Waveform-Agile Sensing for Range and DoA Estimation in MIMO Radars

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Abstract—We propose an agile sensing algorithm to optimally select the transmission waveform of a multiple-input, multiple-output (MIMO) radar system in order to improve target localization. Specifically, we first derive the Cramér-Rao lower bound (CRLB) for the joint estimation of the antenna reflection coefficients and the range and direction-of-arrival of a stationary target using MIMO radar with colocated antennas. The resulting CRLB, that is a function of the transmitted waveform, is then compared to the estimation performance of a maximum-likelihood estimator. We configure waveform parameters to minimize the trace of the predicted estimation error covariance by assuming that the covariance of the observation noise is approximated by the CRLB for high signal-to-noise ratios. In particular, we optimally select the duration and phase function parameters of generalized frequency-modulated (GFM) waveforms to minimize the estimation mean-squared error under constraints of fixed transmission energy and constant time-bandwidth product.

I. INTRODUCTION

MIMO radar systems are emerging as a promising new technology as they have the potential to enhance target detection and identification performance [1]. In addition to other benefits [2]–[5], MIMO radar systems provide the flexibility of transmitting a completely different waveform on each of its colocated antennas. This flexibility can lead to agile sensing and waveform diversity and thus improve target detection and estimation performance by optimally designing the transmission waveforms.

Waveform selection techniques have been developed in order to maximize desirable performance metrics. In [3], beamforming was used to improve the output signal-to-noise ratio (SNR) and increase detection and estimation performance for MIMO radars with colocated antennas. Optimal waveform design for MIMO radars was achieved using an information theoretic approach in [6]. In [7], a procedure was developed for designing the waveform that maximized the signal-to-interference plus-noise ratio for target detection. In [8], the CRLB was used to design a random transmission waveform to achieve a desired beam pattern.

In our work, we combine the use of MIMO radar with colocated antennas with waveform agile sensing by optimally selecting a different waveform to transmit on each antenna. In particular, we incorporate the waveform-dependent CRLB for the joint estimation of range and direction-of-arrival (DoA) of a stationary target, and we make use of generalized frequency-modulated (GFM) waveforms with time-varying phase functions. For $N_T$ transmission antennas, our aim is to optimally select $\Phi = [\phi_1^{T}, \phi_2^{T}, \ldots, \phi_{N_T}^{T}]^{T}$, where the waveform parameter vector $\phi_i^{T}$ consists of the duration $\lambda_i$, frequency-modulation (FM) $b_i$ and phase function $\xi_i(t/t_r)$ of the GFM waveform transmitted by the $i$th antenna. Here, $t_r$ is a normalization time constant, and $T$ denotes vector transpose. In order to localize a stationary target, the transmission antenna waveform parameters are selected such that the trace of the predicted estimation error covariance is minimized. The error covariance is obtained under the assumption that the covariance of the observation noise can be closely approximated by the CRLB when the SNR is high. Thus, minimizing the waveform-dependent CRLB corresponds to minimizing the estimation mean-squared error (MSE). Using a maximum likelihood estimator, we provide simulation results to demonstrate that the estimate covariance is in close agreement to the CRLB, thus validating the use of the CRLB as the optimization performance criterion.

The paper is organized as follows. We discuss the structure of the waveform used in the configuration process in Section II. The received waveform for the MIMO radar system is provided in Section III, together with the computation of the CRLB for the joint estimation of the antenna transmission coefficients and the range and DoA of the target. The optimization criteria for configuring linear FM (LFM) waveforms and GFM waveforms with varying phase functions are considered in Section IV, and simulation results are provided in Section V.

II. WAVEFORM STRUCTURE

We choose the transmission waveform from each of the co-located antennas of a MIMO radar system to be a GFM
waveform that is given by [9]

\[ s(t) = a(t) e^{j2\pi b t/t_r}. \]  

(1)

Here, \( a(t) \) is the amplitude envelope function and \( b \) is the FM rate of the GFM. By varying the phase function \( \xi(t/t_r) \), we can obtain different time-frequency waveform signatures that uniquely characterize the GFM. The time-frequency signature corresponds to the waveform instantaneous frequency obtained using the time-derivative of the phase function. For example, when \( \xi(t/t_r) = (1/2)(t/t_r)^2 \), the GFM has a linear instantaneous frequency given by \( d/dt(\xi(t/t_r)) = t/t_r^2 \), and it simplifies to an LFM chirp, a waveform commonly used in radar applications. When \( \xi(t/t_r) = \ln(t/t_r) \), the instantaneous frequency is hyperbolic and the corresponding waveform is the hyperbolic FM (HFM) chirp; this is a waveform similar in time-frequency structure to the waveforms used by bats for echo-location [9]. Note that GFM waveforms with nonlinear phase function were shown in [10] to provide more accurate signatures of different GFMs can be affected in different ways due to the low energy content at the tails of the Gaussian function. Although the ideal choice for the envelope would be a rectangular function, this function is not differentiable and thus the corresponding waveform-dependent CRLB cannot be obtained in closed form. As an alternative, we choose to use a sigmoid envelope function. This function is differentiable and thus can lead to a closed form CRLB expression. In addition, it closely approximates a rectangular window in the time domain with an out-of-band frequency roll-off that is much higher than that of a rectangular window (as shown in Fig. 1). The sigmoid function is defined as

\[ a(t) = \alpha \left[ \frac{1}{1 + e^{-q t}} - \frac{1}{1 + e^{-q(t-\lambda)}} \right], \]  

(2)

where the parameter \( \alpha \) is chosen such that \( s(t) \) in (1) has unit energy, \( \lambda \) is the duration of the waveform, and \( q \) is a design parameter that controls the roll-off rate of the window in the frequency domain.

III. RECEIVER MODEL AND CRLB

A. Received Signal Model

We consider a MIMO radar system with \( N_T \) colocated transmitter antennas and \( N_R \) receiver antennas. The transmitted signal \( s_i(t) \), \( i = 1, \ldots, N_T \), of the \( i \)th antenna is sampled every \( T_s \) seconds to obtain \( s_i[n] = s_i(nT_s) \), \( n = 1, \ldots, N \). The resulting transmitted signal matrix is given by \( S = [s_{12} \cdots s_{1N}] \) where \( s_n = [s_1[n] s_2[n] \cdots s_{N_T}[n]]. \) The time-delayed (by \( \tau \) ) signal due to a single target is given by \( s_i(\tau)[n] = s_i(nT_s - \tau) \); the delayed sampled signal matrix is thus given by \( S(\tau) = [s_1(\tau) s_2(\tau) \cdots s_{N_T}(\tau)]. \)

The signal received by the \( N_R \) receivers is given by the \( N_R \times N \) matrix

\[ R = \beta a(\theta) v^T(\theta) S(\tau) + W, \]  

(3)

where the steering vectors \( v(\theta) \) and \( a(\theta) \) of the transmitter and receiver antennas, respectively, depend on the DoA \( \theta \), \( \beta \) is the reflection coefficient of the target, and \( W \) is a \( N_R \times N \) noise matrix whose elements are snapshots of zero-mean additive Gaussian noise. Vectorizing \( W \), we obtain \( w = \text{vec}[W] = [w_1^T w_2^T \cdots w_N^T]^T \), where \( w_n = [w_1[n] w_2[n] \cdots w_{N_T}[n]]^T \) and \( w[n] \) is the \( n \)th sample of the Gaussian noise added to the waveform transmitted by the \( i \)th antenna. The noise covariance matrix can be expressed as \( C_w = E\{ww^H\} = C_T \otimes C_S \) where \( C_T \) and \( C_S \) are the corresponding temporal and spatial noise covariance matrices, \( E\{\cdot\} \) denotes statistical expectation, \( H \) denotes complex conjugate transpose, and \( \otimes \) is the Kronecker product. Following the same notation, we vectorize \( R \) such that \( r = [r_1^T r_2^T \cdots r_N^T]^T \) where \( r_n = [r_1[n] r_2[n] \cdots r_{N_T}[n]]^T \) is the \( n \)th snapshot of the received signal. Note that (3) can also be extended to multiple targets.
B. Joint CRLB Derivation

We want to estimate the location of a target by estimating the reflection coefficient, range, and DoA of the target. We denote the parameter vector to be jointly estimated as \( \Psi = [\beta_R \beta_I \tau \theta]^T \), where we denote \( \beta = \beta_R + j \beta_I \). Then, the probability density function \( p(\tau; \Psi) \) of the received data given the unknown parameter vector is a joint Gaussian density with mean vector \( \mu(\Psi) = \beta a(\theta) v^T(\theta) S(\tau) \) and covariance matrix \( C_w \). The \( i \)th element of the Fisher information matrix \( I(\Psi) \) for estimating \( \Psi \) is given by [11]

\[
I(\Psi)_{ij} = 2 \text{Re} \left\{ \frac{\partial \mu^H(\Psi)}{\partial \Psi_i} C_w^{-1} \frac{\partial \mu(\Psi)}{\partial \Psi_j} \right\},
\]

where \( i, j = 1, 2, 3, 4 \), \( \text{Re}\{\cdot\} \) denotes the real part and \( \Psi_i \) is the \( i \)th element of the parameter vector \( \Psi = [\beta_R \beta_I \tau \theta]^T \). The joint CRLB is thus given by \( \text{CRLB}_\Psi = (I(\Psi))^{-1} \).

We define \( \mathbf{A}(\theta) = a(\theta)v^T(\theta) \), and the diagonal elements of the Fisher information matrix are expressed as \( I(\Psi)_{11} = I_{\beta_R}, \quad I(\Psi)_{22} = I_{\beta_I}, \quad I(\Psi)_{33} = I_{\tau}, \quad \) and \( I(\Psi)_{44} = I_\theta \), then we can obtain these elements in closed form as

\[
I_{\beta_R} = |\beta|^2 2 \text{Re} \left\{ \text{Tr} \left\{ \mathbf{S}^H(\tau) \frac{\partial^2\mathbf{A}(\theta)}{\partial \theta^2} \mathbf{C}_S^{-1} \mathbf{A}(\theta) \mathbf{S}(\tau) \mathbf{C}_T^{-1} \right\} \right\},
\]

\[
I_{\beta_I} = |\beta|^2 2 \text{Re} \left\{ \text{Tr} \left\{ \mathbf{S}^H(\tau) \mathbf{A}(\theta) \mathbf{C}_S^{-1} \mathbf{A}(\theta) \mathbf{S}(\tau) \mathbf{C}_T^{-1} \right\} \right\},
\]

\[
I_{\tau} = |\beta|^2 2 \text{Re} \left\{ \text{Tr} \left\{ \frac{\partial^2\mathbf{A}(\theta)}{\partial \tau^2} \mathbf{A}(\theta) \mathbf{C}_S^{-1} \mathbf{A}(\theta) \mathbf{S}(\tau) \mathbf{C}_T^{-1} \right\} \right\},
\]

The other Fisher information matrix elements can be similarly derived in closed form. Note that we obtain direct DoA measurements and the range information is obtained from \( c \tau/2 \), where \( c \) is the speed of light in the air. Thus, the parameters of interest directly depend on the CRLB on estimating \( \Psi \) as the CRLB is obtained as inverse of the Fisher information matrix. As a result, if the error covariance of the estimate is in close agreement with the CRLB, then the CRLB can be used as the performance metric in the waveform selection algorithm.

IV. WAVEFORM SELECTION

Under high SNR conditions, minimizing the parameter estimation MSE corresponds to minimizing the CRLB (which was shown to depend on the transmitted waveforms). As a result, selecting the optimal waveform for each antenna in order to minimize the MSE corresponds to searching over a library of waveforms and their parameters such that the trace of the error covariance matrix (corresponding to the trace of the CRLB) is minimized. The library of waveforms is formed by varying the phase function \( \xi(t/t_r) \), FM rate \( b \), and duration \( \lambda \) of the GFM waveform in (1) and (2).

In this paper, we consider three types of GFM waveforms: LFM waveforms with phase function \( \xi(t/t_r) = (t/t_r)^2 \) and linear instantaneous frequency, HFM waveforms with phase function \( \xi(t/t_r) = \ln(t/t_r) \) and hyperbolic instantaneous frequency, and exponential frequency-modulated (EFM) waveforms with phase function \( \xi(t/t_r) = e^{t/t_r} \) and exponential instantaneous frequency. In the first simulation case, we consider a library consisting of only LFM waveforms with varying FM rates and durations. In the second simulation case, we consider a library consisting of all three GFM waveforms, thus allowing the phase functions as well as the durations and FM rates to vary. Once the optimal waveform for each antenna is chosen, the optimal waveforms are transmitted and the received waveform is processed using a maximum likelihood estimator to jointly estimate the delay and DoA of the target.

For a fair comparison between the different possible selection waveforms, we fix the time-bandwidth of each waveform in the library as well as the transmitted energy of all the antenna waveforms. Fixing the total transmitted energy is equivalent to fixing the total transmission time of all the waveforms given by \( \lambda = \sum_{i=1}^{N_T} \lambda_i \), where \( \lambda_i \) is the duration of the waveform transmitted by the \( i \)th antenna. We define \( \Delta f_i \) to be the frequency sweep of the \( i \)th transmit waveform (that is, the difference between the maximum and minimum frequencies in the waveform). Then, we choose \( L \) combinations of \( \lambda_i \) values that fix \( \lambda_i \Delta f_i \) to a constant value for varying \( \Delta f_i \). The value of \( L \) depends on how fine we want the grid resolution of the search in the selection algorithm to be.

For the LFM only library, the \( L \) combinations consist of the same type of waveform. When the library includes LFM, HFM and EFM waveforms with varying durations and FM rates, we again form \( L \) combinations of possible durations for each combination of \( N_T \) possible phase functions (including the combination of using the same phase function on different antennas). The waveform selection algorithm then chooses the combination of phase functions and durations that results in the minimum CRLB trace.

V. SIMULATION RESULTS

We tested the waveform selection algorithm for the joint estimation of the delay and DoA of a single target with 15 km range and 30° DoA. The MIMO radar in the simulations consisted of \( N_T = 3 \) colocated antennas, and the carrier frequency was \( f_c = 10 \) GHz. The allowable waveform duration ranged between 10 \( \mu s \) to 70 \( \mu s \), and \( \lambda = 90 \) \( \mu s \). The maximum allowable frequency sweep was constrained to 12 MHz.

When the waveform library consisted only of LFM waveforms, the duration grid size was chosen to be 10 \( \mu s \), and the trace of the CRLB was computed for each of the combinations. Fig. 2 shows that the trace of the CRLB did change for different combinations of LFM waveform durations. When we included LFM, HFM and EFM waveforms with different durations in the library, many more combinations were computed. In particular, as shown in Fig. 3, the trace of the CRLB is shown for different phase and duration combinations.

The estimation of DoA strongly depends on the energy of the transmitted signal whereas the delay estimation is affected by the type of signal transmitted. As a result, when we consider
the CRLB of the delay for different waveforms, the differences in magnitude are more pronounced. This is demonstrated in Fig. 4 when the GFM waveform library is used. The CRLB for the delay estimate for the best and worst combinations is shown to vary by a factor of about 400 in magnitude.

In Fig. 5, we demonstrate that the CRLB can be used as a performance metric for the maximum likelihood estimator (MLE). Specifically, we show that the variance of the delay and DoA MLE estimates are in close agreement with their CRLB.

The results of the waveform-agile estimation algorithm are demonstrated in Fig. 6. Specifically, we compared the the CRLB and MSE performance of the waveform selection algorithm with the CRLB and MSE performance of fixed LFM waveforms. The LFM waveforms were chosen to have short durations (for good delay estimates) and wide bandwidths (for good Doppler estimates). They also satisfied the energy and time-bandwidth product constraints. As we can see in Fig. 6, the CRLB of the delay estimation using waveform selection outperformed the CRLB of the fixed LFM waveform. Also, the MSE performance when transmitting the selected waveforms outperformed the performance of transmitting the fixed LFM waveforms.

VI. CONCLUSION

We developed a waveform-agile algorithm for locating a target using a MIMO radar system with colocated antennas. In particular, we designed different time-varying signatures for multiple transmit antennas in order to minimize the estimation error covariance of the parameters of a stationary
target. Using high SNR assumptions, the estimation error covariance is approximated by the waveform-dependent CRLB for the joint estimation of range and direction-of-arrival (DoA) of the target. Simulation results demonstrated the effectiveness of the waveform-agile localization algorithm.

**REFERENCES**


