MONTE-CARLO BASED ESTIMATION METHODS FOR RAPIDLY-VARYING SEA CLUTTER

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ABSTRACT

We consider two Monte-Carlo based methods for characterizing the scattering function of rapidly-varying sea clutter. The first method uses multiple particle filtering to estimate the clutter space-time covariance marix by exploiting the structure of the matrix. This method is then compared to a baseline approach that estimates the clutter covariance matrix based on the Weibull distribution approximation. Both methods are evaluated by formulating a detection problem that simulates a small moving target in heavy sea clutter.

***Index Terms*—** Multiple particle filtering, covariance matrix estimation, Weibull distribution, Monte Carlo simulations.

1. INTRODUCTION

Waveform design and diversity techniques have been successfully used in radar applications to improve target detection and tracking performance. However, in order to achieve this improvement, it is essential to have accurate information on the characteristics of the propagating medium. This can be a difficult problem when the target is moving in heavy sea clutter and the signal-to-clutter ratio (SCR) is low, even if the target has large radar cross-section. Although methods exist for estimating sea clutter statistics under slowly-varying conditions [1, 2], they do not work well when the clutter scene varies quickly.

In this paper, we compare two Monte Carlo based methods for estimating the space-time covariance matrix of sea clutter for rapidly-varying radar scenes. The first method approximates the clutter characteristics according to a Weibull distribution, and then estimates the parameters of the distribution at each time step of the sea clutter variation. This follows the approach in [3] that approximates the distribution of short-windowed data sets as a Weibull distribution and estimates the local statistics for each range cell. The second method is based on formulating the clutter scene using a scattering function based representation in the range-Doppler space, in terms of the time-frequency shifts caused by the scene on the transmitted waveform, and then estimating the scattering function covariance matrix. Using this formulation, we proposed a method in [4] that uses the multiple particle filtering sequential Monte Carlo method [5] to sequentially estimate the dynamic state of the system and to overcome the system's high dimensionality problem. The effectiveness of both methods is demonstrated by formulating a detection problem for a moving target in high sea clutter.

The paper is organized as follows. In Section 2, we discuss the Weibull approximation approach to estimate sea clutter statistics. In Section 3, we formulate the dynamic radar scene problem to estimate the covarariance of the scattering function. In Section 4, we provide the multiple particle filtering method for estimating the sea clutter space-time covariance matrix. Simulation results illustrating detection performance when comparing the two methods are presented in Section 5.

2. CLUTTER STATISTICS APPROXIMATION USING WEIBULL DISTRIBUTIONS

The statistical properties of rapidly-varying sea clutter have been modeled using different distributions in the literature. When a radar has an adequate high resolution that can resolve fine structures on the sea surface, then sea clutter returns are not well-modeled using a Gaussian distribution [6]. Instead, sea clutter has been modeled to have gamma, inverse gamma, compound Gaussian, Weibull, or log-normal distributions [7–9]. In [3], the authors demonstrated the time-varying nature of sea clutter observed at low grazing angles and tried to model this variability with a Weibull distribution.

The probability density function for a Weibull-distributed

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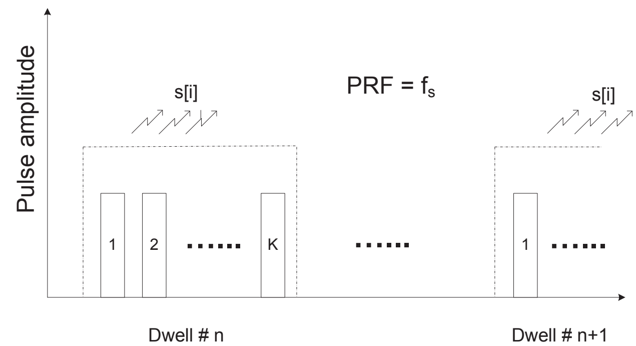

Fig. 1. K identical pulses are transmitted in each dwell and their returns are used to estimate clutter statistics.

variable z is given by

$$p(z;\lambda,\kappa) = \begin{cases} \frac{\kappa}{\lambda}\left(\frac{z}{\lambda}\right)^{\kappa-1} e^{-(z/\lambda)^{\kappa}}, & z \geq 0 \\ 0, & z < 0 \end{cases},$$

where $\lambda > 0$ is a dispersion parameter and $\kappa > 0$ is a shape parameter used to describe the spikiness of the data. The mean of the Weibull distribution is given by $\lambda\,\Gamma(1+(1/\kappa))$, where $\Gamma(\cdot)$ is the Gamma function. Using the Weibull distribution approximation, spiky sea clutter data can have a higher probability of occurrence than the mean of the distribution. This is an important consideration (due to the high magnitudes of some sea clutter components) not possible using a Gaussian distribution approximation.

For rapidly-varying radar scenes, the work in [3] computed local estimates of the mean and variance of short-segments of measurements assumed to be Weibull-distributed. The aim of the authors was to adjust the threshold from range cell to range cell in order to have a constant false alarm rate over the varying clutter for detecting a stationary target. For our application, we estimate the parameters of the Weibull distribution at each time step as we consider a moving target. In particular, for the sea clutter observation model, we consider a radar that transmits a burst of K pulses using f_s Hz pulse repetition frequency (PRF). We assume that the same signal $s_n[i]$, $i = 0, 1, \cdots, N_s - 1$ is transmitted repeatedly in the nth dwell, as demonstrated in Figure 1. The Weibull parameters are then estimated at each time step, and thus for each range, using maximum likelihood estimation (MLE) techniques and the multiple measurements obtained from each dwell.

3. RAPIDLY-VARYING RADAR SCENE

3.1. Received signal formulation

In order to formulate the observation from each dwell in terms of a time-varying representation of transformed transmitted pulses, we consider the return signal in the nth dwell, at time step n. The return signal in the nth dwell is sampled at f_b Hz to yield a sequence $y_n[k,m] = y[k,m] = y(k, m\,\Delta T_b)$, $k = 0, 1, \cdots, K-1$, $m = m_0, m_1, \cdots, m_{M_n-1}$, where $\Delta T_b = 1/f_b$ is the *fast* sampling interval, m_0 is the lowest range bin in the validation gate at time step n, and the validation gate at dwell n consists of M_n range bins. In a rapid-varying radar scenario, we combine the scatterers according to the range bin and pulse number of the reflectivity in each burst. In the nth cell, the complex reflectivity of the aggregate scatterers is denoted by $x_n[k,m] = x[k,m]$, where k is the pulse number and m is the range bin from the sensor. Note that, when a target is present in a range-pulse cell, the reflectivity $x_n[k,m]$ includes both clutter and targets. The radar return is thus modeled as

$$y_n[k,m] = \sum_{i=0}^{N_s-1} x_n[k, m-i]\, s_n[i] + v_n[k,m] \quad (1)$$

where $v_n[k,m] = v_n[k,m]$ is white Gaussian observation noise at the nth dwell.

Let the reflectivity matrix at the nth dwell be a matrix B_n that consists of the reflectivities $x[k,m]$ at the nth dwell:

$$B_n = \begin{bmatrix} x_n[m_0,0] & x_n[m_0,1] & \ldots & x_n[m_0,K-1] \\ x_n[m_1,0] & x_n[m_1,1] & \ldots & x_n[m_1,K-1] \\ \vdots & \vdots & \vdots & \vdots \\ x_n[m_{N_v-1},0] & x_n[m_{N_v-1},1] & \ldots & x_n[m_{N_v-1},K-1] \end{bmatrix}$$

where $N_v = M_n + N_s - 1$. This reflectivity matrix has N_v rows with pulse numbers (slow time) increasing along each row, and K columns with range bin (fast time or delay) increasing along each column.

The transmitting matrix, P_n, at the nth dwell is defined as an M_n by N_v matrix consisting of the transmitted signal $s_n[i] = s[i]$:

$$P_n = \begin{bmatrix} s[0] & s[1] & \cdots & s[N_s-1] & 0 & \cdots & \\ 0 & s[0] & s[1] & \cdots & s[N_s-1] & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \\ 0 & 0 & \cdots & \cdots & \cdots & s[N_s-2] & s[N_s-1] \end{bmatrix}$$

where, the signal begins at the pth element in the pth row, $p = 1, 2, \cdots, M_n$.

Using the above notation, the observation in (1) can be rewritten as an $M_n \times K$ observation matrix

$$Y_n = P_n B_n + V_n \quad (2)$$

with elements $y_n[m,k]$ described in (1). Here, V_n is the $M_n \times K$ noise matrix at dwell n with elements $v_n[m,k]$.

3.2. Scattering function

Using a continuous-time representation for a linear time-varying system $\mathcal{L}$, the scattering function of the system,

$$A_{\mathcal{L}}(\tau,\nu) = \int h_{\mathcal{L}}(t,\tau)\, e^{-j2\pi\nu t} dt\,,$$

is expressed in terms of the system's time-varying impulse response $h_{\mathcal{L}}(t,\tau)$. The scattering function provides a measure of time shifts by τ and frequency shifts ν that the system causes on the transmitted signal [10]. After discritization, this function yields the scattering matrix A_n that is related to B_n via a Fourier transformation (FT). Specifically,

$$A_n = B_n\, D \tag{3}$$

where D is the $K \times K$ discrete FT matrix:

$$D = \frac{1}{\sqrt{K}} \begin{bmatrix} 1 & \dots & 1 & \dots & 1 \\ e^{j2\pi\frac{(K-1)}{2K}} & \dots & 1 & \dots & e^{-j2\pi\frac{(K-1)}{2K}} \\ e^{j2\pi\frac{2(K-1)}{2K}} & \dots & 1 & \dots & e^{-j2\pi\frac{2(K-1)}{2K}} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ e^{j2\pi\frac{(K-1)(K-1)}{2K}} & \dots & 1 & \dots & e^{-j2\pi\frac{(K-1)(K-1)}{2K}} \end{bmatrix}.$$

The scattering matrix A_n provides the range-Doppler description of the scatterer complex reflectivities in matrix B_n. Each element in A_n describes the states of the scatterers. In each range bin, static scatterers stay in the middle row, the first $(K-1)/2$ elements present the negative Doppler shifts and are related to the scatterers moving away from the sensor with different velocities, and the last $(K-1)/2$ elements present the positive Doppler shifts and are related to the scatterers moving toward the sensor with different velocities. According to the characteristics of sea clutter, most scatterers reside around the middle of each row (zero Doppler).

In this work, the scatterers and target are assumed to move with constant velocities. As shown in Figure 2, as the scatterers move, some of them that do not stay in the middle columns of A_n will move out of the validation gate, and some range-Doppler cells will become empty. To better describe the evolution of the scattering matrix, an evolution matrix F is introduced, that describes the movement of the scatterers and the population of the empty cells. The new scatterer cells that are populated into the scattering matrix are described using exponential weighted summations of the complex reflectivities in the immediate neighborhood of these cells, as illustrated in Figure 2. This procedure will continue along each column until all the empty cells are populated. We vectorize the matrix A_n to form $\mathbf{a}_n = \text{vec}(A_n)$ by stacking the columns of A_n from left to right to form a $KN_v \times 1$ vector The evolution of the vectorized scattering matrix can be expressed as:

$$\mathbf{a}_n = F\mathbf{a}_{n-1} + \mathbf{w}_n\,. \tag{4}$$

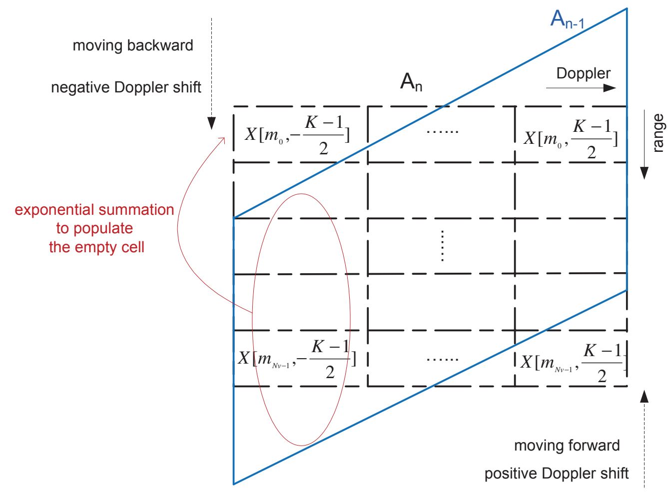

Fig. 2. Evolution of the scattering function.

The zero-mean, complex Gaussian noise $\mathbf{w}_n$ has covariance Q_n, and F incorporates the scatterer movements between dwells and populates the range-Doppler cells that move into the validation gate. The matrix F is a $KN_v \times KN_v$ block-diagonal matrix. Each block matrix F_l, $l = -(K-1)/2, \dots, -1, 0, 1, \dots, (K-1)/2$, is defined based on l. Specifically, when $l = -(K-1)/2, \dots, -1$, the block matrix is given by

$$F_l = \begin{bmatrix} 2^{|l|-1}e^{-|l|\alpha} & \dots & (2^{|l|-1} - |l| + 1)e^{-(N_v+|l|-1)\alpha} \\ \vdots & \vdots & \vdots \\ e^{-\alpha} & \dots & e^{-N_v\alpha} \\ 1 & \dots & 0 \\ 0 & \vdots & \vdots \\ \vdots & \dots & 0 \end{bmatrix}.$$

When $l = 1, \dots, (K-1)/2$, the block matrix is

$$F_l = \begin{bmatrix} 0 & \dots & \vdots \\ \vdots & \vdots & 0 \\ 0 & \dots & 1 \\ e^{-N_v\alpha} & \dots & e^{-\alpha} \\ \vdots & \vdots & \vdots \\ (2^{l-1} - l + 1)e^{-(N_v+l-1)\alpha} & \dots & 2^{l-1}e^{-l\alpha} \end{bmatrix},$$

and when $l = 0$, the scatterers are not moving, and the block matrix is an $N_v \times N_v$ identity matrix $\mathbb{I}_{N_v}$.

3.3. State space model for clutter covariance matrix

We obtain the vector $\mathbf{b}_n = \text{vec}(B_n)$ by vectorizing the matrix B_n. The covariance matrix of the reflectivity vector $\mathbf{b}_n$, $\mathbf{\Sigma}_{\mathbf{b}_n} = \text{E}[\mathbf{b}_n \mathbf{b}_n^H]$, provides both time and space covariance information. Note that H denotes Hermitian transpose and $\text{E}[\cdot]$ is the expectation operator.

Our aim is to estimate the sea clutter statistics by estimating the space-time covariance matrix $\mathbf{\Sigma}_{\mathbf{b}_n}$ of the scatterers. By vectorizing (3), we obtain

$$\mathbf{b}_n = (D \otimes \mathbb{I}_{N_v})\, \mathbf{a}_n \,,$$

where $\otimes$ denotes the Kronecker product. Then,

$$\mathbf{\Sigma}_{\mathbf{b}_n} = (D \otimes \mathbb{I}_{N_v}) \mathbf{\Sigma}_{\mathbf{a}_n} (D^H \otimes \mathbb{I}_{N_v})$$

Thus, $\mathbf{\Sigma}_{\mathbf{b}_n}$ can be obtained by the estimation of $\mathbf{\Sigma}_{\mathbf{a}_n}$.

From (4), the covariance matrix of the scattering vector can be expressed as:

$$\mathbf{\Sigma}_{\mathbf{a}_n} = \text{E}[\mathbf{a}_n \mathbf{a}_n^H] = F \mathbf{\Sigma}_{\mathbf{a}_{n-1}} F^H + Q_n. \tag{5}$$

This equation provides the state dynamic model. The observation in (2) can be written in terms of A_n as:

$$Y_n = P_n A_n D^{-1} + V_n.$$

After vectorization, we obtain

$$\begin{aligned} \mathbf{y}_n &= (D^{-H} \otimes P_n)\mathbf{a}_n + \mathbf{v}_n \\ &= \check{P}_n \mathbf{a}_n + \mathbf{v}_n \,, \end{aligned} \tag{6}$$

where $\mathbf{y}_n = \text{vec}(Y_n)$ and $\check{P}_n = D \otimes P_n$. Note that we used here the vectorization property given by $\text{vec}(GZL) = (L^H \otimes G)\mathbf{z}$, where G, Z, L are three arbitrary matrices and $\mathbf{z} = \text{vec}(Z)$ [11].

By computing the covariance of (6), the observation model is then:

$$\mathbf{\Sigma}_{\mathbf{y}_n} = \check{P}_n \mathbf{\Sigma}_{\mathbf{a}_n} \check{P}_n^H + R_n, \tag{7}$$

where R_n is the covariance of $\mathbf{v}_n$. Using (7), we can obtain $p(\mathbf{y}_n | \mathbf{\Sigma}_{\mathbf{a}_n})$ to update the filter. The covariance matrices Q_n in (5) and R_n in (7) are assumed to follow from a Wishart distribution assumption on $\mathbf{w}_n$ and $\mathbf{v}_n$, respectively.

4. ESTIMATION OF TIME-VARYING SEA CLUTTER STATISTICS

Particle filtering is a sequential Monte Carlo method that is based on sampling from proposal distributions to approximate probability density functions. Particle filtering is often an attractive alternative to Kalman filtering when the system equations are nonlinear or random processes are non-Gaussian [12, 13]. However, when the dimensionality of the state space is large, which is the case for our estimation problem, then a huge set of particles is needed to provide accurate estimation results. As a result, the computational complexity of the algorithm becomes prohibitive. As discussed in [5], multiple particle filtering can be used to overcome this dimensionality problem.

4.1. Multiple particle filtering sequential Monte Carlo Approach

Let the dynamic and measurement models of a system be expressed as:

$$\begin{aligned} \boldsymbol{\alpha}_n &= f_n(\boldsymbol{\alpha}_{n-1}, \boldsymbol{\omega}_{n-1}) \\ \boldsymbol{\beta}_n &= h_n(\boldsymbol{\alpha}_n, \boldsymbol{\gamma}_n)\,, \end{aligned} \tag{8}$$

where $\boldsymbol{\alpha}_n$ is the high-dimensional system state vector at time step n, f_n and h_n are (possibly nonlinear) functions, and $\boldsymbol{\omega}_n$ and $\boldsymbol{\gamma}_n$ are noise vectors. Using the multiple particle filtering approach in [5], $\boldsymbol{\alpha}_n$ is divided into L subvectors:

$$\boldsymbol{\alpha}_n = \begin{bmatrix} \boldsymbol{\alpha}_{1,n} \\ \boldsymbol{\alpha}_{2,n} \\ \vdots \\ \boldsymbol{\alpha}_{L,n} \end{bmatrix}.$$

Each $\boldsymbol{\alpha}_{l,n},\ l = 1, 2, \cdots, L$, is estimated using a different particle filter. The weights at time step n are updated by:

$$w_{l,n}^{(i)} \propto w_{l,n-1}^{(i)} \cdot \frac{p(\boldsymbol{\beta}_n | \boldsymbol{\alpha}_{l,n}^{(i)}, \tilde{\boldsymbol{\alpha}}_{-l,n}) p(\boldsymbol{\alpha}_{l,n}^{(i)} | \boldsymbol{\alpha}_{l,n-1}^{(i)}, \hat{\boldsymbol{\alpha}}_{-l,n-1})}{\pi_l(\boldsymbol{\alpha}_{l,n}^{(i)} | \boldsymbol{\alpha}_{l,n-1}^{(i)}, \hat{\boldsymbol{\alpha}}_{-l,n-1}, \boldsymbol{\beta}_n)}, \tag{9}$$

where $l = 1, 2, \ldots, L$ is the index corresponding to the multiple particle filters, i is the particle index, and $\tilde{\boldsymbol{\alpha}}_{-l,n}$ and $\hat{\boldsymbol{\alpha}}_{-l,n-1}$ are the predicted and estimated values of all states (except $\boldsymbol{\alpha}_{l,n}$) at time step n.

4.2. Estimation of the covariance matrix of the radar scene scattering function

In our formulation, the covariance matrix $\mathbf{\Sigma}_{\mathbf{a},n}$ of the scattering function is the unknown state in the state-space formulation. We notice that (5) and (7) correspond to the dynamic and measurement models, respectively. In order to use sequential Monte Carlo methods to estimate the unknown state, we first vectorize the dynamic state and measurement models to obtain

$$\begin{aligned} \tilde{\mathbf{\Sigma}}_{\mathbf{a}_n} &= (F \otimes F)\tilde{\mathbf{\Sigma}}_{\mathbf{a}_{n-1}} + \tilde{\mathbf{Q}}_n \\ \tilde{\mathbf{\Sigma}}_{\mathbf{y}_n} &= (\check{P} \otimes \check{P})\tilde{\mathbf{\Sigma}}_{\mathbf{a}_n} + \tilde{\mathbf{R}}_n, \end{aligned}$$

where

$$\tilde{\boldsymbol{\Sigma}}_{\mathbf{a}_n} = \text{vec}(\boldsymbol{\Sigma}_{\mathbf{a}_n}) \quad \tilde{\boldsymbol{\Sigma}}_{\mathbf{y}_n} = \text{vec}(\boldsymbol{\Sigma}_{\mathbf{y}_n})$$
$$\tilde{\mathbf{Q}}_n = \text{vec}(\mathbf{Q}_n) \quad \tilde{\mathbf{R}}_n = \text{vec}(\mathbf{R}_n)\,.$$

After vectorization, the dimensionality of the covariance vector $\tilde{\boldsymbol{\Sigma}}_{\mathbf{a}_n}$ is $\Xi = (KN_v)^2$. The value of Ξ can be quite large, even if we consider a small number of pulses. For example, if we use $K = 9$ pulses and $M_n = 10$ range bins, then even if we reduce the signal length to $N_s = 6$, we still obtain a small value for $N_v = M_n + N_s - 1 = 15$ and thus $\Xi = 18,225$. Then, the state dynamic formulation suffers from high dimensionality that prevents direct implementation of particle filtering (or even Kalman filtering estimation even if the transformations are linear). We therefore apply the multiple particle filtering method [5].

The state vector $\boldsymbol{\alpha}_n$ in (8) corresponds to the vectorized covariance matrix, $\tilde{\boldsymbol{\Sigma}}_{\mathbf{a}_n}$. According to the multiple particle filtering method, the state vector is divided into K independent subvectors:

$$\tilde{\boldsymbol{\Sigma}}_{\mathbf{a}_n} = \begin{bmatrix} \boldsymbol{\Lambda}_{1,n}, \\ \boldsymbol{\Lambda}_{2,n}, \\ \vdots, \\ \boldsymbol{\Lambda}_{K,n} \end{bmatrix},$$

where each vector $\boldsymbol{\Lambda}_{k,n},\ k = 1, 2, \cdots, K$, has dimension KN_v^2, where K is the number of pulses in each burst. The function f_n in (8) is then $F \otimes F$, which is a block diagonal matrix

$$F \otimes F = \begin{bmatrix} F_1 \otimes F & 0 & \cdots & 0 \\ 0 & F_2 \otimes F & \cdots & 0 \\ & & \cdots & \\ 0 & 0 & \cdots & F_K \otimes F \end{bmatrix}.$$

The term F_k is defined in Section 3.2.

Therefore, at any time step n, $L = K$ particle filters are applied simultaneously. For the kth subsystem, the estimation of this segment of the current state is obtained using the dynamic and measurement models

$$\boldsymbol{\Lambda}_{k,n} = (F_k \otimes F)\boldsymbol{\Lambda}_{k,n-1} + \mathbf{V}_{k,n}$$
$$\tilde{\boldsymbol{\Sigma}}_{\mathbf{y}_n} = (\check{P} \otimes \check{P})\tilde{\boldsymbol{\Sigma}}_{\mathbf{a}_n} + \tilde{\mathbf{R}}_n\,.$$

The weight for the ith particle is updated according to

$$w_{k,n}^{(i)} \propto w_{k,n-1}^{(i)} \cdot \frac{p(\boldsymbol{\Sigma}_{\mathbf{y}_n}|\boldsymbol{\Lambda}_{k,n}^{i}\tilde{\boldsymbol{\Lambda}}_{-k,n}^{(i)})p(\boldsymbol{\Lambda}_{k,n}^{(i)}|\boldsymbol{\Lambda}_{k,n-1}^{(i)}\hat{\boldsymbol{\Lambda}}_{-k,n-1})}{\pi_k(\boldsymbol{\Lambda}_{k,n}^{(i)}|\boldsymbol{\Lambda}_{k,n-1}^{(i)}, \hat{\boldsymbol{\Lambda}}_{-k,n-1}, \tilde{\boldsymbol{\Sigma}}_{\mathbf{y}_n})}$$

where

$$\tilde{\boldsymbol{\Lambda}}_{-k,n} = [\hat{\boldsymbol{\Lambda}}_{1,n}^T \cdots \hat{\boldsymbol{\Lambda}}_{k-1,n}^T\, \hat{\boldsymbol{\Lambda}}_{k+1,n}^T \cdots \hat{\boldsymbol{\Lambda}}_{k,n}^T]^T$$

and

$$\hat{\boldsymbol{\Lambda}}_{j,n}^T = \sum_i w_{j,n-1}^{(i)} \boldsymbol{\Lambda}_{j,n}^{(i)}\,.$$

The observation used in the kth particle filter has a complex Gaussian distribution with zero-mean and covariance matrix $\Sigma_{\mathbf{y}_n}$.

5. SIMULATIONS FOR COMPARING MONTE CARLO METHODS

In this section, we compare the estimation performance of the two Monte Carlo based methods by detecting a moving target in heavy sea clutter using one sensor.

For our simulations, we choose a total of $K = 9$ pulses to transmit in each dwell and the validation gate size is $M_n = 11$. The waveform transmitted is chosen to be a linear frequency-modulated (LFM) chirp signal with $N_s = 6$. For the multiple particle filtering approach, $L = 9$ particle filters run simultaneously, and each particle filter uses 50 particles. The simulated sea clutter is generated by following a compound-Gaussian model [7] assumption, where, at each time step n, 200 Monte Carlo simulations are run to obtain the space-time covariance matrix of the reflectivity vector. The range bin size and Doppler resolution are chosen such that, from one dwell to another, the scatterer in the lth column, $l = -(K-1)/2, \cdots, 0, \cdots, (K-1)/2$, of the scattering function moves l bins. The target is assumed to move with a constant velocity of 20 m/s. Target returns from all pulses and range bins are combined to form observation vector. The amplitudes of the target returns are sampled from a zero-mean, complex Gaussian process with variance σ^2, which is assumed known and determined by the specified SCR values and clutter energy.

We use the generalized likelihood ratio test (GLRT) detector [14] together with the observations in order to detect the range bin location of the target, utilizing each pulse of each burst. The estimation is performed using the Weibull distribution approximation at each time step or by estimating the space-time covariance matrix of the sea clutter at each time step, using the multiple particle filtering Monte Carlo method. The resulting detection performance is shown in Figure 3. As it can be seen, the direct estimation of the space-time covariance matrix using the multiple particle filtering approach yields higher detection performance results for a fixed probability of false alarm when compared to the Weibull approximation.

6. CONCLUSION

In this paper, we compared two Monte Carlo based methods used to estimate heavy sea clutter statistics. The first method is based on approximating the clutter at each time step to have a Weibull distribution whose parameters are estimated

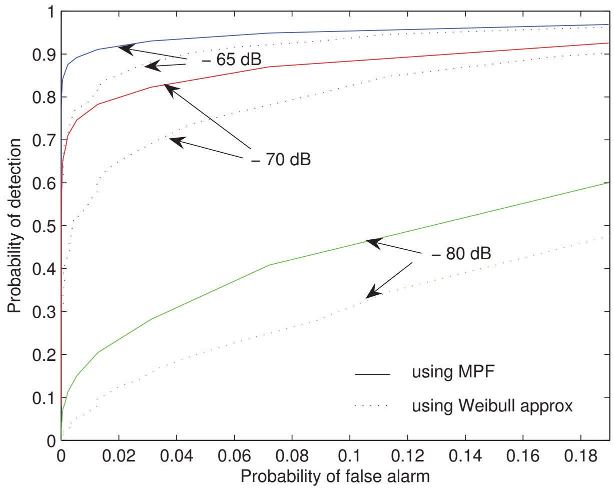

Fig. 3. Receiver operating characteristic (ROC) curves for a GLRT detector that is operating at various SCR values (given in dB) using the multiple particle filtering and the Weibull approximation Monte Carlo methods.

using maximum-likelihood techniques. The second method estimates the space-time covariance matrix of the sea clutter. The estimation is based on vectorizing the scattering matrix from the radar scene representation and then estimating the covariance matrix of the corresponding scattering vector. The method uses sequential Monte Carlo methods to estimate the dynamic state in the dynamic state-space representation of the radar scene. As the dimensionality of the unknown state is very high, we exploit the special structure of the state-space formulation to apply multiple particle filtering. The detection improvement of the multiple particle filtering when compared to the Weibull approximation is demonstrated in a simulation for detecting a moving target in heavy sea clutter for different SCR values.

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