}_n\in\mathcal{S}_n.\end{aligned}$$

Fig. 5 shows the computer simulation of a linear array of 21 elements that are separated by half wavelength. The main beam is at 10° with width 15° and polarization $(\mu,\nu)=(60^o,30^o)$. Compared to the beampattern without null region, the beampattern with null region in $[-42^o,-24^o]$ has a power gain of 13.62 dB (bottom figure), while the beampattern without null region has a power gain of 14.26 dB (top figure).

3.2.4. *Sidelobe polarization control*

Previously considered design criteria from Section 3.2.1–3.2.3 are on the sibelobe power only. In fact, we can also accommodate the control of sidelobe polarization in the problem formulation. The sidelobe polarization control are beneficial

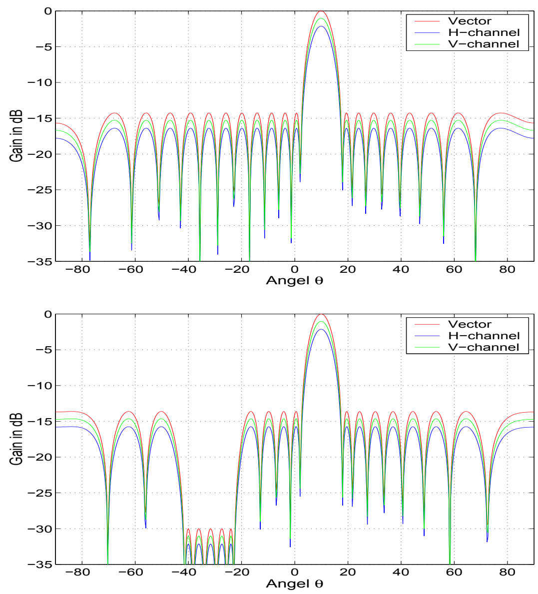

Fig. 5. Top: without null region; Bottom: with null region.

for the cases, *e.g.*, when there are known interference or jamming sources from certain arrival directions. Sidelobe polarization can be controlled such that the interference of the known sources on the beam is minimized.

Assume that we want to control the polarization of the sidelobes at directions $\{\mathbf{r}_k : k = 1, 2, \ldots, K\}$. The corresponding polarizations are $\{(\mu_k, \nu_k) : k = 1, 2, \ldots, K\}$ respectively. We can thus add the following linear constraints $E^{(H)}(\mathbf{r}_k)/E_k^{(V)}(\mathbf{r}_k) = \gamma_k e^{j\delta_{\phi_k}}$, where $\gamma_k, \delta_{\phi_k}$ is determined by the polarizations $\{(\mu_k, \nu_k)\}$ (c.f. Section 2.1). Adding these constraints into (5), we obtain

$$
\begin{aligned}
\min_{\boldsymbol{\xi};\ \tau} \quad & \tau \\
\text{s.t.} \quad & \begin{bmatrix} E^{(H)}(\mathbf{r}_0) \\ E^{(V)}(\mathbf{r}_0) \end{bmatrix} = \begin{bmatrix} \cos\mu & \sin\mu \\ -\sin\mu & \cos\mu \end{bmatrix} \begin{bmatrix} \cos\nu \\ j\sin\nu \end{bmatrix} \\
& ||E^{(H)}(\mathbf{r}_s)||^2 + ||E^{(V)}(\mathbf{r}_s)||^2 \le \tau^2; \quad \mathbf{r}_s \in \mathcal{S}_{\mathbf{r}} \\
& E^{(H)}(\mathbf{r}_k) = \gamma_k e^{j\delta_{\phi_k}} E_k^{(V)}(\mathbf{r}_k); \quad 1 \le k \le K.
\end{aligned}
$$

The simulation of showing the performance of sidelobe control is given in Fig. 6. We use a linear array of 15 elements that are separated by half wavelength. The main beam is at 10° with width 15° and polarization $(\mu, \nu) = (60^o, 30^o)$. In addition, the sidelobe at $\theta = -24^o$ needs to have polarization $(\mu_1, \nu_1) = (30^o, -30^o)$. Compared with the result in Fig. 4, the beampattern with sidelobe polarization control has a power gain of 9.03 dB, while the beampattern without sidelobe polarization control has a power gain of 9.09 dB.

4. SUMMARY

We considered the synthesis of polarized beampattern using an array of co-aligned vector antennas. We formulated the problem in a convex form with design variables to be the amplitudes and phases of the weights in the antenna elements. We have shown that the vector array and scalar array have the same capability of suppressing the sidelobe power density, whereas the vector array has the additional advantage of controlling both the main beam and sidelobe polarizations. Although we have only considered the transmit beamforming in an active array, similar results can be generalized to the receive beamforming in passive vector sensor arrays.

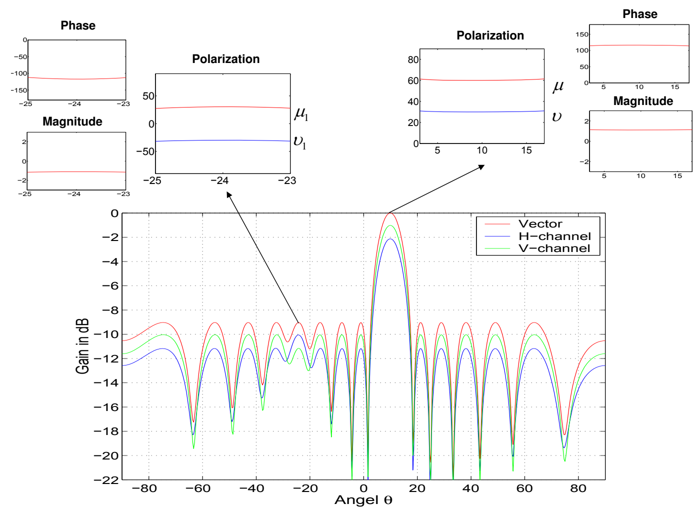

Fig. 6. Simulation of sidelobe control.

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