2007 USNC / URSI North American Radio Science Meeting

July 22 - 26, 2007, Ottawa, Canada

An Incremental Theory of Diffraction formulation for the scattering by a thin elliptical cylinder, a strip, or a slit in a conducting surface

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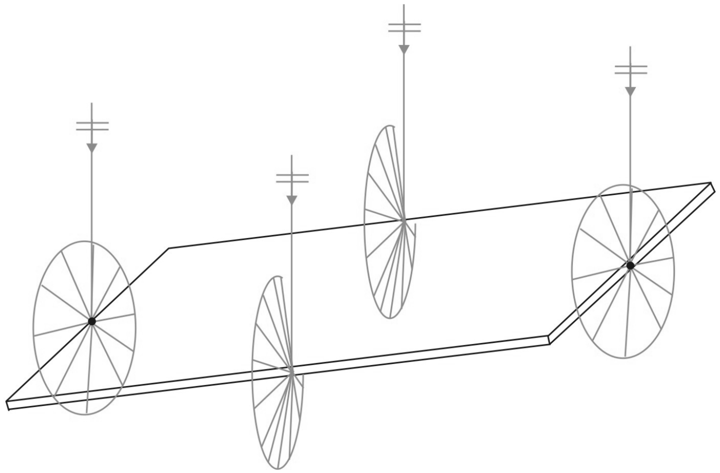
MOTIVATIONS

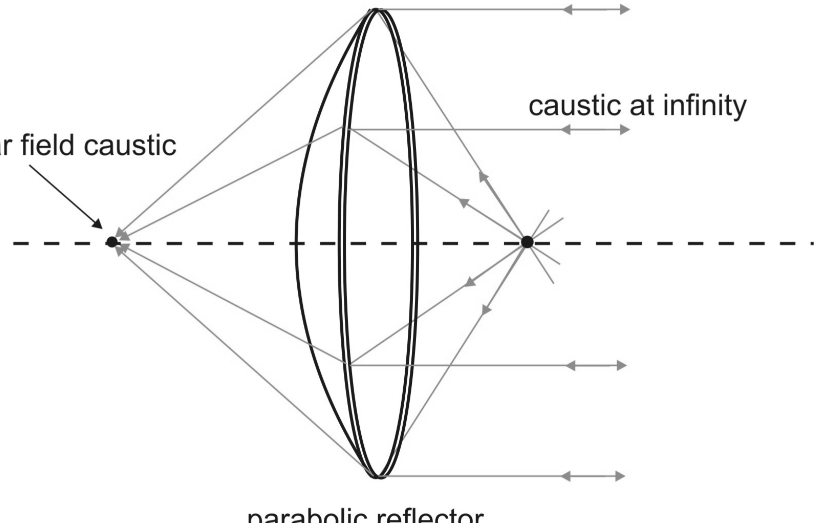
- The Incremental Theory of Diffraction (ITD) formulation has been introduced to evaluate the field scattered by moderately sized, local circular cylinder-shaped structures.

Advantages

→ overcomes difficulties in applying in ray-field descriptions close and at **caustics**

→ smoothly blends into the field predicted by UTD (when applicable)





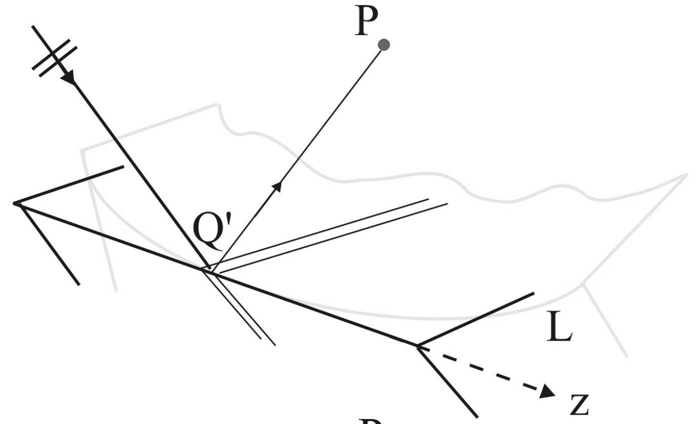
→ improves upon the field estimate whenever a stationary phase condition has not yet been established

→ augments Physical Optics (fringe formulation)

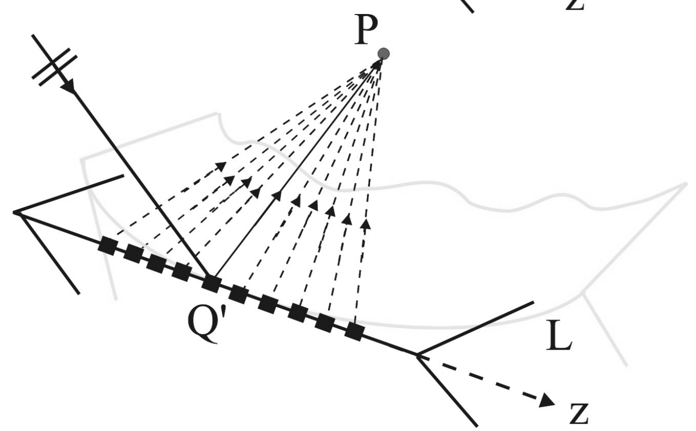
BACKGROUND

- Incremental field contributions may be deduced from either the currents or the field of local canonical problems

  - Elementary Edge Waves (PTD)



  - Incremental Theory of Diffraction (ITD)

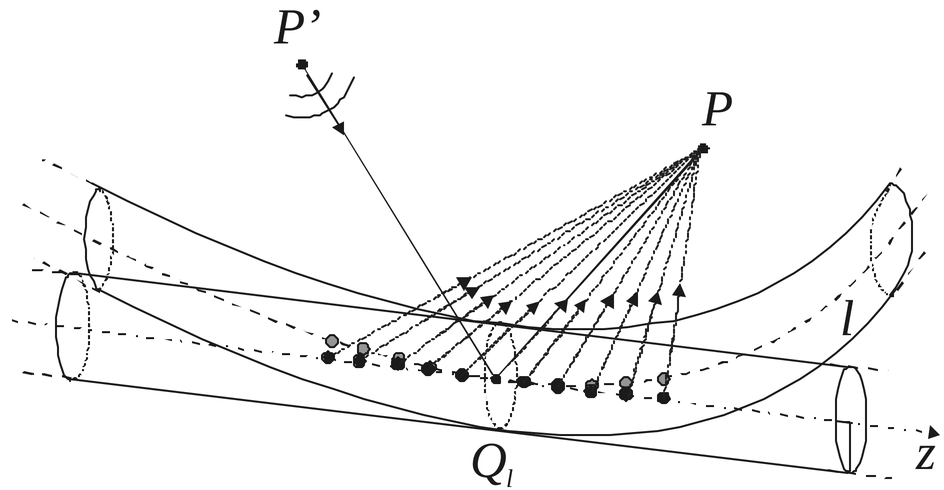


- To extend the applicability of the ITD approach

ELLIPTIC CYLINDER SHAPED CONFIGURATION

ITD LOCALIZATION PROCESS

$$\Psi^t\left(P\right)=\Psi^i\left(P\right)+\Psi^s\left(P\right)$$



- Scattered field representation

$$\Psi^s\left(P\right)=\int_l \psi\left(Q_l\right)dl$$

- The incremental field $\psi(Q_l)$ may be deduced from an appropriate local canonical problem tangent at Q_l.

For this purpose, we need to find a convenient field representation for the exact solution of the local canonical problem

$$\Psi_c^s\left(P\right)=\int_{-\infty}^{\infty}\psi_c\left(z''\right)dz''$$

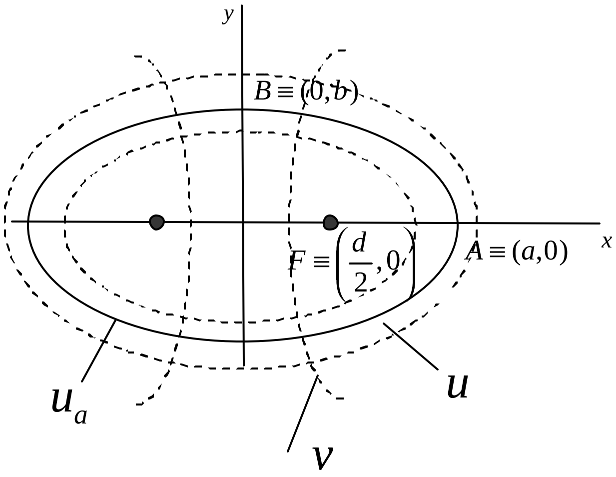
- Then, at high-frequency, it is assumed that

$$y\left(Q_l\right)=y_c\left(z''\right)\Big|_{z''=0}$$

INCREMENTAL FIELD CONTRIBUTION

- Spectral synthesis

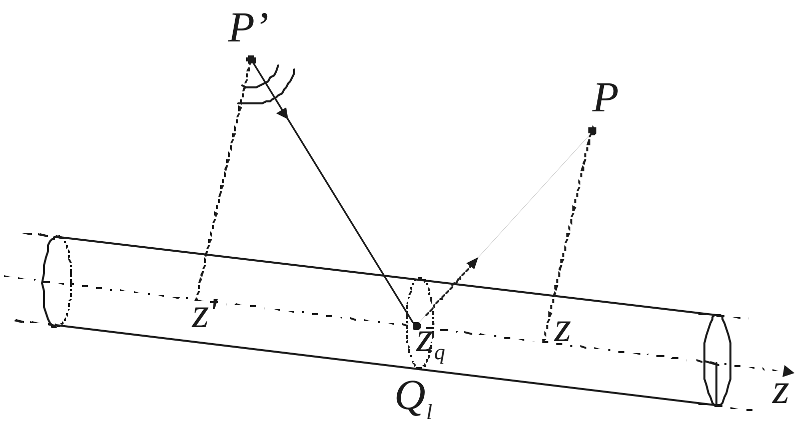
$$x = \frac{d}{2}\cosh u \cos v$$
$$y = \frac{d}{2}\sinh u \sin v$$
$$e = d/2a$$



$$G^{3D}(r,r') = \frac{1}{2p}\int_{-\infty}^{\infty} \overbrace{U(k_r,u,v)\cdot U(k_r,u',v')}^{G^{2D}(k_r,u,v,u',v')} e^{-jk_z(z-z')}\,dk_z$$

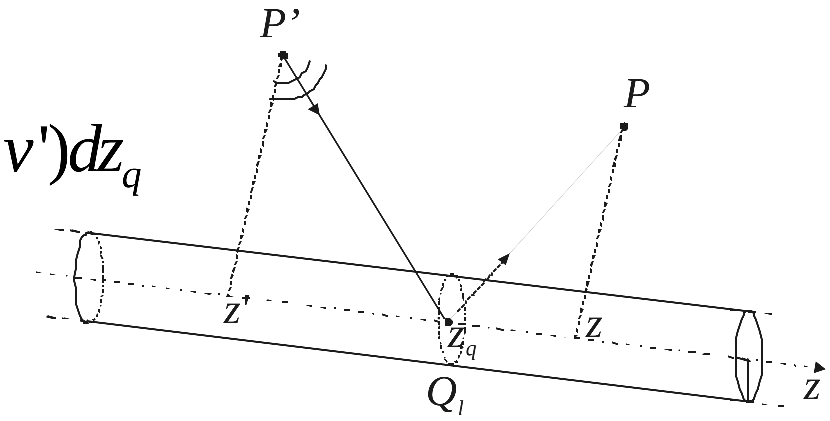
- By Fourier analysis, this spectral integral representation is interpreted as the spatial convolution product of two functions

$$G^{3D}(r,r') = \int_{-\infty}^{\infty} u(z_q - z,u,v)\cdot u(z'-z_q,u',v')\,dz_q$$



INCREMENTAL FIELD CONTRIBUTION

$$G^{3D}\left(r,r'\right)=\int_{-\infty}^{\infty} u(z_q - z,u,v)\cdot u(z'-z_q,u',v')dz_q$$



- This above spatial integral representation allows to directly define the local incremental field contribution

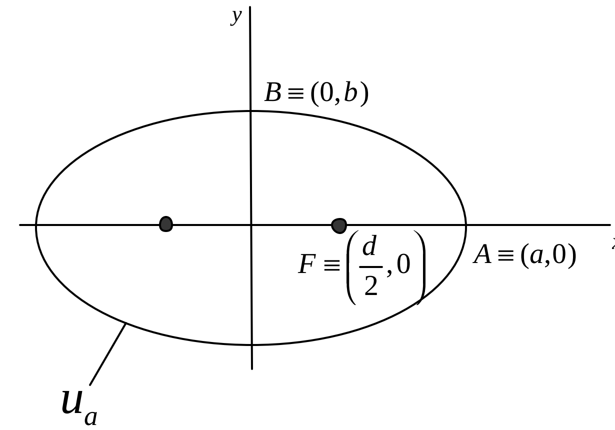
$$y\,(z_q)\Big|_{z_q=0} = u(-z,u,v)\cdot u(z',u',v') = F^{-1}\left[U(k_r,u,v)\right]\cdot F^{-1}\left[U(k_r^{'},u',v')\right]$$

Inverse Fourier transform

LOCAL CANONICAL PROBLEM

- Canonical solution (spectral synthesis) for an infinite elliptic cylinder with both source and observation point at finite distance

$$G(\mathbf{r},\mathbf{r}') = G^{e}(\mathbf{r},\mathbf{r}') + G^{o}(\mathbf{r},\mathbf{r}')$$



$$G^{e,o}(\mathbf{r},\mathbf{r}') = -j\sum_{n=0}^{\infty}\frac{1}{\Omega_n^{(e,o)}}G_n^{e,o}(\mathbf{r},\mathbf{r}')$$

$$G_n^{e,o}(\mathbf{r},\mathbf{r}') = \frac{1}{2p}\int_{-\infty}^{\infty} C_n^{e,o}\left(k_r''\frac{d}{2},u_a\right) S_{(e,o)_n}\left(k_r''\frac{d}{2},v\right) R_{(e,o)_n}^{(4)}\left(k_r''\frac{d}{2},u\right)\cdot$$
$$S_{(e,o)_n}\left(k_r''\frac{d}{2},v'\right) R_{(e,o)_n}^{(4)}\left(k_r''\frac{d}{2},u'\right) e^{-jk_z''(z-z')}\,dk_z''$$

$$k_r'' = \sqrt{k^2 - \left(k_z''\right)^2}$$

LOCAL CANONICAL PROBLEM

- Canonical solution (spectral synthesis) for an infinite elliptic cylinder with both source and observation point at finite distance

$$G_n^{e,o}(\mathbf{r},\mathbf{r}') = \frac{1}{2p}\int_{-\infty}^{\infty} C_n^{e,o}\left(k_r''\frac{d}{2},u_a\right) S_{(e,o)_n}\left(k_r''\frac{d}{2},v\right) R_{(e,o)_n}^{(4)}\left(k_r''\frac{d}{2},u\right)\cdot$$

$$S_{(e,o)_n}\left(k_r''\frac{d}{2},v'\right) R_{(e,o)_n}^{(4)}\left(k_r''\frac{d}{2},u'\right) e^{-jk_z''(z-z')}dk_z''$$

$$C_n^{e,o}\left(k_r''\frac{d}{2},u_a\right) = \frac{\mathrm{R}_{(e,o)_n}^{(1)}\left(k_r''\frac{d}{2},u_a\right)}{\mathrm{R}_{(e,o)_n}^{(4)}\left(k_r''\frac{d}{2},u_a\right)}$$

soft b.c.

first derivative

$$C_n^{e,o}\left(k_r''\frac{d}{2},u_a\right) = \frac{\mathrm{R}_{(e,o)_n}^{(1)'}\left(k_r''\frac{d}{2},u_a\right)}{\mathrm{R}_{(e,o)_n}^{(4)'}\left(k_r''\frac{d}{2},u_a\right)}$$

hard b.c.

- **modal series representation**, rapidly converges for **small** (moderately sized) **elliptic cylinders**

THE APPROPRIATE SPECTRUM FUNCTIONS

- The GF is cast in the form of the product of 2 spectrum functions, depending only on either the incidence or the observation aspects (soft case)

$$C_n^{e,o}\left(k_r''\frac{d}{2},u_a\right)=\frac{\mathrm{R}^{(1)}_{(e,o)_n}\left(k_r''\frac{d}{2},u_a\right)}{\mathrm{R}^{(4)}_{(e,o)_n}\left(k_r''\frac{d}{2},u_a\right)}=s_n^{e,o}\left(k_r''\frac{d}{2},u_a\right)s_n^{e,o}\left(k_r''\frac{d}{2},u_a\right)$$

$$G_n^{e,o}(\mathrm{r},\mathrm{r}')=\frac{1}{2p}\int_{-\infty}^{\infty}\left[s_n^{e,o}\left(k_r''\frac{d}{2},u_a\right)S_{(e,o)_n}\left(k_r''\frac{d}{2},v\right)R^{(4)}_{(e,o)_n}\left(k_r''\frac{d}{2},u\right)\right]\left[s_n^{e,o}\left(k_r''\frac{d}{2},u_a\right)S_{(e,o)_n}\left(k_r''\frac{d}{2},v'\right)R^{(4)}_{(e,o)_n}\left(k_r''\frac{d}{2},u'\right)\right]e^{-jk_z''(z-z')}dk_z''$$

$\Downarrow$ $U_n\left(k_r'',u,v\right)$ $\qquad$ $\Downarrow$ $U_n\left(k_r'',u',v'\right)$

- Use of the ITD FT-convolution process yields

$$u_n(-z,u,v)\cdot u_n(z',u',v')=\frac{1}{2p}\int_{-\infty}^{\infty}U_n\left(k_r'',u,v\right)e^{-jk_z z}dk_z\cdot\frac{1}{2p}\int_{-\infty}^{\infty}U_n\left(k_r'',u',v'\right)e^{-jk_z' z'}dk_z'$$

HIGH-FREQUENCY EXPRESSIONS

$$u_n(-z,u,v) = \frac{1}{2p}\int_{-\infty}^{\infty} s_n^{e,o}\left(k_r''\frac{d}{2},u_a\right)S_{(e,o)_n}\left(k_r\frac{d}{2},v\right)R_{(e,o)_n}^{(4)}\left(k_r\frac{d}{2},u\right)e^{-jk_z z}dk_z$$

- Asymptotic expression of $R_{(e,o)_n}^{(4)}(c,u)$

$$R_{(e,o)_n}^{(4)}\left(k_r\frac{d}{2},u\right) \sim \frac{1}{\sqrt{k_r\frac{d}{2}\cosh u}}e^{-j\left[k_r\frac{d}{2}\cosh u-(2n+1)p/4\right]} \qquad k_r\frac{d}{2}\cosh u \to \infty$$

- Since $\frac{d}{2}\cosh u \to r$ for large u

$$R_{(e,o)_n}^{(4)}\left(k_r\frac{d}{2},u\right) \sim \frac{1}{\sqrt{k_r r}}e^{-j\left[k_r r-(2l+1)p/4\right]} \qquad u \to \infty$$

HIGH-FREQUENCY EXPRESSIONS

- Useful representation of $u_n(-z,u,v)$ for large u

$$u_n(-z,u,v) \approx \frac{e^{j(2l+1)p/4}}{2p} \int_{C_q} s_n^{e,o}\left(k_r'' \frac{d}{2}, u_a \right) S_{(e,o)_n}\left(k_r \frac{d}{2}, v \right) \frac{1}{\sqrt{r \sin b}}\, e^{-jkr\cos(q-b)} \sqrt{k \sin q}\, dq$$

- Asymptotic evaluation for large r

$$u_n(-z,u,v) \approx \frac{e^{j(n+1)p/2}}{\sqrt{2p}}\, s_n^{e,o}\left(k \sin b \frac{d}{2}, u_a \right) S_{(e,o)_n}\left(k \sin b \frac{d}{2}, v \right) \frac{e^{-jkr}}{r}$$

- By analogy

$$u_n(z',u',v') \approx \frac{e^{j(n+1)p/2}}{\sqrt{2p}}\, s_n^{e,o}\left(k \sin b' \frac{d}{2}, u_a \right) S_{(e,o)_n}\left(k \sin b' \frac{d}{2}, v' \right) \frac{e^{-jkr'}}{r'}$$

HIGH-FREQUENCY EXPRESSIONS

- Incremental contribution for a soft elliptic cylinder

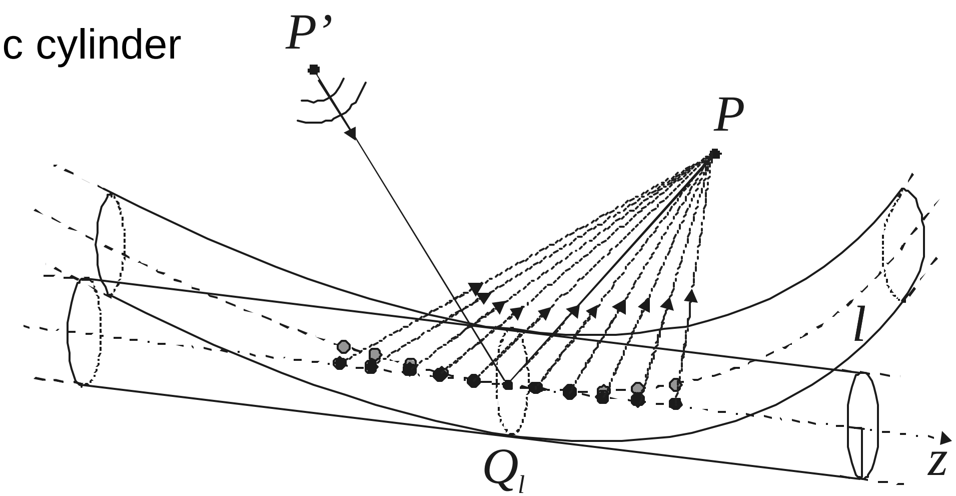
$$y_n^{e,o}(z_q)\Big|_{z_q=0} = \frac{e^{j(n+1)p}}{2p}\left(\frac{\mathrm{R}^{(1)}_{(e,o)_n}\left(k\sin b\,\frac{d}{2},u_a\right)}{\mathrm{R}^{(4)}_{(e,o)_n}\left(k\sin b\,\frac{d}{2},u_a\right)}\right)^{\frac{1}{2}} S_{(e,o)_n}\left(k\sin b\,\frac{d}{2},v\right)\cdot$$

$$\left(\frac{\mathrm{R}^{(1)}_{(e,o)_n}\left(k\sin b'\,\frac{d}{2},u_a\right)}{\mathrm{R}^{(4)}_{(e,o)_n}\left(k\sin b'\,\frac{d}{2},u_a\right)}\right)^{\frac{1}{2}}\cdot S_{(e,o)_n}\left(k\sin b'\,\frac{d}{2},v'\right)\frac{e^{-jkr}}{r}\frac{e^{-jkr'}}{r'}$$

- Scattered scalar field by the actual elliptic cylinder

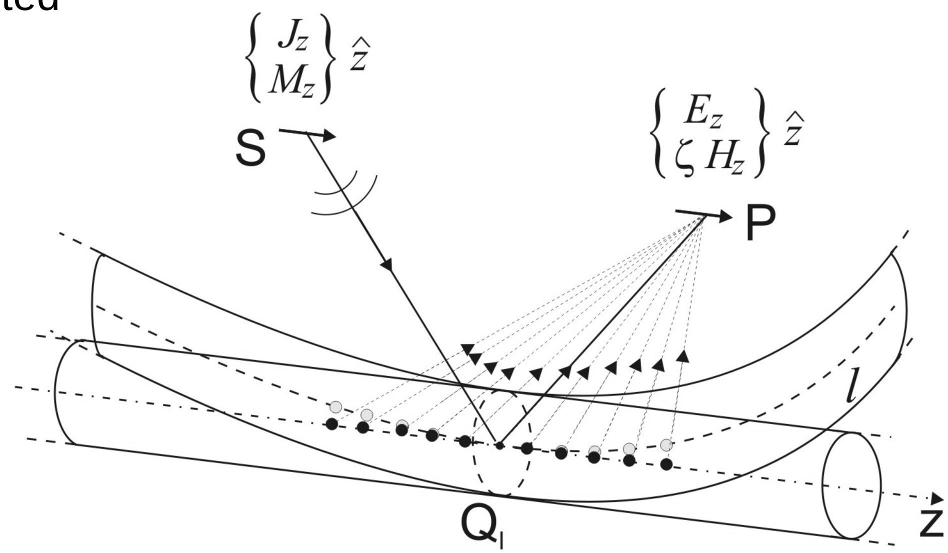
$$\Psi^{e,o}\left(\boldsymbol{r},\boldsymbol{r}'\right) = -j\sum_{n=0}^{\infty}\frac{1}{\Omega_n^{(e,o)}}\int_l y_n^{e,o}(Q_l)\,dl$$

$$\Psi\left(\boldsymbol{r},\boldsymbol{r}'\right) = \Psi^{e}\left(\boldsymbol{r},\boldsymbol{r}'\right) + \Psi^{o}\left(\boldsymbol{r},\boldsymbol{r}'\right)$$



ELECTROMAGNETIC CASE

- In the electromagnetic case for $\hat{z}$ directed dipole illumination and observation



$\Psi^{e,o}_{s,h}$ ⟶ EM Vector Potentials

soft hard

$\Psi^{e,o}_{s}$ ⟶ A^{j}_{z}

$\Psi^{e,o}_{h}$ ⟶ F^{m}_{z}

$$\left(E^{j}_{z}, z\, H^{m}_{z}\right) = -jw \frac{k_r^2}{k^2} \Psi^{e,o}_{s,h}$$

Thus, $\left(d E^{s}_{z}, z\, d H^{s}_{z}\right)_n$ is obtained after replacing in $y^{e,o}_{n}$

$U_n\left(k_r^{"}, u^{(')}, v^{(')}\right)$ by $k_r^{(')} U_n\left(k_r^{"}, u^{(')}, v^{(')}\right)$

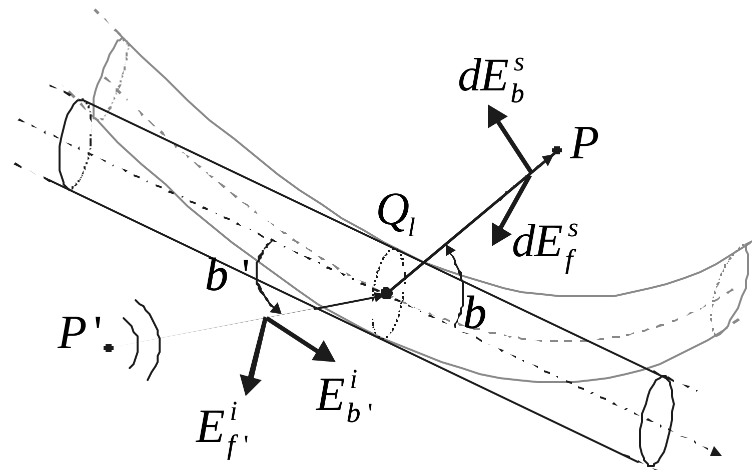
THE DYADIC SCATTERING COEFFICIENT

- **Dyadic closed-form high-frequency** expression for the ITD incremental scattered field contribution

$$\begin{pmatrix} dE_b(P) \\ dE_f(P) \end{pmatrix} = \begin{pmatrix} D_s(b, b', f, f') & 0 \\ 0 & D_h(b, b', f, f') \end{pmatrix} \begin{pmatrix} E^i_{b'}(Q_l) \\ E^i_{f'}(Q_l) \end{pmatrix} \frac{e^{-jkr}}{4p\, r}$$

$$D_{s,h}(b, b', f, f') = -j \sum_{n=0}^{\infty} \frac{1}{\Omega_n^{(e)}} y^{\,e}_{(s,h)_n}(Q_l) - j \sum_{n=0}^{\infty} \frac{1}{\Omega_n^{(o)}} y^{\,o}_{(s,h)_n}(Q_l)$$

- The expected transitional behavior of the field is reconstructed by numerical integration of the incremental contributions along the curved axis of the actual cylindrical configuration.



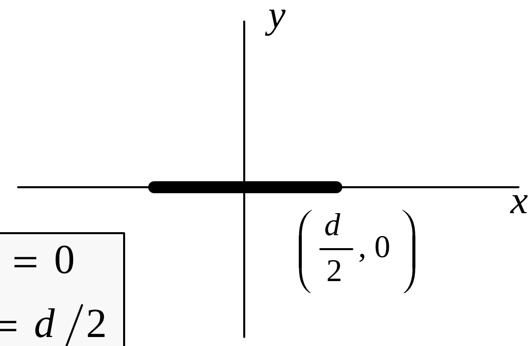
$$y^{\,e,o}_{(s,h)_n}(Q_l) = \frac{e^{j(n+1)p}}{2p} s^{\,e,o}_{(s,h)_n}\left(k \sin b \frac{d}{2}, u_a\right) s^{\,e,o}_{(s,h)_n}\left(k \sin b' \frac{d}{2}, u_a\right) S_{(e,o)_n}\left(k \sin b \frac{d}{2}, v\right) S_{(e,o)_n}\left(k \sin b' \frac{d}{2}, v'\right)$$

INCREMENTAL SCATTERING BY A STRIP

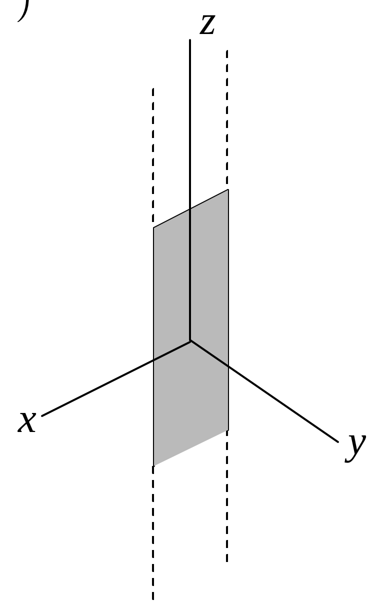
- ITD coefficient for a strip
- the odd part vanishes (soft case)
- the even part vanishes (hard case)

$$D_s\left(b, b', f, f'\right) = -j\sum_{n=0}^{\infty}\frac{1}{\Omega_n^{(e)}} y_{s_n}^{e}(Q_l)$$

$$D_h\left(b, b', f, f'\right) = -j\sum_{n=0}^{\infty}\frac{1}{\Omega_n^{(o)}} y_{h_n}^{o}(Q_l)$$



$$u_a = 0$$
$$a = d/2$$
$$e = 1$$



$$y_{s_n}^{e}(Q_l) = \frac{e^{j(n+1)p}}{2p} s_{s_n}^{e}\left(k\sin b\,\frac{d}{2}, 0\right) s_{s_n}^{e}\left(k\sin b'\,\frac{d}{2}, 0\right) S_{e_n}\left(k\sin b\,\frac{d}{2}, v\right) S_{e_n}\left(k\sin b'\,\frac{d}{2}, v'\right)$$

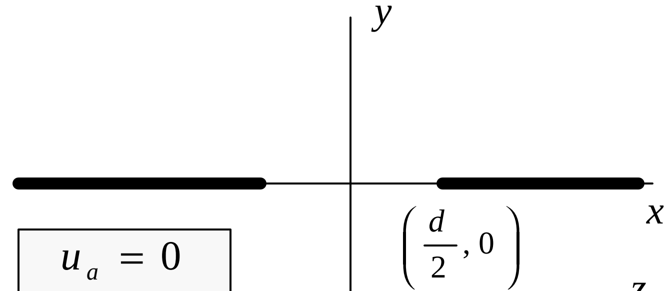
$$y_{h_n}^{o}(Q_l) = \frac{e^{j(n+1)p}}{2p} s_{h_n}^{o}\left(k\sin b\,\frac{d}{2}, 0\right) s_{h_n}^{o}\left(k\sin b'\,\frac{d}{2}, 0\right) S_{o_n}\left(k\sin b\,\frac{d}{2}, v\right) S_{o_n}\left(k\sin b'\,\frac{d}{2}, v'\right)$$

$$s_{s_n}^{e}\left(k\sin b^{(')}\,\frac{d}{2}, 0\right) = \left(\frac{R_{e_n}^{(1)}\left(k\sin b^{(')}\,\frac{d}{2}, 0\right)}{R_{e_n}^{(4)}\left(k\sin b^{(')}\,\frac{d}{2}, 0\right)}\right)^{\frac{1}{2}}$$

$$s_{h_n}^{o}\left(k\sin b^{(')}\,\frac{d}{2}, 0\right) = \left(\frac{R_{o_n}^{(1)'}\left(k\sin b^{(')}\,\frac{d}{2}, 0\right)}{R_{o_n}^{(4)'}\left(k\sin b^{(')}\,\frac{d}{2}, 0\right)}\right)^{\frac{1}{2}}$$

INCREMENTAL SCATTERING BY A SLIT

- ITD coefficient for a slit (dual of the strip)
- the even part vanishes (soft case)
- the odd part vanishes (hard case)



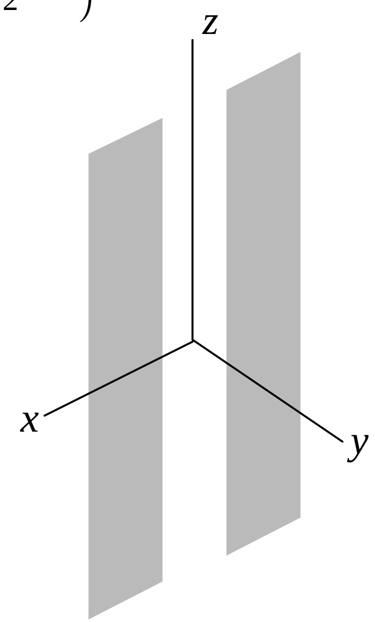
$u_a = 0$

$a = d/2$

$e = 1$

$$D_s\left(b,b',f,f'\right) = -j\sum_{n=0}^{\infty}\frac{1}{\Omega_n^{(o)}}y^{o}_{s_n}(Q_l)$$

$$D_h\left(b,b',f,f'\right) = -j\sum_{n=0}^{\infty}\frac{1}{\Omega_n^{(e)}}y^{e}_{h_n}(Q_l)$$



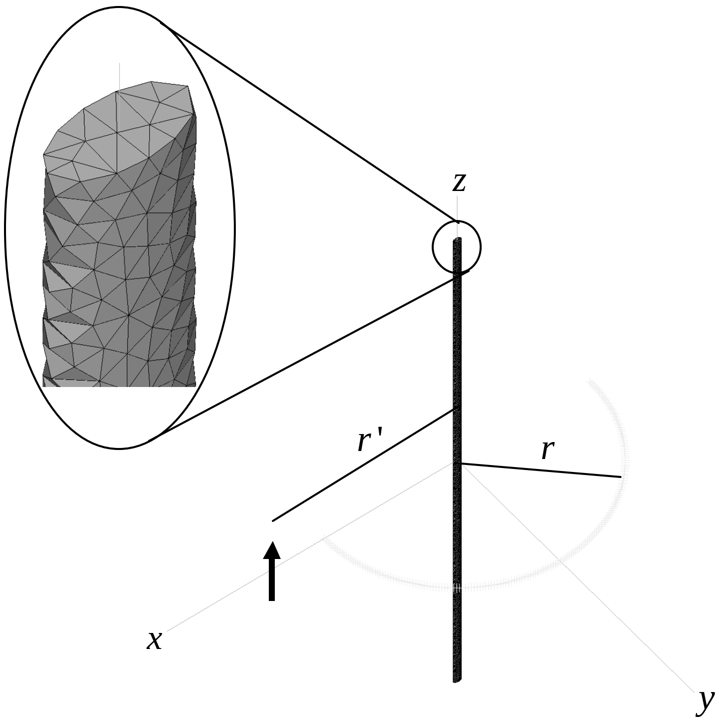
$$y^{o}_{s_n}(Q_l) = \frac{e^{j(n+1)p}}{2p}s^{o}_{s_n}\left(k\sin b\,\frac{d}{2},0\right)s^{o}_{s_n}\left(k\sin b'\,\frac{d}{2},0\right)S_{o_n}\left(k\sin b\,\frac{d}{2},v\right)S_{o_n}\left(k\sin b'\,\frac{d}{2},v'\right)$$

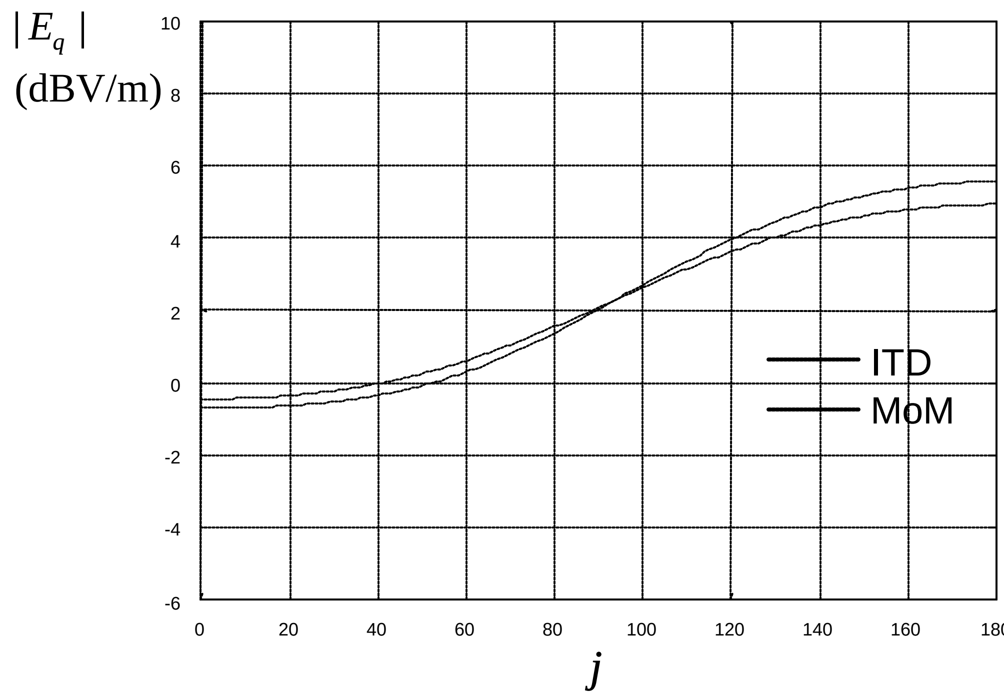
$$y^{e}_{h_n}(Q_l) = \frac{e^{j(n+1)p}}{2p}s^{e}_{h_n}\left(k\sin b\,\frac{d}{2},0\right)s^{e}_{h_n}\left(k\sin b'\,\frac{d}{2},0\right)S_{e_n}\left(k\sin b\,\frac{d}{2},v\right)S_{e_n}\left(k\sin b'\,\frac{d}{2},v'\right)$$

$$s^{o}_{s_n}\left(k\sin b^{(')}\frac{d}{2},0\right) = \left(\frac{\mathrm{R}^{(1)'}_{o_n}\left(k\sin b^{(')}\frac{d}{2},0\right)}{\mathrm{R}^{(4)'}_{o_n}\left(k\sin b^{(')}\frac{d}{2},0\right)}\right)^{\frac{1}{2}} \qquad s^{e}_{h_n}\left(k\sin b^{(')}\frac{d}{2},0\right) = \left(\frac{\mathrm{R}^{(1)}_{e_n}\left(k\sin b^{(')}\frac{d}{2},0\right)}{\mathrm{R}^{(4)}_{e_n}\left(k\sin b^{(')}\frac{d}{2},0\right)}\right)^{\frac{1}{2}}$$

NUMERICAL RESULTS

- Results from simulations are compared with a **MoM** solution (Feko™)
- excitation: **electric hertzian dipole**
- **#1** straight uniform elliptic cylinder – azimuthal scan
- $f\,'=0$



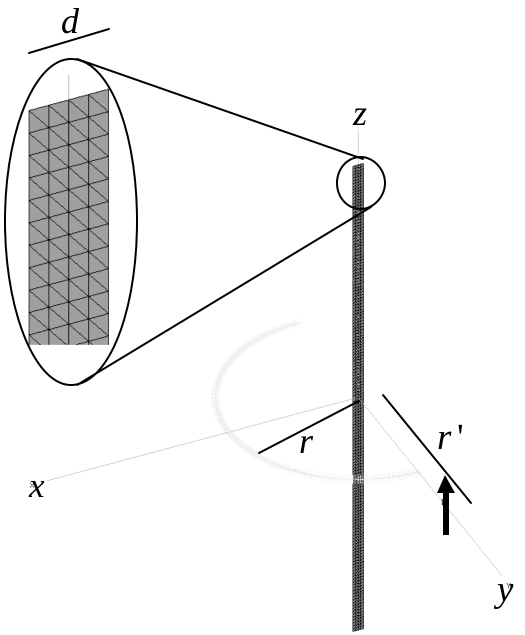


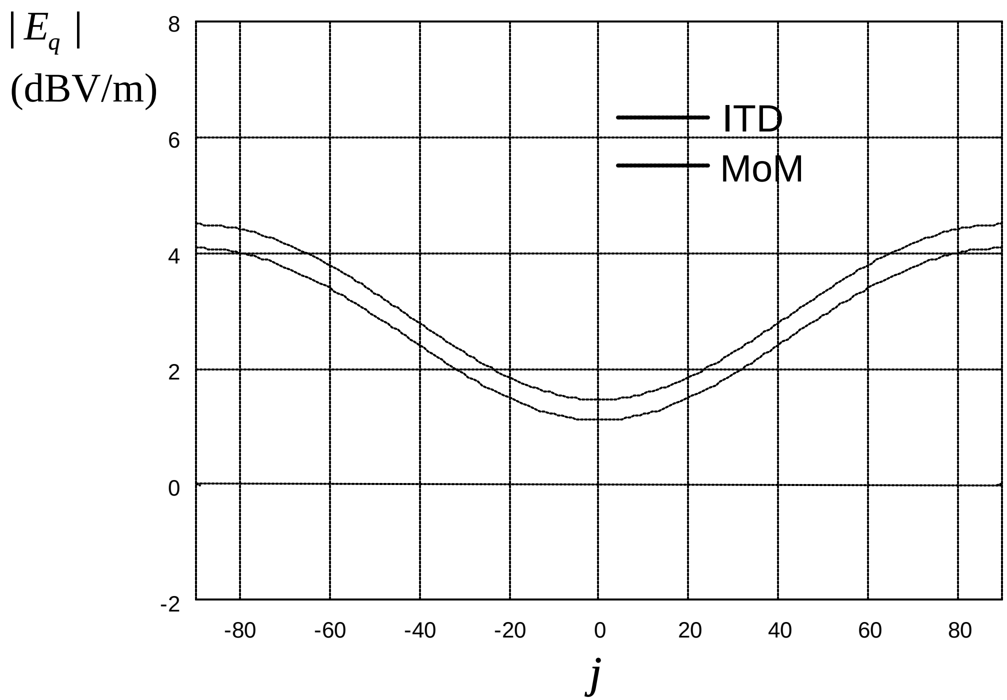
$a = 0.15\,l \qquad b = 0.075\,l \qquad d = 0.26\,l$

$e = \dfrac{d}{2a} = 0.866 \qquad r = 5\,l \qquad r\,' = 7\,l$

NUMERICAL RESULTS

- **#2** strip – azimuthal scan
- $\phi' = \frac{\pi}{2}$

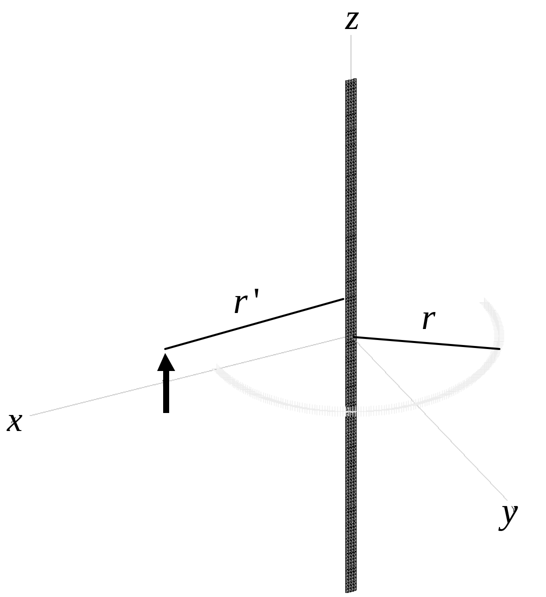


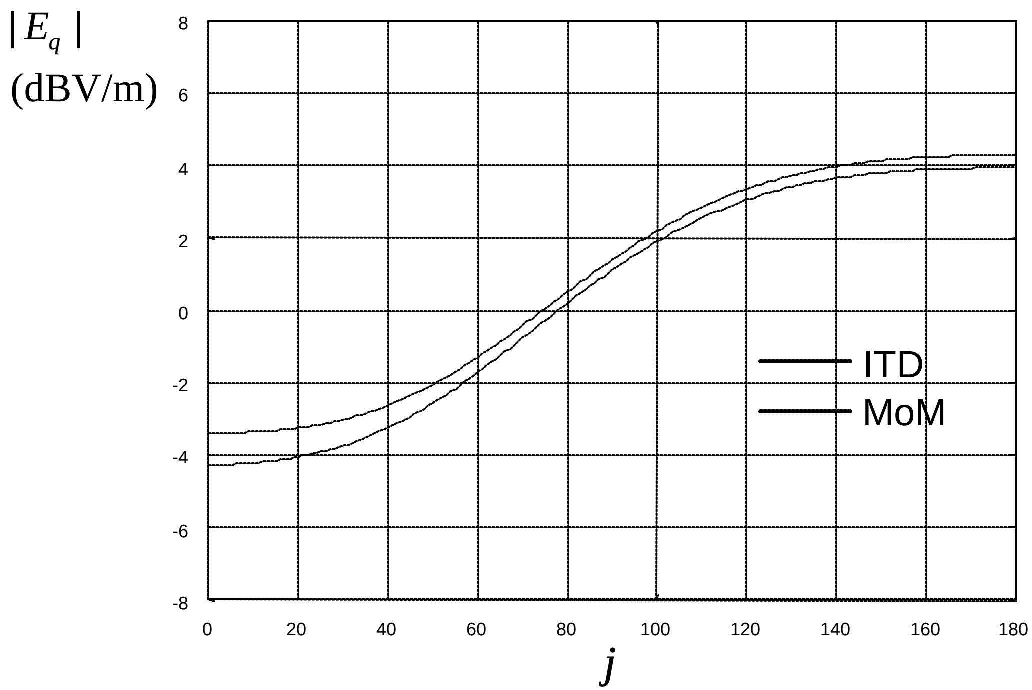


$d = 0.4\,\lambda \qquad r = 5\,\lambda \qquad r' = 7\,\lambda$

NUMERICAL RESULTS

- **#3** strip – azimuthal scan
- $f\,' = 0$





$d = 0.4\,l$ $r = 5\,l$ $r\,' = 7\,l$

CONCLUDING REMARKS

- ITD provides a quite general procedure for defining incremental field contributions from the exact solution of canonical problems with a uniform cylindrical configuration.

- Here, this procedure has been applied to elliptic cylinders, and to strips and slits as particular cases (EM case).

- Explicit closed form, high-frequency expressions of a dyadic incremental scattering coefficient have been obtained for moderately sized elliptic cylinders, strips and slits.

- Comparisons with results obtained by MoM simulations have shown that the proposed incremental representation may be used to estimate the scattered field from moderately sized elliptic structures.