# A Novel Polyphase Code for Sidelobe Suppression

*(Invited Paper)*

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***Abstract*— A binary code sequence attained by modification of a Golay code is proposed. This code has sidelobes which are out of phase with the main lobe, a property which facilitates their removal. Unknown delay–dependent phase terms proclude the multiplexing of a Golay pair in frequency. However the modified Golay code is found to have an autocorrelation whose square is complementary with that of the code's original Golay partner. This enables complementary behavior to be achieved when the modified code is multiplexed at equal offsets both above and below the partner. Applications include sidelobe removal in radar returns.**

## I. Introduction

It is necessary in several diverse applications to localise a received signal pulse in time. The problem arises in radar, since the delay of a returned radar pulse corresponds to the range of the object which reflected it. The problem also arises in communications, where it is necessary to synchronise a mobile handset with a base station's pilot signal.

Localisation is typically performed by matched–filtering a received signal with the transmitted waveform. This process compresses the returned pulse in time. Ideally the output of the matched filter would be an impulse at the sought–after delay. In reality the matched filter output will contain sidelobes.

Sidelobes can be mitigated by the use of amplitude–weighting in the filters [5], or by post–processing filter output [6], [9]. Another strategy is to modulate the signal pulse with a phase code having good autocorrelation properties. Binary codes and polyphase codes (eg. [1], [3], [7]) have been used for this purpose.

Golay complementary codes [8] have properties conducive to use in radar and communications systems. The sum of autocorrelations of each of a Golay code pair is a delta function. This property can be used for the complete removal of sidelobes from radar signals, by transmitting each code, match–filtering the returns and combining them.

This study introduces a new code derived by modification of a Golay code sequence. This code has interesting properties which facilitate the removal of sidelobes in matched filter output.

## II. Review of Golay Codes

Two discrete binary sequences of length $N$, $p_1(n)$ and $p_2(n)$, are termed Golay complementary sequences if the sum of their autocorrelations is zero except at zero lag, *i.e.*

$$R_{p_1}(k) + R_{p_2}(k) = 2N\delta(k) \tag{1}$$

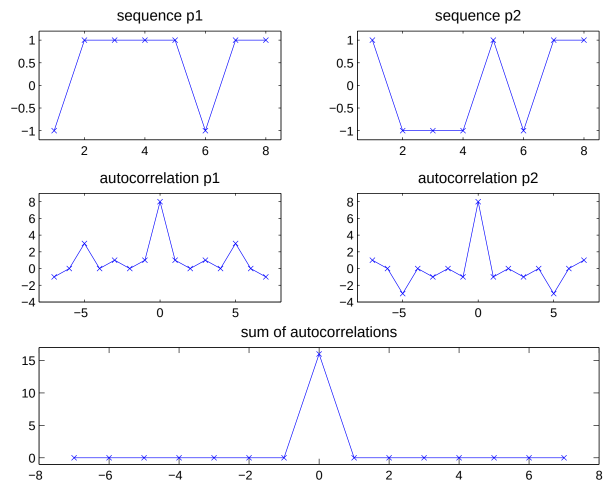


Fig. 1. Pair of Golay sequences, their autocorrelations, and sum of autocorrelations

A length–8 Golay pair and its complementary property is illustrated in figure 1. The following properties hold.

$$p_1(n) \in \{1, -1\}, \; n = 1 \ldots N \tag{2}$$
$$p_2(n) \in \{1, -1\}, \; n = 1 \ldots N \tag{3}$$
$$R_{p_1}(k) + R_{p_2}(k) = 2N, \quad k = 0 \tag{4}$$
$$R_{p_1}(k) = -R_{p_2}(k), \quad k \neq 0 \tag{5}$$
$$R_{p_1}(k)^2 = R_{p_2}(k)^2 \tag{6}$$

It is also the case that

$$R_{p_1}(2k) = R_{p_2}(2k) = 0 \;\; \forall k \neq 0 \tag{7}$$

provided that the Golay sequences are constructed in a standard manner from a length–2 seed and are not permuted.

Individual Golay sequences have relatively flat spectra. The peak–to–mean envelope power ratio of a Golay sequence can be shown to be bounded by length of the sequence [2]. This has application in OFDM power control.

## III. Modified Golay Code

Let $p_1(n)$ and $p_2(n)$ be a Golay complementary pair. Define a sequence $q$ in terms of $p_2$ as

$$q(n) = p_2(n)e^{i\frac{\pi}{2}n} \tag{8}$$

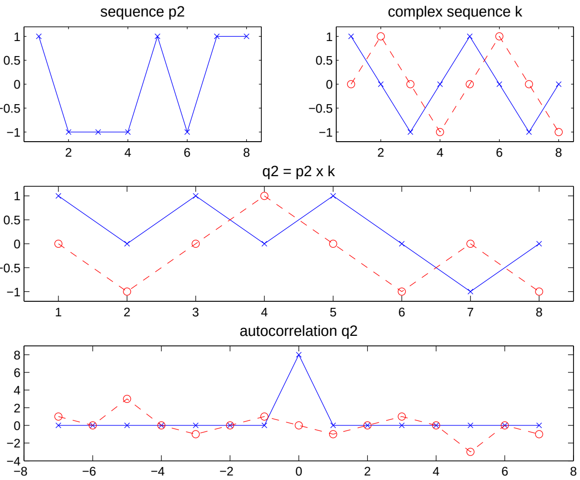

Fig. 2. Modification to Golay sequence

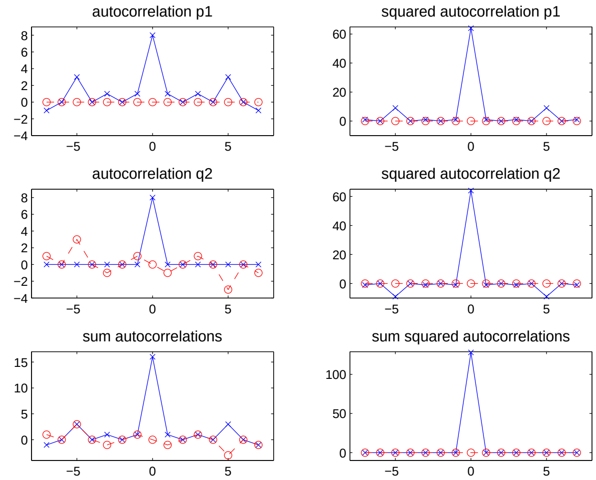

Fig. 3. Complementarity property of modified Golay sequence

Then

$$R_q(k) = R_{p_2}(k)e^{i\frac{\pi}{2}k} \tag{9}$$

It follows from eq. (7) that the only nonzero real value occurs at $k = 0$.

$$\Re(R_q(k)) = N\delta(k) \tag{10}$$

$$\Im(R_q(k)) = \begin{cases} R_{p_2}(k), & k \neq 0 \\ 0, & k = 0 \end{cases} \tag{11}$$

Figure 2 illustrates the formation of a modified code from an original length–8 Golay sequence (top left) and a complex phasor sequence (top right, imaginary values plotted with dashed line). The resulting code and its autocorrelation are shown in the bottom 2 plots. Observe that the only non–zero real value in the autocorrelation occurs at zero lag. All other non–zero values are imaginary.

It is evident that $q(n)$ is not complementary with $p_1(n)$. However consider the square of the autocorrelation function in eq. (9)

$$R_q(k)^2 = R_{p_2}(k)^2 e^{i\pi k} \tag{12}$$

$$= \begin{cases} -R_{p_1}(k)^2 & \text{if k is odd} \\ 0 & \text{if k is even, } k \neq 0 \\ R_{p_1}(k)^2 & \text{if } k = 0 \end{cases} \tag{13}$$

and thus

$$R_q(k)^2 + R_{p_1}(k)^2 = 2N^2\delta(k) \tag{14}$$

In other words the squares of the autocorrelation functions of $p_1(n)$ and $q(n)$ exhibit complementary behaviour. Note that if $p_1(n)$ was subjected to modification in the same manner as $p_2$, the resulting code would be Golay–complementary with $q(n)$. In other words complementarity is preserved by multiplying each of 2 complementary codes by $e^{i\frac{\pi}{2}k}$, but modifying only one of the codes results in a pair which is complementary in the square.

The complex autocorrelation of an original length–8 Golay code and its square are displayed in the top row of fig. 3. Corresponding plots for the modified code (derived from the second member of the Golay pair) are displayed in the second row. Each plot in the final row is the sum of the two plots above. Observe that the modified Golay pair is complementary in the square only.

## IV. Signal Model

Consider a RF pulse which is phase–modulated by an arbitrary code sequence $p(n)$.

$$s_p(t) = \sum_{n=0}^{N-1} p(n)\Omega\left(\frac{t}{T} - n - 0.5\right) \tag{15}$$

where $T$ is the chip length and $\Omega(t)$ is an arbitrary pulse–shaping function with support approximately on $(-0.5, 0.5)$.

$$\int_{-0.5}^{0.5} \Omega(t)^2 \approx 1 \tag{16}$$

The continuous autocorrelation of $s_q(t)$ is

$$R_{s_p}(\tau) = \int_{-\infty}^{\infty} s_p(t)s_p^*(t-\tau)\,dt \tag{17}$$

$$= \int_{-\infty}^{\infty} \sum_{n=0}^{N-1}\sum_{m=0}^{M-1} p(n)p^*(m) \times \Omega\left(\frac{t}{T} - n - \frac{1}{2}\right)\Omega\left(\frac{t+\tau}{T} - m - \frac{1}{2}\right) dt$$

$$= T\sum_{k=-\infty}^{\infty}\sum_{n=0}^{N-1} p(n)p(n-k)\gamma(k,\tau) \tag{18}$$

where $\gamma(k,\tau)$ is the integral

$$\gamma(k,\tau) = \int_{-\infty}^{\infty} \Omega(t)\Omega(t + k - \frac{\tau}{T})\,dt \tag{19}$$

Note that the sum over $m$ has changed to a sum over all possible lags $k$, where $k = n - m$. It is assumed that $p(k) = 0,\ \ k < 0$ or $k \geq N$. From (16) it follows that

$$R_{s_p}(\tau) \approx T\sum_{k=-\infty}^{\infty} \Pi\left(\frac{1}{2}(k - \frac{\tau}{T})\right)\gamma(k,\tau)R_p(k) \tag{20}$$

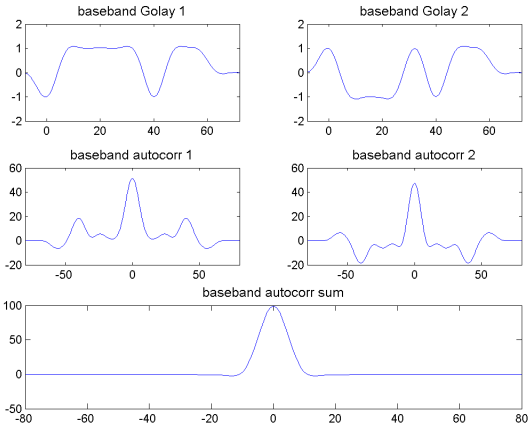


Fig. 4. Baseband Golay sequences, autocorrelations and sum

where $\Pi(x)$ is the rectangle function

$$\Pi(x) = \begin{cases} 1 & \text{if } |x| < \frac{1}{2} \\ 0 & \text{otherwise} \end{cases} \tag{21}$$

For any given value of $\tau$ there will be at most two terms for which $\Pi(\cdot) \neq 0$,

$$k_1(\tau) = \left\lceil \frac{\tau}{T} \right\rceil - 1 \quad \text{and} \quad k_2(\tau) = \left\lceil \frac{\tau}{T} \right\rceil \tag{22}$$

So eq. (20) could be rewritten as

$$R_{s_p}(\tau) = T\left(\gamma(k_1(\tau),\tau)R_p(k_1(\tau)) + \gamma(k_2(\tau),\tau)R_p(k_2(\tau))\right) \tag{23}$$

The Golay pair of fig. 1 has been baseband modulated as described above, using a raised cosine for the pulse shaping function $\Omega$, and presented in figure 4. Observe that the main lobe in the autocorrelation sum is no longer a spike, but extends to either side of the maximum by a distance equal to the chip length. The shape of this lobe is inherited from the pulse–shaping function.

## V. Single Modified Golay Code

The modified Golay code sequence $q(n)$ is used to produce a phase–coded pulse $s_q(t)$. This is carrier modulated at $\omega_c$ and transmitted. Some time later the signal is received and demodulated, giving

$$y_q(t) = ae^{-\omega_c d} s_q(t-d) \tag{24}$$

where $d$ is the delay inherent in transmission.

The cross–correlation of $y_q$ with the transmitted pulse $s_q$ is

$$R_{s_q y_q}(\tau) = ae^{\omega_c d} R_{s_q y_q}(\tau - d) \tag{25}$$

This will have arbitrary phase due to the delay–dependent phase factor. However it follows from (10–11) that $R_{s_q y_q}(d)$ will be out of phase with the rest of the cross–correlation sequence. This property motivates the following algorithm for sidelobe removal.

Assume that a segment of the received signal containing the return from a solitary target can be isolated. The segment is match–filtered by computation of the cross–correlation of the segment with the transmitted baseband waveform. The sampled filter output is thus

$$x(n) = R_{s_q y_q}(nT_s) \tag{26}$$

where $T_s$ is the sample rate. A matrix $X$ can be formed from $x$ by seperating the real and imaginary components

$$X_{n1} = \Re(x(n)) \tag{27}$$
$$X_{n2} = \Im(x(n)) \tag{28}$$

and from this a sample covariance matrix $C_x$ may be formed

$$C_x = X^T X \tag{29}$$

Eigendecomposition yields two eigenvectors $v_1$ and $v_2$ and corresponding eigenvectors $\lambda_1$ and $\lambda_2$, $\lambda_1 > \lambda_2$. Since the main lobe of $x$ contains most of the power, the vector $v_1$ corresponding to the largest eigenvalue can be taken as an estimate of the complex phase associated with the main lobe. Similarly $v_2$ which is orthogonal to $v_1$ is an estimate of sidelobe phase. The main lobe may be recovered by using $v_1$ to project the data matrix onto the main lobe subspace

$$\hat{x}_m = X v_1 \tag{30}$$

and the offset of the main lobe within the segment may be computed from

$$k_{max} = \arg\max \hat{x}_m \tag{31}$$

The above algorithm is suitable for removal of sidelobes when only one pulse is present in the time segment, as will be the case in pilot channels of communication systems. If other pulses are present, as is likely to be the case in radar returns, the above algorithm can be used to isolate the strongest return. Power from other weaker returns will be present in the output, but whether or not this corresponds to mainlobes or sidelobes will depend upon the delay phase factor associated with each return.

A single waveform based on code $q$ was simulated in Gaussian noise with SNR of -10 dB. This was match–filtered and a segment of the output encapsulating the pulse is given in the top subplot of fig. 5. Eigenvectors for projection of the signal onto mainlobe and sidelobe components were computed as described above, and the filter output postprocessed accordingly. The magnitudes of the mainlobe and sidelobe components are presented in the second and third subplots respectively.

## VI. Frequency–Seperated Code Pair

Consider a Golay pair $p_1$ & $p_2$ used to form phase–coded pulses $s_1$ & $s_2$ and transmitted simultaneously, one at carrier $\omega_c$ and the other offset in frequency at $\omega_c + \omega_b$.

$$y_1(t) = ae^{-i\omega_c d} s_1(t-d) \tag{32}$$
$$y_2(t) = ae^{-i(\omega_c+\omega_b)d} s_2(t-d) \tag{33}$$

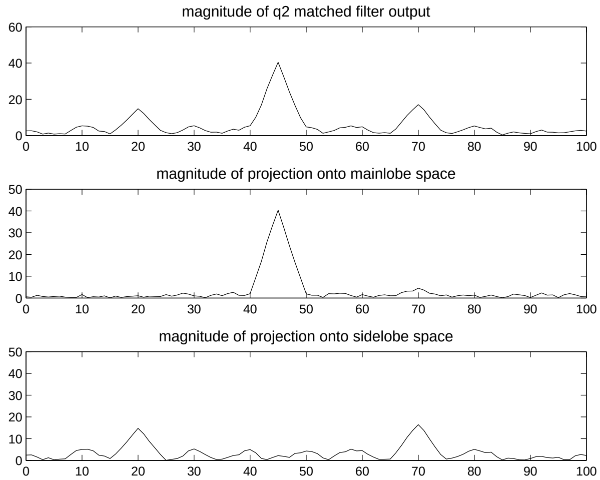


Fig. 5. Matched filter output and projections

These are match–filtered by cross–correlating with the transmitted waveforms

$$R_{s_1y_1}(\tau) = ae^{-i\omega_c d}R_{s_1}(\tau - d) \quad (34)$$
$$R_{s_2y_2}(\tau) = ae^{-i(\omega_c+\omega_b)d}R_{s_2}(\tau - d) \quad (35)$$

and summed to produce

$$R_{s_1y_1}(\tau) + R_{s_2y_2}(\tau) = ae^{-i\omega_c d}\left(R_{s_1}(\tau - d) + R_{s_2}(\tau - d)e^{-i\omega_b d}\right) \quad (36)$$

This in general does not cancel at points distant from the true delay $d$ due to the range–dependent phase factor on the second term. Therefore seperating two codes in frequency destroys their complementarity.

Consider what happens when the second code is transmitted twice, offset an equal amount both above and below carrier. Equation (33) is replaced by two equations

$$y_{2a}(t) = ae^{-i(\omega_c+\omega_b)d}s_2(t - d) \quad (37)$$
$$y_{2b}(t) = ae^{-i(\omega_c-\omega_b)d}s_2(t - d) \quad (38)$$

The product of correlations of the two offset signals with $s_2$ is

$$R_{s_2y_{2a}}(\tau) \times R_{s_2y_{2b}}(\tau) = a^2e^{-i2\omega_c d}R^2_{s_2}(\tau - d) \quad (39)$$

thereby recovering the square of the autocorrelation independently of the offset phase. The square of the cross–correlation of $y_1$ with $s_1$ is

$$R^2_{s_1y_1}(\tau) = a^2e^{-i2\omega_c d}R^2_{s_1}(\tau - d) \quad (40)$$

and when added to (39) produces

$$\Upsilon(\tau) = R^2_{s_1y_1}(\tau) + R_{s_2y_{2a}}(\tau) \times R_{s_2y_{2b}}(\tau)$$
$$= a^2e^{-i2\omega_c d}\left(R^2_{s_1}(\tau - d) + R^2_{s_2}(\tau - d)\right) \quad (41)$$

Recall the square–complementary property of the modified Golay code with its original unmodified partner (eq. 14). If the modified Golay code $q$ is used in place of $p_2$, and $s_2$ is formed from this, then (41) expands to

$$\Upsilon(\tau) = T^2a^2e^{-i2\omega_c d}\left[\gamma^2(k_1)2N^2\delta(k_1)+ \gamma^2(k_2)2N^2\delta(k_2) + 2N\gamma(k_1)\gamma(k_2)\times (R_p(1)(1 + i)\delta(k_2) + R_p(1)(1 - i)\delta(k_1))\right] \quad (42)$$

where the $\tau$ argument is omitted from $\gamma(k)$, $k_1$ and $k_2$ for convenience. This is clearly zero except when $k_1 = 0$ or $k_2 = 0$. In other words sidelobes have been completely removed.

### A. Cross terms

Now assume that there are two overlapping returns with delays $d_1$ and $d_2$, $|d_1 - d_2| < 2NT$. The seperated return signals are

$$y_1(t) = ae^{-i\omega_c d_1}s_1(t - d_1) + \tilde{a}e^{-i\omega_c d_2}s_1(t - d_2) \quad (43)$$
$$y_{2a}(t) = ae^{-i(\omega_c+\omega_b)d_1}s_2(t - d_1) + \tilde{a}e^{-i(\omega_c+\omega_b)d_2}s_2(t - d_2) \quad (44)$$
$$y_{2b}(t) = ae^{-i(\omega_c-\omega_b)d_1}s_2(t - d_1) + \tilde{a}e^{-i(\omega_c-\omega_b)d_2}s_2(t - d_2) \quad (45)$$

The cross–correlations of these with the transmitted signal are

$$R_{s_1y_1} = ae^{-i\omega_c d_1}R_{s_1}(\tau - d_1) + \tilde{a}e^{-i\omega_c d_2}R_{s_1}(\tau - d_2) \quad (46)$$
$$R_{s_2y_{2a}} = ae^{-i(\omega_c+\omega_b)d_1}R_{s_2}(\tau - d_1) + \tilde{a}e^{-i(\omega_c+\omega_b)d_2}R_{s_2}(\tau - d_2) \quad (47)$$
$$R_{s_2y_{2b}} = ae^{-i(\omega_c-\omega_b)d_1}R_{s_2}(\tau - d_1) + \tilde{a}e^{-i(\omega_c-\omega_b)d_2}R_{s_2}(\tau - d_2) \quad (48)$$

Note that $R_{s_{2a}} = R_{s_{2b}}$ and is written above as $R_{s_2}$. Computing the sum of squared cross–correlations,

$$\Upsilon(\tau) = R^2_{s_1y_1}(\tau) + R_{s_2y_{2_a}}(\tau)R_{s_2y_{2_b}}(\tau) \quad (49)$$
$$= a^2e^{-i2\omega_c d_1}\left(R^2_{s_1}(\tau - d_1) + R^2_{s_2}(\tau - d_1)\right) + \tilde{a}^2e^{-i2\omega_c d_2}\left(R^2_{s_1}(\tau - d_2) + R^2_{s_2}(\tau - d_2)\right) + 2a\tilde{a}e^{-i\omega_c(d_1+d_2)}\chi(\tau) \quad (50)$$

where $\chi(\tau)$ is dependent upon cross terms,

$$\chi(\tau) = R_{s_1}(\tau - d_1)R_{s_1}(\tau - d_2) + R_{s_2}(\tau - d_1)R_{s_2}(\tau - d_2)\cos\omega_b(d_1 - d_2) \quad (51)$$

The first term of $\Upsilon(\tau)$ is due solely to the target at delay $d_1$ and the second term is due solely to the target at $d_2$. Each has the form of $R^2_{s_1} + R^2_{s_2}$ and thus will cause a single main lobe as described by eq. (42). The third term is due to cross terms in the square operation and depends upon $d_1$, $d_2$ and $\omega_b$.

A modified Golay code pair $p_1(n)$ & $q(n)$ is used in a frequency–offset radar pulse system as described above. The return signals from two nearby reflectors in Gaussian noise are simulated. The matched filter outputs $y_1(t)$ and $y_{2a}(t)$ are shown in the uppermost subplots of fig. 6. The delay–independent squares of cross–correlations are presented

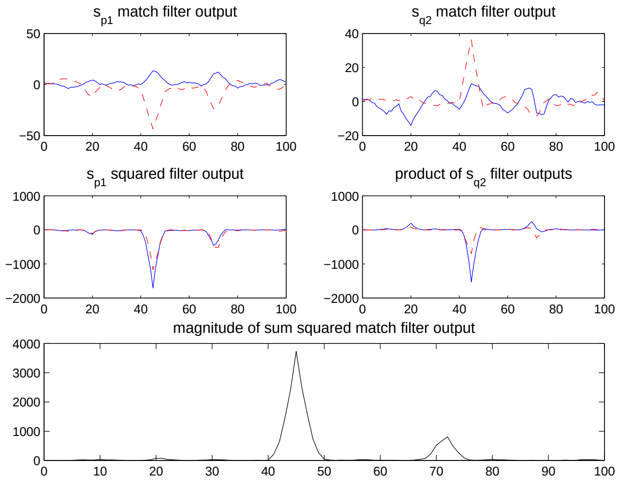


Fig. 6. Use of modified Golay pair for sidelobe removal, two targets

immediately beneath. The bottom subplot is the sum of the squared cross–correlations. Observe that two distinct peaks are clearly present. The smaller of the two returns is situated in a sidelobe of the stronger return and was not obvious in either matched filter output. The effect of cross terms is insignificant at points not near either peak.

## VII. Conclusion

This paper has described a polyphase code derived from one member of a Golay complementary pair. The autocorrelation sequence of this code is imaginary at all points except zero lag. This property facilitates the suppression of sidelobes in matched filter output when the new sequence is used to phase–code a signal pulse. The new sequence achieves complementary behaviour with the other member of the original Golay pair when the squares of autocorrelation functions are considered. Transmission of the modified code at two equal offset frequencies allows the square of the matched filter output to be recovered independently of offset phase. This allows sidelobes to be completely removed. However cross–terms are introduced when two returns are closely seperated.

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