PERFORMANCE OF BEAMPATTERN SYNTHESIS USING HIGH-DIMENSIONAL VECTOR ANTENNA ARRAYS

Jin-Jun Xiao and Arye Nehorai

Department of Electrical and Systems Engineering
Washington University, St. Louis, MO 63130
Emails: {xiao, nehorai}@ese.wustl.edu

ABSTRACT

We consider the synthesis of a spatially directional beam with full polarization control using an array of electromagnetic vector antennas (EMVA), among which each antenna consists of $p \geq 2$ orthogonal electric or magnetic dipole elements. We have shown in [1] that when $p = 2$, the vector array enables polarization control of the synthesized beam while the spatial power pattern remains the same as that achieved by the scalar array. In this paper, we study the high-dimensional case when $p \geq 3$. Our results indicate that the vector antenna array with $p \geq 3$ improves the power gain of the main beam (over the sidelobes), namely the gain is shown to be linearly proportional to the vector-antenna dimension p, in addition to enabling full polarization control of the beampattern. This implies that EMVA virtually increases the array size by exploiting the full electromagnetic (EM) field components, in addition to offering the freedom to control the beampattern polarization.

Index Terms**—** Waveform polarization, beampattern synthesis, electromagnetic vector antenna

1. INTRODUCTION

It is well known that exploiting waveform polarization improves the performance of active sensing systems such as radar [3, 4] and increases the capacity of communication systems. We consider the beampattern synthesis using an array of vector antennas. Each vector antenna in the array consists of $p \geq 2$ orthogonal electric or magnetic dipole elements. Our aim is to synthesize beampatterns with not only desired spatial power pattern, but also desired polarization. In traditional beampattern synthesis of a scalar array, radio signals from a set of small non-directional antenna elements are combined with different weights to achieve the beam spatial directionality. The beam emitted by such arrays are of a fixed polarization and can not controlled. To obtain a beam with full polarization control, we use an array of vector antennas. We design complex weights for individual array elements to achieve a beam with both desired spatial power density and desired polarizations. Our goals are to explore the potential of EMVA from the transmitter point of view. By exploiting the full EM field components, we study how vector antennas can achieve the polarization control and improve the spatial power pattern of the array.

The subject of the present paper which is different than the previous work in beamforming beampattern synthesis [5, 6] is that our work jointly designs the waveform polarization and the spatial power pattern, whereas the existing literature focuses on the latter only. We design the amplitudes and phases of the electric fields emitted from these dipole antennas to achieve the desired polarization control and spatial power pattern. By formulating the problem in an efficiently solvable convex optimization form, we can easily adopt various design criteria on the spatial power pattern and polarization by adding corresponding constraints. We have shown in [1] that when $p = 2$, the vector array enables polarization control of the synthesized beam while the spatial power pattern remains the same as that achieved by the scalar array. In this paper, we study the performance of the beampattern synthesis when $p \geq 3$. Our results indicate that for $p \geq 3$, vector antenna not only allows full polarization control of the beampattern, but also improves the power gain of the main beam, where the gain is linearly proportional to vector antenna dimensionality p. This implies that EMVA has advantage of both controlling the beampattern polarization and increasing the array size if full EM fields components are exploited.

2. PROBLEM FORMULATION

In this section we formulate the problem of the optimal beampattern synthesis with full polarization control. Consider a narrowband plane wave $\mathbf{E} = [E_H, E_V]^T = [\xi_H, \xi_V]^T s(t)$ traveling in a uniform medium in the 3-D space electric field, where ξ_H, ξ_V are two complex numbers, and $s(t)$ is the scalar complex envelope of the waveform. For ease of visualization, one efficient method of describing the waveform polarization is the so-called polarization ellipse. According to [2, Lemma 2.1], every non-zero $\boldsymbol{\xi} \stackrel{\text{def}}{=} [\xi_H,\ \xi_V]^T \in \mathbb{C}^2$ has a unique

The work is supported in part by the Department of Defense under the Air Force Office of Scientific Research MURI Grant FA9550-05-0443.

representation

$$\boldsymbol{\xi} = ||\boldsymbol{\xi}||e^{j\psi} \begin{bmatrix} \cos\mu & -\sin\mu \\ \sin\mu & \cos\mu \end{bmatrix} \begin{bmatrix} \cos\nu \\ j\sin\nu \end{bmatrix} \quad (1)$$

where $\psi \in (-\pi, \pi]$, $\mu \in (-\frac{\pi}{2}, \frac{\pi}{2}]$, $\nu \in [-\frac{\pi}{4}, \frac{\pi}{4}]$. Moreover, ψ, μ, ν in (1) are uniquely determined if and only if $\xi_H^2 + \xi_V^2 \neq 0$. Thus in the remaining part of the paper, we use the pair of angles (μ, ν) to describe the waveform polarization.

2.1. Vector antenna array

The antenna array we consider consists of N EMVA located at N positions in $\mathbb{R}^3$ with coordinates $\mathbf{x}_n : 1 \leq n \leq N$. Each antenna in the array further has p orthogonal dipole elements, where $1 \leq p \leq 6$. The antennas are driven by the same carrier signal with wavelength λ and convex envelope $s(t)$. The antenna currents (or weights) on the p dipoles in the n-th EMVA are denoted by (see Fig. 1)

$$\boldsymbol{w}_n = [w_n^{(1)},\ w_n^{(2)}, \ldots, w_n^{(p)}]^T; \quad 1 \leq n \leq N \quad (2)$$

We further introduce $\boldsymbol{w}$ to be the concatenation of all $\boldsymbol{w}_n$, *i.e.*,

$$\boldsymbol{w} = [\boldsymbol{w}_1^T,\ \boldsymbol{w}_2^T,\ \ldots, \boldsymbol{w}_N^T]^T.$$

It is easy to see that the dimensionality of $\boldsymbol{w}$ is $pN \times 1$.

Given the location $\mathbf{x}_n : 1 \leq n \leq N$ of the N antennas. The array response, as a function of spatial direction $\mathbf{r}$, can be expressed as $\mathbf{a}(\mathbf{r}) = [e^{-j\psi_1(\mathbf{r})}, e^{-j\psi_2(\mathbf{r})}, \ldots, e^{-j\psi_N(\mathbf{r})}]^T$, where $\psi_n(\mathbf{r}) = k\mathbf{r} \cdot \mathbf{x}_n$ and $k = 2\pi/\lambda$ is the wave number. To get the antenna array response of each vector antenna in the array, we suppose $\mathbf{r} = [\cos\theta\cos\phi,\ \cos\theta\sin\phi,\ \sin\theta]^T$ is a unit vector representing a spatial direction in $\mathbb{R}^3$. We further choose

$$\begin{aligned} \mathbf{r}_H &= [-\sin\phi,\ \cos\phi,\ 0], \\ \mathbf{r}_V &= [-\cos\phi\sin\theta,\ -\sin\phi\sin\theta,\ \cos\theta]. \end{aligned} \quad (3)$$

It is easy to see that $(\mathbf{r}, \mathbf{r}_H, \mathbf{r}_V)$ forms a right hand coordinate system. For a plane wave traveling along $\mathbf{r}$, its electric field is orthogonal to $\mathbf{r}$ and lies in the plane spanned by $(\mathbf{r}_H, \mathbf{r}_V)$.

We consider the type of vector antennas consisting of orthogonal electric and magnetic dipoles. Such vector antenna can consist of up to six dipoles: three electric dipoles and three magnetic along the x, y, and z-axis. In the coordinate system $(\mathbf{r}_H, \mathbf{r}_V)$, each of the six dipoles in the far field has the following responses [2]:

$$\begin{aligned} \mathbf{v}_x^{(E)}(\mathbf{r}) &= \begin{bmatrix} -\sin\phi & -\cos\phi\sin\theta \end{bmatrix} \\ \mathbf{v}_x^{(M)}(\mathbf{r}) &= \begin{bmatrix} -\cos\phi\sin\theta & \sin\phi \end{bmatrix} \end{aligned} \quad (4)$$

$$\begin{aligned} \mathbf{v}_y^{(E)}(\mathbf{r}) &= \begin{bmatrix} \cos\phi & -\sin\phi\sin\theta \end{bmatrix} \\ \mathbf{v}_y^{(M)}(\mathbf{r}) &= \begin{bmatrix} -\sin\phi\sin\theta & -\cos\phi \end{bmatrix} \end{aligned} \quad (5)$$

$$\begin{aligned} \mathbf{v}_z^{(E)}(\mathbf{r}) &= \begin{bmatrix} 0 & \cos\theta \end{bmatrix} \\ \mathbf{v}_z^{(M)}(\mathbf{r}) &= \begin{bmatrix} \cos\theta & 0 \end{bmatrix} \end{aligned} \quad (6)$$

For a vector antenna consisting of p dipole elements. We use $\mathbf{V}(\mathbf{r})$ to denote the antenna response. We will consider antenna types with $2 \leq p \leq 6$. Thus, in terms of the vector antenna response $\mathbf{V}(\mathbf{r})$ (which has dimension $p \times 2$), we further obtain that the vector antenna array response is $\mathbf{A}(\mathbf{r}) = \mathbf{a}(\mathbf{r}) \otimes \mathbf{V}(\mathbf{r})$, which has dimensionality $pN \times 2$.

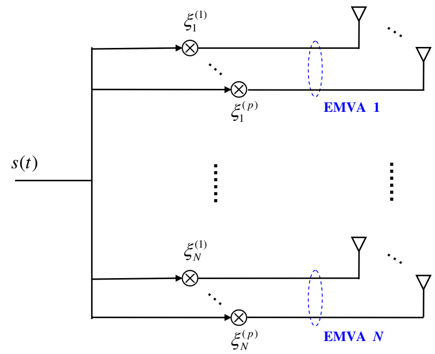

Fig. 1. Vector antenna array

The normalized electrical field emitted from the antenna array (by ignoring the common carrier and the baseband signal $s(t)$) can be expressed as $\mathbf{E}(\mathbf{r}) = \mathbf{A}(\mathbf{r})^T\boldsymbol{w}$. Let us use $E(\mathbf{r}; H)$ and $E(\mathbf{r}; V)$ to denote the decomposition of $\mathbf{E}(\mathbf{r})$ along H and V. We immediately have

$$\begin{aligned} E(\mathbf{r}; H) &= \mathbf{A}(\mathbf{r}; H)^T\boldsymbol{w}, \\ E(\mathbf{r}; V) &= \mathbf{A}(\mathbf{r}; V)^T\boldsymbol{w}. \end{aligned}$$

Along $\mathbf{r}$, the wave polarization is determined by the ratio between $E(\mathbf{r}; V)$ and $E(\mathbf{r}; H)$, and the radiated energy can be expressed as $||\mathbf{E}(\mathbf{r})||^2 = |E(\mathbf{r}; H)|^2 + |E(\mathbf{r}; V)|^2$.

2.2. Polarized beampattern synthesis

The problem of beampattern synthesis is to design the antenna weights $\boldsymbol{w}$ to achieve a desired wave pattern. In a scalar array, the goal is merely to control spatial power pattern. However, for the array of vector antennas, we can further achieve the control of beam polarization. Specifically, our goal is to synthesize a beampattern with the following properties:

i) The mainbeam, assumed pointing at a direction $\mathbf{r}_0$, has desired power P and polarization (μ, ν);

ii) The power of sidelobes in an interested region (denoted by $\mathcal{S}_\mathbf{r}$) are suppressed.

Compared with the scalar array, the beampattern synthesis in a vector array enforces an additional polarization constraint on the main beam.

There are various criteria in suppressing the sidelobe power while maintaining the mainlobe power and polarization. Two conventional criteria are the total sidelobe power minimization or the maximum sidelobe power minimization.

We focus on the latter only in this paper and this leads to the following optimization problem:

$$\begin{aligned} \min_{\boldsymbol{w}} \quad & \max_{\mathbf{r}_s \in \mathcal{S}_{\mathbf{r}}} ||\mathbf{E}(\mathbf{r}_s)||^2 \\ \text{s.t.} \quad & \mathbf{E}(\mathbf{r}_0) = \sqrt{P}e^{j\phi} \begin{bmatrix} \cos\mu & -\sin\mu \\ \sin\mu & \cos\mu \end{bmatrix} \begin{bmatrix} \cos\nu \\ j\sin\nu \end{bmatrix} \\ & \mathbf{E}(\mathbf{r}) = \mathbf{A}(\mathbf{r})^T \boldsymbol{w}, \quad \forall\, \mathbf{r} \end{aligned}$$

To obtain the optimal weights $\boldsymbol{w}$ for the Np antenna elements, we need to solve the above optimization problem. To remove the "max" operation in the objective function and transform it into a standard function, we introduce an auxiliary variable τ. We further assume $\sqrt{P}e^{j\phi} = 1$ without loss of generality. Then we obtain the following problem

$$\begin{aligned} \min_{\boldsymbol{w},\,\tau} \quad & \tau \\ \text{s.t.} \quad & ||\mathbf{E}(\mathbf{r}_s)||^2 \leq \tau; \quad \forall\, \mathbf{r}_s \in \mathcal{S}_{\mathbf{r}} \qquad (7) \\ & \mathbf{E}(\mathbf{r}_0) = \begin{bmatrix} \cos\mu & -\sin\mu \\ \sin\mu & \cos\mu \end{bmatrix} \begin{bmatrix} \cos\nu \\ j\sin\nu \end{bmatrix} \\ & \mathbf{E}(\mathbf{r}) = \mathbf{A}(\mathbf{r})^T \boldsymbol{w}, \quad \forall\, \mathbf{r} \end{aligned}$$

It is easy to see that the above problem is convex, and is in fact a second order cone programming (SOCP) [7]. SOCP is a special class of convex optimization problem, and therefore enjoys all the advantages of convexity. There are well-developed numerical methods to solve a general convex optimization problem, among which the most well known one is the interior point method. the numerical examples in Section 3, we adopt an optimization toolbox: SeDuMi [8] to solve the SOCP formulated above. SeDuMi, which stands for Self-Dual-Minimisation, is a software package that solves optimization problems over symmetric cones using the primal-dual interior-point methods.

3. NUMERICAL RESULTS

In this section we give numerical examples to show the performance of beampattern synthesis. We will consider both cases when $p = 2$ and when $p \geq 3$. Since the formulated problem in (7) is convex, we adopt the optimization toolbox: SeDuMi [8] to solve for the optimal solution.

3.1. 2-D arrays: vector vs. scalar

The analysis in [1] implies that for the 2-D vector arrays with co-aligned electric and magnetic dipoles, the achievable spatial power pattern is identical to that achieved by the scalar arrays consisting of either electric or magnetic dipoles. However, the vector array has the advantage of enabling the control of the polarization.

Fig. 2 shows the computer simulation of an 18 antennas in the array. The antennas are located at the three axes with coordinates:

$$\{\mathbf{x}_n : 1 \leq n \leq 18\} = \left\{ \left[(m-3.5)\frac{\lambda}{2}, 0, 0\right]^T : 1 \leq m \leq 6 \right\}$$
$$\bigcup \left\{ \left[0, (m-3.5)\frac{\lambda}{2}, 0\right]^T : 1 \leq m \leq 6 \right\}$$
$$\bigcup \left\{ \left[0, 0, (m-3.5)\frac{\lambda}{2}\right]^T : 1 \leq m \leq 6 \right\} \qquad (8)$$

where λ is the wavelength. Still in the vector array, each antenna consists of one electric dipole and one magnetic dipole both are pointing along the z-axis. The two scalar arrays are compromised of electric dipoles only or magnetic dipoles only. The main beam points at direction $\mathbf{r}_0 = [\theta(\mathbf{r}_0), \phi(\mathbf{r}_0)] = [45^o, 45^o]$ with beamwidth 5^o for both the elevation and azimuth angles. As can be seen, three arrays achieve the same spatial power pattern, while the vector array in addition achieve a mainbeam control where in this example, the polarization of the mainbeam is set to be $(\mu, \nu) = [-30^o, 45^o]$.

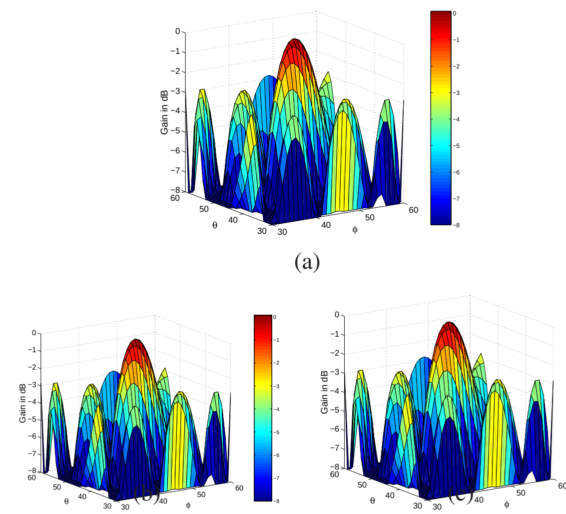

Fig. 2. Comparison of power gain of the beampattern synthesis: (a) vector array; (b) array of electric dipoles; (c) array of magnetic dipoles.

3.2. High-dimensional arrays: power gain vs. sensor dimensionality

In this subsection we analyze by simulations the performance of beampattern synthesis using arrays of high dimensional antenna arrays. We still use the array size 18 with antennas located at the x, y, and z-axis as given in (8). We will consider arrays of vectors antennas with $2 \leq p \leq 6$. For each p, we choose dipole elements according to Table 1. We note that at each row, the symbol "$\surd$" denotes a certain dipole element is included in the antenna. For example, when $p = 2$, one electric dipole and one magnetic dipole at the z-axis are selected. Correspondingly, the antenna response $\mathbf{V}(\mathbf{r}) = [\mathbf{v}_z^{(E)}(\mathbf{r}),\ \mathbf{v}_z^{(M)}(\mathbf{r})]$ (c.f., (4)–(6)). In addition, in these examples, we choose $\mathbf{r}_0 = [\theta(\mathbf{r}_0), \phi(\mathbf{r}_0)] = [45^o, 45^o]$. The polarization of the mainbeam is set to be $(\mu, \nu) = [30^o, 45^o]$.

Table 1. Vector antenna dipole elements for $2 \leq p \leq 6$ that are used in the simulations.

	$\mathbf{v}_x^{(E)}$	$\mathbf{v}_y^{(E)}$	$\mathbf{v}_z^{(E)}$	$\mathbf{v}_x^{(M)}$	$\mathbf{v}_y^{(M)}$	$\mathbf{v}_z^{(M)}$
$p=2$			$\surd$			$\surd$
$p=3$	$\surd$	$\surd$	$\surd$			
$p=4$		$\surd$	$\surd$		$\surd$	$\surd$
$p=5$	$\surd$	$\surd$	$\surd$		$\surd$	$\surd$
$p=6$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$

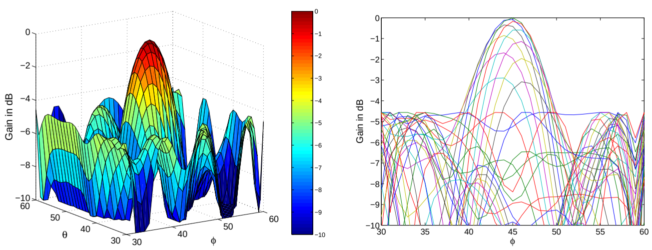

Fig. 3. Illustration of mainbeam power gain: $p=3$.

The achieved spatial power patterns for $p = 3, 4, 6$ are given in Fig. 3–Fig. 4 respectively. For each case, the top figure plots the two-dimensional power pattern in plane of the elevation and azimuth angles (θ, ϕ). In the bottom figure, each curve shows the the slice of the power pattern versus θ for fixed ϕ. As cab ne seen, once the sensor dimensionality p increases, the mainbeam power gain versus the sidelobes increases. As plotted in Fig. 5, all three curves for different array size $N = 6, 12, 18$ show that such gain is almost linearly proportional to p. We thus conclude that EMVA array has advantage of i) enabling the control of the beampattern polarization; and ii) virtually increasing the array size since multiple EM fields at each antenna is exploited.

4. CONCLUSION

In this paper we have considered the performance of beampattern synthesis with polarization constraints using an array of multiple vector antennas consisting of electric and magnetic dipole elements. We have shown that vector antenna array with antenna dimension $p \geq 3$ not only enables polarization control but also improves the power gain of the synthesized spatial beampattern. Our numerical results indicate that the power gain achieved by the array is linearly proportional to p. This reveals that for a vector antenna array, it not only enables polarization control, but also virtually increases the array size by exploiting multiple EM fields at each physical point.

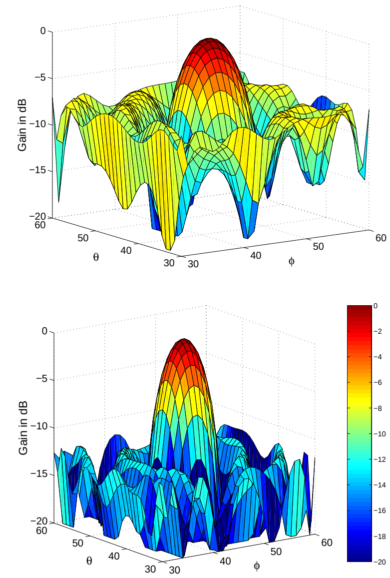

Fig. 4. Illustration of mainbeam power gain: $p=4$ (top) and $p=6$ (bottom).

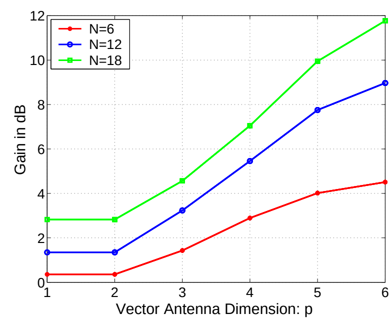

Fig. 5. Power gain versus antenna dimensionality p.

5. REFERENCES

[1] J.-J. Xiao and A. Nehorai, "Optimal beam pattern synthesis of a polarized array," *Proc. of IEEE Statistical Signal Processing Workshop*, Madison, WI, Aug. 2007.

[2] A. Nehorai and E. Paldi, "Vector-sensor array processing for electromagnetic source localization," *IEEE Trans. on Signal Processing*, vol. 42, pp. 376-398, Feb. 1994.

[3] M. Hurtado and A. Nehorai, "Polarimetric detection of targets in heavy inhomogeneous clutter," to appear in *IEEE Trans. on Signal Processing*.

[4] M. Hurtado, T. Zhao, and A. Nehorai, "Adaptive polarized waveform design for target tracking based on sequential Bayesian inference," *IEEE Trans. on Signal Processing*, vol. 56, no. 3, pp. 1120-1133, Mar. 2008.

[5] H. Lebret and S. Boyd, "Antenna array pattern synthesis via convex optimization," *IEEE Trans. on Signal Processing*, vol. 45, no. 3, pp. 526-532, Mar. 1997.

[6] J. Li and P. Stoica, *Robust Adaptive Beamforming*, John Wiley & Sons, New York, NY, 2005.

[7] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge Univ. Press, Cambridge, U.K., 2003.

[8] J. F. Sturm, "Using SeDuMi 1.02, a Matlab toolbox for optimization over symmetric cones," *http://www.optimization-online.org/DB_HTML/2001/10/395.html*.