Managing Multi-Modal Sensor Networks Using Price Theory †

Phani Chavali, Student Member, IEEE and Arye Nehorai∗, Fellow, IEEE

Department of Electrical and Systems Engineering
Washington University in St. Louis
One Brookings Drive, St. Louis, MO 63130, USA
Email: {chavalis, nehorai}@ese.wustl.edu
Phone: 314-935-7520 Fax: 314-935-7500

ABSTRACT

We propose a unified framework for sensor management in multi-modal sensor networks, which is inspired by the trading behavior of economic agents in commercial markets. Each sensor node (SN) acts as a seller who wants to sell the data it collects, to the sensor network manager (SM) who acts as a buyer. The resources and the data are priced by looking to balance global supply and demand, with the SN required to purchase resources for producing the data, and the SM required to purchase data to accomplish his tasks. We model this interaction as a double sided market, with both consumers and producers, and propose an iterative double auction mechanism for computing the equilibrium of such a market. We relate the equilibrium point to the solutions of sensor selection (SS), resource allocation (RA) and data fusion (DF) problems, which constitute the sensor management. The proposed framework will enable the system to determine the kind and the amount of data that should be produced, and to combine the data that is produced at each SN. To illustrate this framework, we consider the problem of multiple-target tracking as an example. Numerical simulations demonstrate the effectiveness of the proposed method, and show that appropriate sensor management will result in an accurate estimate of the number of targets in the scene, higher correct identifications of the targets, and a lower mean-squared error in the estimates of their positions and velocities.

Index Terms

† Copyright (c) 2012 IEEE. Personal use of this material is permitted. However, permission to use this material for any other purposes must be obtained from the IEEE by sending a request to pubs-permissions@ieee.org. This work was supported by the Department of Defense under the AFOSR Grant FA9550-11-1-0210 and the ONR Grant N000140810849.
multi-modal sensors, multi-target tracking, resource allocation, sensor selection, data fusion, price theory, auctions

I. INTRODUCTION

The use of sensor\textsuperscript{1} networks is gaining momentum for various applications, including situation assessment [1], enemy intent understanding and prediction, aircraft health monitoring [2], and crisis management [3]. It has been an active area of research in the past few years. With the advances in the development of inexpensive sensing devices such as acoustic, optical, seismic, radar, and biological sensors, it has become feasible to deploy several kinds of sensors to obtain information about various targets, activities and events. In some cases, humans act as sensors when they can provide information about the scene. However, due to varying uncertainty in the environment, and due to the possibility of sensor failures, a single sensor is incapable of reliably obtaining information at all times. A multi-modal sensor network consists of multiple sensors, which are placed in the region of interest, with no restrictions on spatial and temporal overlapping to obtain multi-modal data. Multi-modal sensors have been used in the past for cardiovascular diagnosis [4], 3D image reconstruction [5], temperature inference [6], and target tracking [7]. When the information from different sensors is appropriately combined, the performance of the overall system improves significantly compared to the performance obtained by using a single sensing modality. However, employing multi-modal sensors results in other problems.

First, the multi-modal data collected across the sensors is highly diverse, incomparable, and sometimes inconsistent. The sensing system should understand how to combine the information provided by multiple sensors, and it should be capable of adaptively changing the way it combines this information depending on the situation. Second, gathering information using the sensors incurs cost: physical cost, computational cost, and communication cost. Sensing applications, however, are constrained by limited resources. Hence, the sensing system should decide which sensors it should use to produce the data, and how it should distribute the limited resources to the sensors, such that the system achieves the best performance within the cost constraints. These considerations give rise to three important aspects in sensor management: sensor selection (SS) [8], resource allocation (RA) and data fusion (DF) [9]. Sensor management can thus be defined as a process that manages and coordinates the use of sensors

\textsuperscript{1}Sensors could refer to any devices/means of acquiring information.
by adaptively selecting the sensors, distributing the resources among the sensors, and combining the information obtained from various sensors with an overall goal of improving the system performance.

In this paper, we propose a framework for sensor management that is inspired by the trading behavior of agents in a commercial market and we use an economic price theory-based approach [10]. The idea of using a market-based approach is not new [11], and it has been used to solve the resource allocation problems in the past. In our work, however, we use this approach for the entire sensor management process, by jointly addressing SS, RA, and DF. We model both the sensor nodes and the sensor manager as self interested agents in a double-sided market. This kind of modeling enables us to obtain a joint solution to all three problems related to sensor management. We model each sensor node (SN) as a producer who wishes to sell the data it produces, and each sensor manager (SM) as a consumer who wants to buy the data from the sensors. The basic idea is that the data is priced by looking to balance global supply vs global demand. The price of the data forces the SM to restrict itself to using the data only from a limited number of sensors. Since the SM must provide a payment for the data it uses, it will be straightforward in describing the utility it obtains from each kind of SN. In a similar way, the SN will purchase resources, for example, power, from the SM and pay in return for the resources. The SN will use these resources to produce the data and it will be straightforward in describing the quantity of the data it can provide with the resources it has. In this way, the proposed approach will provide a natural framework for the SN and the SM to interact and to communicate their utilities dynamically. It also provides a rigorous methodology for incorporating the importance of the information that the sensors collect to the overall system goal.

We describe the proposed framework using multiple-target tracking [12] as an illustrative example. The task of the sensing system in multiple-target tracking is to identify and track the positions, the velocities, and the categories of an unknown number of targets moving in the region of interest. Our motivation for using multiple-target tracking as an example stems from the fact that this problem is encountered in practice in several military and commercial applications such as air traffic control and battle-field surveillance. Multiple-target tracking also poses several major challenges such as an unknown and time-varying number of targets, the inability of the sensors to observe all the targets at all times etc. Tracking systems that rely on a single sensor cannot overcome these difficulties, and they are vulnerable to errors. To improve the performance of the tracking system, a multi-modal sensor network should be used. Earlier works using multi-modal information for target tracking include [7], [13], and [14]. Although the
description we provide is specific to target tracking, our proposed framework can be easily extended to suit other applications that employ multi-modal sensors.

The rest of the paper is organized as follows. In Sec. II, we describe the problems of SS, RA, and DF. In Sec. III, we provide a brief introduction to price theory. We describe an economic market model, the interaction between the agents in a market and the notion of a Walrasian equilibrium. We then propose an iterative double auction algorithm to find an approximate equilibrium of the market. In Sec. IV, we describe our proposed approach for sensor management to solve the multiple target tracking problem using multi-modal sensors. We employ a multi-static radar, an infrared camera, and a human scout to collect the information about the target scene. We derive the state transition model for the state vector, and the statistical measurements models for all the sensors and we use Monte-carlo sampling methods to obtain the Bayesian estimates of the state vector. We provide some numerical examples in Sec. V, and we summarize the work in Sec. VI.

We use the following notations in the paper. We denote vectors by boldface lowercase letters, e.g., $a$, and matrices by boldface uppercase letters, e.g., $A$. For a matrix $A$, we use $a_i$ to represent the $i^{th}$ column of $A$ and $[A]_{ij}$ to represent the element in the $i^{th}$ row and the $j^{th}$ column. $(A)^T$, $(A)^H$, and vec($A$) denote the transpose, conjugate transpose and vector form of the matrix $A$, respectively. The $i^{th}$ element of a vector $a$ is denoted by $a_i$. The Kronecker product of two matrices, $A$ and $B$, is denoted as $A \otimes B$. $I_M$ and $0_{M \times N}$ denote an identity matrix of order $M$ and a zero matrix of size $M \times N$, respectively. $\mathcal{N}(x; \mu, \Sigma)$ and $\mathcal{CN}(x; \mu, \Sigma)$ denote a normal distribution and a complex normal distribution in variable $x$, with mean $\mu$ and covariance matrix $\Sigma$, respectively. $\mathbb{R}$ denotes the set of all real numbers and and $\mathbb{N}$ denotes the set of all natural numbers.

II. PROBLEM DESCRIPTION

The objective of a typical sensor network is to estimate a set of parameters $\theta \in \Theta$, referred as the state vector, using the measurements obtained by the sensors. The inference about the underlying situation or event can be then drawn based on the estimate of the state vector. Consider a sensor network with $P$ sensors, possibly of different modalities, labeled as $S = \{1, 2, \ldots, P\}$. At time $t$, each of these sensors can partially observe the vector $\theta_t$, i.e., the measured data $y_{p,t}$ corresponding to the $p^{th}$ sensor is a function of a subset $\theta_{p,t} \subset \theta_t$ of the state vector, which is represented as a projection defined on $\Theta$. For the $p^{th}$ sensor, we label this projection as $g_p(\theta_p)$. The
measurement vector corresponding to the $p^{th}$ sensor at time $t$ can then be written as

$$y_{p,t} = g_p(\theta_{p,t}; t) + w_{p,t},$$

where $w_{p,t}$ denotes the additive noise. The function $g_p(\cdot)$ depends on the kind of the sensor used, and it is assumed to be known but possibly nonlinear. Let $Y_t = \{y_{1,t}, \ldots, y_{P,t}\}$ denote the measurements obtained by all the sensors at time $t$ and $y_{p,1:t}$ be the measurements obtained by the $p^{th}$ sensor up to time $t$. Given the set $Y_{1:t} = \{y_{1,1:t}, \ldots, y_{P,1:t}\}$, our objective is to estimate the state vector $\theta_t$. A minimum mean-squared error (MMSE) estimate of the state vector can be computed as the mean of the global posterior distribution $p(\theta_t | Y_{1:t})$, given the measurements made by the sensors up to time $t$. We assume the existence of a central fusion center, which we refer to as the sensor manager (SM) that processes the measurements obtained by the individual sensors and computes the global posterior distribution $p(\theta_t | Y_{1:t})$. The SM also decides which sensors should be activated at each time, how the finite resources should be distributed among the activated sensors, and how the information obtained from each sensor should be combined. These considerations give rise to the three aspects of the sensor management: SS, RA and DF.

Let $c_{p,t}$ denote the cost incurred by activating the $p^{th}$ sensor at time $t$, $\pi_t \subset \mathcal{S}$ denote a subset of sensors at time $t$, $\mathcal{P}(\mathcal{S})$ denote the power set of $\mathcal{S}$, $\{u_i\}_{i=1}^3$ denote a set of utility functions that characterizes the system performance at time $t$, and $r_{p,t}$ denote the resources allocated to the $p^{th}$ sensor at time $t$. Using these notations, we define SS, RA, and DF as follows.

The problem of SS is to select a subset of sensors from a given set possible sensors, with the constraints on the total cost and coverage. It can be formulated as the following constrained optimization problem:

$$\text{SS} : \quad \pi_t^* = \arg \max_{\pi_t \in \mathcal{P}(\mathcal{S})} u_1^1,$$

subject to $\bigcup_{p \in \pi_t} \theta_{p,t} = \theta$ and $\sum_{p \in \pi_t} c_{p,t} \leq \eta_c$.  

In this formulation, the first constraint corresponds to the coverage constraint, which ensures that every component of the state vector is covered by at least one sensor, and the second constraint ensures that the overall cost incurred by employing the sensors is within a predefined threshold. The parameter $\eta_c$ corresponds to the constraint on the cost.

The problem of RA concerns allocating the available and limited resources to the sensors to maximize the utility...
each of them provides to the sensor network. It can be formulated as the following joint constrained optimization problem:

$$\text{RA} : \quad \{ r_{p,t}^* \}_{p=1}^P = \arg \max_{\{ r_{p,t} \}_{p=1}^P} \{ u_t^2 \},$$

subject to \( \sum_{p \in \pi_t} r_{p,t} \leq \eta_t \). \hspace{1cm} (3)

The parameter \( \eta_t \) corresponds to the total available resources.

The problem of DF is to find an optimal way to combine the information that is obtained from various sensors. Since the data collected by the sensors is, in general, inconsistent, it should be combined at the decision level [15]. In this paper, we compute the global posterior distribution as a weighted sum of the local posterior distributions due to the individual sensors. This method of data fusion is called the linear opinion pool [16]. The problem of the DF then corresponds to finding the optimal weights, and it can be formulated as the following constrained optimization problem:

$$\text{DF} : \quad \mu_t^* = \arg \max_{\mu_t \in \mathbb{R}^P} u_t^3,$$

subject to \( \sum_{p \in \pi_t^*} \mu_{p,t} = 1 \), and

$$p(\theta_t \mid Y_{1:t}) = \sum_{p \in \pi_t^*} \mu_{p,t} p(\theta_t \mid y_{p,1:t}), \hspace{1cm} (4)$$

where the weight \( \mu_{p,t} \) represents a subjective measure of the reliability of the data collected by the \( p^{th} \) sensor, and the utility function \( u_t^3 \) is a function of the global posterior distribution \( p(\theta_t \mid Y_{1:t}) \).

SS, RA, and DF have been studied independently in the past, and several techniques using various cost and utility functions, such as the reduction in the uncertainty of the state vector, volume of the confidence ellipsoid of the state vector etc., have been used. SS has been addressed in [17]-[21], RA has been studied in [22]-[23] and DF in [24]. The list is not exhaustive. All these earlier methods neglect the importance of dynamically changing environment on the utility functions that each SN and SM use. Further, there was no unified framework for obtaining a joint solution to SS, RA, and DF at the same time. Our approach addresses these concerns by providing a framework, where the utility function of the SNs and the SMs is updated periodically. Each node of the sensor network acts as a self-interested economic agent that operates in a virtual market. A global efficient behavior is enforced by adjusting the price vector, which decides how each node behaves. The equilibrium point of this virtual market is
then related to the solutions of SS, RA, and DF.

III. PRICE THEORY AND AUCTIONS: PRELIMINARIES

A. Walrasian Equilibrium

Price theory is a branch of economics that explains the trade of goods and services between different economic agents [10]. Agents fall into two different categories: consumers and producers. Consumers can buy and sell various goods in the market, whereas producers can transform goods of some sort into goods of a different sort. Consider an economic market with $N_c$ consumers, $N_p$ producers, and $K$ indivisible goods. For the $i^{th}$ consumer, his preference for consuming various bundles of goods, denoted as $x_i = [x_{i1}, x_{i2}, \ldots, x_{iK}]^T$ and referred to as the demand vector, with $x_{ik} \in \mathbb{N}$ representing the quantity of the $k^{th}$ good that the $i^{th}$ consumer trades, is specified by a utility function, $u_i : \mathbb{N}^K \rightarrow \mathbb{R}$. The utility function of a consumer ranks various bundles of goods according to his preference.

If $x_{ik} > 0$, then the consumer buys the good, and if $x_{ik} < 0$, the consumer sells the good. Each consumer starts with an initial endowment of goods $e_i = [e_{i1}, \ldots, e_{iK}]^T$, with $e_{ik} \in \mathbb{N}$ representing the quantity of the $k^{th}$ good available for trade with the $i^{th}$ consumer. Given a price vector $p = [p_1, \ldots, p_K]^T$, the objective of the $i^{th}$ consumer, $i = 1, \ldots, N_c$, is to choose an optimal demand vector that maximizes his utility function under the constraint that the total wealth he spends is less than the total wealth he can generate by selling his endowment, given by $\sum_{k=1}^K e_{ik}p_k = p^T e_i$, at price $p$. This feasible set is called the budget set of the consumer. The consumer’s choice for his preferred bundle of goods is obtained by solving the following constrained optimization problem:

$$x_i^* = \arg \max_{x_i \in B_i(p, e_i)} u_i(x_i),$$

where $B_i(p, e_i) = \{x_i \in \mathbb{N}^K : p^T x_i \leq p^T e_i\}, \quad i = 1, \ldots, N_c.$ (5)

Agents of the second type, the producers, will take as input, goods from the consumers and convert them into goods of different sort. For the $j^{th}$ producer, a vector $y_j = [y_{j1}, y_{j2}, \ldots, y_{jK}]^T$, $y_{jk} \in \mathbb{N}$, called the production plan vector, where $y_{jk} > 0$ if the $k^{th}$ good is an output, and $y_{jk} < 0$ if it is an input, defines the amount of goods that the producer takes as input and produces as output. The maximum output of the $k^{th}$ good obtained from the $j^{th}$ producer will be a function of his input goods and of the available technology to produce the good. This is represented by a production function, $v_{jk} : \mathbb{N}^K \rightarrow \mathbb{N}$. Each producer has an initial wealth $w_j$ required to start the production. Given a price vector $p = [p_1, \ldots, p_K]^T$, the objective of the $j^{th}$ producer, $j = 1, \ldots, N_p$, is to choose
a production plan vector that maximizes his profit, subject to the constraints on the maximum amount of goods that he can produce. Define $y^+_j = \{y_{jk} \in y_j | y_{jk} > 0\}$, $y^-_j = \{y_{jk} \in y_j | y_{jk} < 0\}$ and $p^-_j = \{p_k \in p | y_{jk} < 0\}$.

The optimal production plan vector of each producer is obtained by solving the following constrained optimization problem:

$$y^*_j = \arg \max_{y_j \in \mathbb{R}^N} p^T y_j,$$

subject to $y^+_j \leq v_{jk}, \forall k$, and $(p^-_j)^T y^-_j \leq w_j, j = 1, \ldots, N_p.$ (6)

Walras [25] defined a notion of an equilibrium in such consumer-producer economic markets, called the Walrasian equilibrium, which is most commonly used by economists today.

**Definition 1: Walrasian Equilibrium**: The tuples $(\{x^*_i\}_{i=1}^{N_c}, \{y^*_j\}_{j=1}^{N_p}, p^*)$ of the demand vector, the production plan vector and the price vector in an economy form a Walrasian equilibrium, if and only if

1) $x_i$ is a solution to the constrained optimization problem given in Eq. (5) at the price $p$, $\forall i$

2) $y_j$ is a solution to the constrained optimization problem given in Eq. (6) at the price $p$, $\forall j$, and

3) the market is clear at the price $p$, i.e., $\sum_{i=1}^{N_c} x_{ik} = \sum_{j=1}^{N_p} y_{jk}, \forall k$.

Under some mild assumptions on the continuity and the monotonicity of the utility and the production functions, it was shown that the Walrasian equilibrium exists for all economies [26] using a fixed point argument. Under a much stronger assumption called the *gross substitutability*\(^2\) condition, the equilibrium is also proved to be unique. The key result of the price theory is that the Walrasian equilibria, although defined as a solution to utility maximization and the profit maximization problems of individual agents, will produce *Pareto-optimal*\(^3\) allocations. This result is stated as the following two fundamental theorems.

**Theorem 1: First Fundamental Welfare Theorem** - If the triplet $(p, x_i, y_j)$, for $i = 1, 2, \ldots, N_c$ and $j = 1, 2, \ldots, N_p$ is a Walrasian equilibrium, then the allocations $x_i$ and $y_j$ for $i = 1, \ldots, N_c$ and $j = 1, \ldots, N_p$ are Pareto-optimal.

**Theorem 2: Second Fundamental Welfare Theorem** - In a convex economy\(^4\), if $x_i$ and $y_j$ for $i = 1, \ldots, N_c$\(\)\(^2\)If there is an increase in the price of one good, then the net demand for other goods does not decrease

\(^3\)An allocation of goods to agents is defined to be Pareto-optimal if no other allocation of the same goods would be preferred by every agent

\(^4\)The utility and production functions of all the consumers and producers are convex functions
and $j = 1, \ldots, N_p$ represent any set of Pareto-optimal allocations, then there exists a price vector $p \in \mathbb{R}^K$ such that the tuples $\left(\{x_i^*\}_{i=1}^{N_c}, \{y_j^*\}_{j=1}^{N_p}, p^*\right)$ form a Walrasian equilibrium for a suitable choice of initial endowments.

The market equilibrium problem is to compute a price vector, the corresponding demand vectors, and the production plan vectors for all the agents in the economy such that they form a Walrasian equilibrium. This problem is of considerable interest in Economics, and several works have investigated this problem [27]-[28]. However, finding computationally efficient polynomial time algorithms to compute the equilibrium prices and allocations for a general economic model is still a major research area. Over the last few years, there has been a huge effort in the theoretical computer science community to develop efficient algorithms for computing the equilibria [29]-[33]. While few groups have been working on developing polynomial time algorithms for specific markets, the other groups focused on developing algorithms for computing the approximate equilibrium. One such notion of an approximate equilibrium is $\epsilon$-approximate equilibrium. The tuples $\left(\{x_i^*\}_{i=1}^{N_c}, \{y_j^*\}_{j=1}^{N_p}, p^*\right)$ form an $\epsilon$-approximate Walrasian equilibrium if, for $0 < \epsilon < 1$, the optimal solutions $\{x_i^*\}_{i=1}^{N_c}$ and $\{y_j^*\}_{j=1}^{N_p}$ are such that $\sum_{i=1}^{N_c} x_i^* = (1 - \epsilon) \sum_{j=1}^{N_p} y_j^* \forall k$ at price $p^*$, i.e., the market clearing condition is approximately satisfied. In the next subsection, we describe auctions and propose an auction mechanism that can be used to compute an $\epsilon$-approximate equilibrium of the market model described in this subsection.

B. Auctions and Price discovery

Auction algorithms first originated as methods for finding solutions to an assignment problem where several agents were competing for various resources [34]. Since then, auctions have been used for solving a wide variety of problems in the areas of computer science [35], Economics [36] and finance. Auction-based algorithms are used for two important reasons. First, they are intuitive and easy to implement. Second, they provide a general theoretic framework for understanding the interaction between self-interested agents and provide computationally efficient methods for solving the allocation problems among these agents, with the objective of achieving Pareto-optimal outcomes.

There are many types of auctioning mechanisms, each with its own unique characteristics and applications [Chap-10, [37]]. Four primary types of auctions that are widely used are the ascending bid auction (English auction), the descending bid auction (Dutch auction), the first price sealed-bid auction and the second price sealed-bid auction.
(Vickrey auction). Based on these primary types, several secondary type auctions have been derived by making minor modifications. For a detailed descriptions of auctions, interested readers can refer [Chap-10, [37]]. Our auction algorithm is a combination of two auction mechanisms: (i) the double auction [38] and (ii) the combinatorial auction [39].

In a traditional auction, an auctioneer is regarded to be either on the sellers’ side or the buyers’ side. When an auctioneer is on the sellers’ side, his main objective is to maximize the sellers’ profits while minimizing their cost, whereas if the auctioneer is on the buyers’ side, his objective is to maximize buyers’ utility and minimize their purchase cost. Both of these scenarios are considered to be one-sided, and hence a third neutral auctioneer scenario is introduced, where the objective is to strike a balance between the two prior cases, with the main goal of maximizing global welfare. These types of auctions are called double auctions (DA). It was shown that DAs are much more efficient than several one-sided auctions [40]. Combinatorial auctions (CA) are a different class of auction mechanisms, where bidders can place bids on the combinations (or bundles) of goods, instead of being limited to bidding on a single item, as happens in most conventional auctions. This ability to bid on several combinations of goods allows the agents to more accurately express their preferences. Combinatorial double auctions (CDA) [40]-[41], which are a combination of DAs and CAs, are most frequently encountered in market-based economies (for e.g. stock exchange markets), and they represent the advantages of combinatorial auctions by allowing bids to be placed on several combinations of goods, and also the double auctions by considering the requirements of both buyers and sellers in the market. The use of auction-based algorithms for computing the market equilibrium was first proposed in [36] and they have been used extensively since then.

We propose an iterative CDA algorithm that can be used for finding the $\epsilon-$approximate Walrasian equilibrium for the market scenario described in the previous section. We restrict ourselves to the class of linear models, where the utility and the production functions are linear in the demand vector and the production plan vector, respectively, i.e.,

\begin{equation}
    u_i(x_i) = \sum_{k=1}^{K} u_{i,k} x_{i,k}, \quad \forall i.
\end{equation}

\begin{equation}
    v_{j,k}(y_k) = \sum_{y_j \in y} v_{j,k} y_{j,k}, \quad \forall j.
\end{equation}

The run of the algorithm is partitioned into several iterations. Each iteration is further partitioned into three steps.
TABLE I
FINDING WALRASIAN EQUILIBRIUM USING AUCTIONS

<table>
<thead>
<tr>
<th>Iterative CDA Algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initialize $p_k^n = 0$, $\forall k$, and $n = 1$</td>
</tr>
<tr>
<td>Repeat until $\epsilon$-approximate equilibrium is satisfied or $n &gt; N$</td>
</tr>
<tr>
<td>$\forall i, j$, find $x_{ik}^n$ and $y_{jk}^n$ by solving Eqs. (5) and (6), respectively</td>
</tr>
<tr>
<td>if $(\sum_{i=1}^{N_c} x_{ik}^n &gt; \sum_{j=1}^{N_p} y_{jk}^n)$</td>
</tr>
<tr>
<td>set $p_k^{n+1} = p_k^n (1 + \delta_{p_k^n}</td>
</tr>
<tr>
<td>else if $(\sum_{i=1}^{N_c} x_{ik}^n &lt; \sum_{j=1}^{N_p} y_{jk}^n)$</td>
</tr>
<tr>
<td>set $p_k^{n+1} = p_k^n / (1 + \delta_{p_k^n}</td>
</tr>
<tr>
<td>else</td>
</tr>
<tr>
<td>set $p_k^{n+1} = p_k^n$</td>
</tr>
</tbody>
</table>

We start with an arbitrary, but a fixed order for each consumer and producer in the economy. Let $p^n = (p_1^n, \ldots, p_K^n)$ denote the price vector at the $n^{th}$ iteration of the algorithm. In the first step of each iteration, the optimal demand vector of every consumer is evaluated. In order to compute the optimal demand vector of the $i^{th}$ consumer, we will use a branch-and-bound technique to solve Eq. (5) at price $p^n$. In the second step of each iteration, the optimal production plan vector of every producer is evaluated. The optimal production plan vector is obtained by solving Eq. (6) using a branch-and-bound technique. In the third step, the auctioneer will compute the total demand and supply for each of the $K$ goods, and will adjust the price based on the demand and the supply values. The price of the $k^{th}$ good is increased if the demand of the good exceeds the supply, and the price is decreased if the supply exceeds the demand. The increase or the decrease in the price is proportional to the value of the excess demand or the excess supply, respectively, computed at the current price. As the price vector changes, the algorithm recomputes the demand and the production plan vector for each consumer and producer. The algorithm will terminate when an approximate equilibrium is achieved or when the number of iterations exceeds a predetermined threshold. The detailed auction mechanism is shown in Table I.

In this manner, an $\epsilon$–approximate equilibrium of the market is reached via the interactions between the producers and consumers using an auction mechanism. We make several comments about the auction mechanism here. First, we assume that the processing center has sufficient computational and communication resources to execute the
mechanism and maintain the equilibrium in real-time. Second, there are two methods to implement the auction mechanism across the sensor network: distributed implementation and a centralized implementation. In a distributed implementation, each agent controls the physical entity it represents, and is capable of computation. In our market, a distributed implementation means that the consumers and the producers evaluate their respective optimal demand and production plan vectors at a given price, and submit the demand and production plan vectors in the form of bids to an auctioneer. The auctioneer will then adjust the price based on the demand and the supply. In a centralized implementation, the all processing is done at a central unit. In our market, a centralized implementation means that the producers and consumers will inform their utility and production functions to the processing center, which then evaluates their optimal production and demand vectors. The processing center can itself act as an auctioneer, and adjust the prices for the subsequent iterations. The distributed implementation is communication intensive as the producers, consumers, and the auctioneer should exchange bids and prices at each iteration, whereas the centralized implementation is computation intensive as the processing center has to evaluate the optimal demand and production plan vectors for all the agents in the market. In this paper, we adopt a centralized implementation. We will describe the reasons for this choice in the Section IV-D. Third, we assume that all the agents in the market are price takers. This assumption ensures that the equilibrium point is a Pareto-optimal allocation.

IV. TARGET TRACKING USING MULTI-MODAL SENSOR NETWORKS

In this section, we address sensor management in multi-modal networks, using an economic price theory-based approach, by considering multiple target tracking as an illustrative example. Although the description is specific to target tracking, the proposed framework can be easily extended for sensor management in other applications employing multi-modal sensors. In the following subsections, we describe the system model first, and then the state space model. Next, we describe the statistical measurement models for various sensors that we use. We then describe how we can jointly address SS, RA and DF. Finally, we use a particle filter for obtaining the Bayesian estimate of the state vector. We do not address the problem of data association\(^5\) in this paper, as we assume that there is no clutter in the environment.

\(^5\)The problem of data association is to label the measurements as target originated or clutter originated.
A. System Model

We consider a planar region of the battlefield, \( \mathcal{R} \), with an unknown number of moving targets of various categories. At time \( t \), we assume that the number of targets is \( N_t \). The targets are indexed as \( \{1, \ldots, n_t, \ldots, N_t\} \), with the position and velocity of the \( n_t^{th} \) target denoted as \( \rho_{nt} = [\rho_{x,n_t}, \rho_{y,n_t}]^T \in \mathbb{R}^2 \) and \( \dot{\rho}_{nt} = [\dot{\rho}_{x,n_t}, \dot{\rho}_{y,n_t}]^T \in \mathbb{R}^2 \), respectively. We label the target categories using \( \alpha = \{\alpha_1, \ldots, \alpha_{n_t}, \ldots, \alpha_{N_t}\} \), where \( \alpha_{n_t} \in \mathcal{A} \), with \( \mathcal{A} \) being the set of all the target categories. The parameters of interest are the number of targets \( N_t \) in the scene, the position \( \rho_{n_t} \), the velocity \( \dot{\rho}_{n_t} \), and the category \( \alpha_{n_t} \) of each of the \( N_t \) targets. The overall state vector at time \( t \) is obtained by concatenating all the unknown parameters, and we denote it using \( \theta_t = [N_t, \rho_{1}^T, \dot{\rho}_{1}^T, \alpha_1, \cdot \cdot \cdot, \rho_{N_t}^T, \dot{\rho}_{N_t}^T, \alpha_{N_t}]^T \in (\mathbb{R}^2 \times \mathbb{R}^2 \times \mathcal{A})^{N_t} \). The sensing system comprises three kinds of sensors; a multistatic radar with three spatially diverse radar antennas, an infrared camera, and a human scout. The radar receivers measure the backscattered waveforms bouncing off of the targets, the infrared camera obtains a top view of the region of interest using aerial shots, and the human scout provides an intelligence report about the number and the categories of the targets in the region of interest. The human scout is trained to recognize the target categories and their number, the imaging sensors obtain information about the locations of the targets, while a multistatic radar measures both the positions and the velocities of the targets accurately. The goal of the tracking system is to use the partial and complementary information obtained from different sensors to track the state vector over a period of time. In particular, we are interested in obtaining an estimate of \( \theta_t \) based on the measurements made up to time \( t \). In the following subsections, we derive the transition model for the state vector, and develop the statistical measurement models for the sensors used.

B. State-Space Model

We assume, for simplicity, that there can be at most one birth or one death of the targets at each state transition, and we represent the probabilities of the death and the birth of the targets using \( p_d \) and \( p_b \), respectively. Hence, we have

\[
p(N_{t+1}|N_t = n_t) = \begin{cases} 
p_b & \text{if } N_{t+1} = n_t + 1, \\
n_t p_d (1 - p_d)^{n_t - 1} & \text{if } N_{t+1} = n_t - 1, \\
1 - p_b - n_t p_d (1 - p_d)^{n_t - 1} & \text{if } N_{t+1} = n_t. \end{cases}
\]

\[
(9)
\]
Let $\alpha_t = \{\alpha_{t,1}, \alpha_{t,2}, \ldots, \alpha_{t,N_t}\}$ denote the categories of each of the $N_t$ targets, and assume that the number of categories is finite, i.e., $\text{card}(A) = M < \infty$. Let $\alpha^* \in A$ be the category of the new target that appears at time $t + 1$. The probability distribution for the target categories $\alpha_{t+1}$ at time $t + 1$ given $\alpha_t$ and $N_{t+1}$ can be written as

$$p(\alpha_{t+1}|\alpha_t, N_{t+1}) = \begin{cases} \frac{1}{M} & \text{if } N_{t+1} = N_t + 1, \alpha_{t+1} = \alpha_t \cup \alpha^*, \\ \frac{1}{N_t} & \text{if } N_{t+1} = N_t - 1, \alpha_{t+1} = \alpha_t - \alpha_{t,n_t}, \alpha_{t,n_t} \in \alpha_t, \\ \frac{p_d p_b}{N_t M} & \text{if } N_{t+1} = N_t, \alpha_{t+1} = \alpha_t - \alpha_{t,n_t} \cup \alpha^*, \alpha_{t,n_t} \in \alpha_t, \\ 1 - p_d p_b & \text{if } N_{t+1} = N_t, \alpha_{t+1} = \alpha_t. \end{cases}$$

(10)

Let $\chi_t$ denote the set of all the targets present in the scene at time $t$. We now define the state transitions of the targets that are present at both times $t$ and $t + 1$. For such a target, $n_t \in \{\chi_{t+1} \cap \chi_t\}$, define a vector of its position and velocity as $\xi_{t+1,n_t} = [\rho_{t+1,n_t}^T, \dot{\rho}_{t+1,n_t}^T]^T$. Then, given $\xi_{t,n_t}$, we have

$$\xi_{t+1,n_t} = F_{n_t} \xi_{t,n_t} + v_{t,n_t},$$

(11)

where $F_{n_t}$ is the state transition matrix and $v_{t,n_t}$ is the process noise. We assume that the targets follow linear trajectories, and hence the state transition matrix is given as

$$F_{n_t} = \begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

(12)

where $\Delta t$ is the system sampling time. The process noise, $v_{t,n_t}$, is assumed to be Gaussian distributed, with a zero mean and a covariance matrix given by [43]:

$$\Sigma_{v,n_t} = \epsilon_{n_t} \begin{bmatrix} \frac{1}{2} \Delta t^3 & 0 & \frac{1}{2} \Delta t^2 & 0 \\ 0 & \frac{1}{2} \Delta t^3 & 0 & \frac{1}{2} \Delta t^2 \\ \frac{1}{2} \Delta t & 0 & \Delta t & 0 \\ 0 & \frac{1}{2} \Delta t & 0 & \Delta t \end{bmatrix},$$

(13)

where $\epsilon_{n_t}$ is the intensity of the process noise for the $n_t$th target. Hence, we have

$$p(\xi_{t+1,n_t}|\xi_{t,n_t}) = \mathcal{N}(\xi_{t+1,n_t}; F_{n_t} \xi_{t,n_t}, \Sigma_{v,n_t}).$$

(14)
The state transition of the vector $\theta_t$ can be obtained by using the chain rule as

$$p(\theta_{t+1}|\theta_t) = p(\xi^*_{t+1})p(\alpha_{t+1}|\alpha_t, N_{t+1})p(N_{t+1}|N_t) \prod_{n_t \in \chi_{t+1}\cap \chi_t} p(\xi_{t+1,n_t}|\xi_{t,n_t}),$$  \hspace{1cm} (15)

where $p(\xi^*_{t+1})$ is the probability density of the position and velocity vector of the new target initiated at time $t+1$.

C. Measurement Model

In this subsection, we describe the sensor measurement models for the multistatic radar, the infrared camera and the human scout. At time $t$, let $Y_t = \{y_{p,t}\}^5_{p=1}$ be the measurements obtained from the sensors. For $p = 1, 2, 3$, the measurements $y_{p,t}$ correspond to the measurements from the three radar antennas, $y_{4,t}$ corresponds to the measurement obtained from the infrared camera, and $y_{5,t}$ corresponds to the measurement obtained from the human scout.

**Multistatic Radar:** We transmit a coherent train of multiple pulses with a pulse repetition period of $t_p$ seconds from the radars:

$$s(t') = \sum_{l=0}^{L-1} a_l(t' - lt_p),$$  \hspace{1cm} (16)

where $a_l(t')$ is the transmitted signal in the $l^{th}$ pulse. The discrete version of the signal $a_l(t')$ is assumed to be of length $G$ and it is denoted as $a_l$. Let $\tau_{p,q,n,t}$ be the total time taken for the signal to travel from the $q^{th}$ radar to the $n^{th}$ target and back to the $p^{th}$ radar, and $\nu_{p,q,n,t}$ be the Doppler frequency shift due the $n^{th}$ target and $p^{th}, q^{th}$ antennas. The parameters $\tau_{p,q,n,t}$ and $\nu_{p,q,n,t}$ depend on the position and velocity of the $n^{th}$ target and the positions of the radars. We have

$$\tau_{p,q,n,t} = \frac{1}{c}\left\{R_{q,n,t} + R_{p,n,t}\right\},$$  \hspace{1cm} (17)

and

$$\nu_{p,q,n,t} = \frac{f_c}{c}\left\{\dot{R}_{q,n,t} + \dot{R}_{p,n,t}\right\},$$  \hspace{1cm} (18)

where $c$ is the speed of propagation, $f_c$ is the carrier frequency, $R_{p,n,t}$ and $R_{q,n,t}$ are the range of the target from the $p^{th}$ antenna and the $q^{th}$ antenna, respectively, and $\dot{R}_{p,n,t}$ and $\dot{R}_{q,n,t}$ are the corresponding range rates. Let the
The corresponding discrete-time signal can be obtained by sampling the received signal and considering only one-to-one. The noise is assumed to be circularly symmetric, complex, white, and following a Gaussian distribution. We assume that the mapping from the target category to the target scattering coefficient is known and one-to-one. The noise is assumed to be circularly symmetric, complex, white, and following a Gaussian distribution. The corresponding discrete-time signal can be obtained by sampling the received signal and considering only $N_s$ samples.

$$y_p(nt_a) = \sum_{q=1}^{3} \beta_{p,q,n_t} \sqrt{\gamma_q \zeta_{p,q,n_t}} \sum_{l=0}^{L-1} a_l(n t_a - l t_p - \tilde{\tau}_{p,q,n_t}) e^{j2\pi \nu_{p,q,n_t} l t_p} + w_p(n t_a).$$  

Here $\tilde{\tau}_{p,q,n_t} = f_s \tau_{p,q,n_t}$ is the delay in the discrete domain. Since $a(t')$ is a narrow pulse, in arriving at Eq. (21), within each $a(t')$, we have approximated the term $e^{j2\pi \nu_{p,q,n_t} l t_p}$ as $e^{j2\pi \nu_{p,q,n_t} l t_p}$ (a constant). Simplifying Eq. (21), we get

$$y_{p,n_t} = \sum_{q=1}^{3} \beta_{p,q,n_t} \sqrt{\gamma_q \zeta_{p,q,n_t}} \left( \Upsilon(p,q,n_t) \otimes \Gamma(p,q,n_t) \right) s + w_p,$$  

where

- $y_{p,n_t}$ is a $L N_s \times 1$ received signal vector at the $p^{th}$ antenna due to the signal bouncing off the $n_t^{th}$ target
- $\Upsilon(p,q,n_t)$ is an $L \times L$ Doppler modulation matrix defined as $\text{diag}\{1, e^{j2\pi \nu_{p,q,n_t} t_p}, \ldots, e^{j2\pi \nu_{p,q,n_t} (L-1) t_p}\}$
- $\Gamma(p,q,n_t)$ is a $N_s \times G$ time shift matrix defined as $\begin{bmatrix} 0_{\nu_{p,q,n_t} \times G} & I_G; & 0_{N_s - \nu_{p,q,n_t} \times G} \end{bmatrix}$
- $s$ is a $L G \times 1$ column vector obtained by stacking the transmitted signal in each pulse i.e., $s = [a_0^{T}, a_1^{T}, \ldots, a_{L-1}^{T}]^{T}$
• $w_p$ is a $LN_s \times 1$ complex additive white Gaussian noise at the $p^{th}$ receiver with a zero mean and a known covariance matrix $\Sigma_{w,p} = \sigma_{w,p}^2 I_{LN_s}$

The received signal due to all the targets can now be expressed in a matrix form as

$$y_{p,t} = \sum_{n_t=1}^{N_t} \sum_{q=1}^{3} \beta_{p,q,n_t} \sqrt{\gamma_q \zeta_{p,q,n_t}} \left( Y(p, q, n_t) \otimes \Gamma(p, q, n_t) \right) s + w_p = \Phi_{p,t} \beta_{p,t} + w_p,$$

(23)

where

• $y_{p,t}$ is a $LN_s \times 1$ received signal vector at the $p^{th}$ antenna due all the targets
• $\Phi_{p,t}$ is a $LN_s \times 3N_t$ matrix defined as $\Phi_{p,t} = [\sqrt{\gamma_1 \zeta_{p,1,1}} \phi_{p,1,1}, \ldots, \sqrt{\gamma_3 \zeta_{p,3,1}} \phi_{p,3,1}, \ldots, \sqrt{\gamma_3 \zeta_{p,3,N_t}} \phi_{p,3,N_t}]$
• $\beta_{p,t}$ is a $3N_t \times 1$ vector defined as $\beta_{p,t} = [\beta_{p,1,1}, \ldots, \beta_{p,1,N_t}, \ldots, \beta_{p,3,1}, \ldots, \beta_{p,3,N_t}]^T$

Hence we have

$$p(y_{p,t} | \theta_t) = C N(y_{p,t}; \Phi_{p,t} \beta_{p,t}, \Sigma_{w,p}), \quad \text{for } p = 1, 2, 3.$$  

(24)

Infrared Camera: We use a measurement model similar to the one proposed in [44] and [1] for the infrared camera. The output $y_{4,t}$ of an infrared camera, which is an $R \times C$ matrix of pixel values, is modeled as a noisy version of an ideal image $I_0$ convolved with the point-spread function of the camera. For simplicity, we assume the point-spread function to be a delta function and the ideal image to be of the form

$$I_0(z) = \sum_{n_t=1}^{N_t} T_{n_t} \delta(z - \rho_{n_t}),$$

(25)

where $z$ is a two-dimensional pixel location in the matrix $y_{4,t}$, and $T_{n_t}$ is a constant that depends on the target category. Assuming that the infrared camera receives a strong signal, we use a near-field approximation and express the likelihood of the output of the camera as

$$p(y_{4,t} | \theta_t) = N(y_{4,t}; \gamma_4 I_0, \Sigma_{ir,1} \otimes \Sigma_{ir,2}),$$

(26)

where $\gamma_4$ is a constant that depends on the quality of the camera, $\Sigma_{ir,1}$ and $\Sigma_{ir,2}$ are the covariance matrices of the measurement noise along the rows and the columns, respectively.

Human scout: The measurement report given by the scout is an $M$-dimensional vector with its $m^{th}$ entry representing the number of targets of category $m$. The total number of targets at time $t$, as counted by the scout, is given as

$$N_{hs,t} = \sum_{m=1}^{M} y_{5,t,m},$$

(27)
where \( y_{5,t,m} \) is the \( m \)th entry of the vector \( y_{5,t} \). Let \( N_{\text{max}} \) be an upper bound on \( N_{\text{hs},t} \). We obtain the probability masses for \( N_{\text{hs},t} \), denoted \( g(N_{\text{hs},t}) \), by evaluating a Gaussian density of mean \( N_t \), which is the actual number of targets, and variance \( \sigma_t^2 \) followed by normalizing:

\[
g(N_{\text{hs},t} = k) = \frac{N_t^k}{\sqrt{2\pi \sigma_t^2}} \exp\left(-\frac{(k-N_t)^2}{2\sigma_t^2}\right)
\]

The variance \( \sigma_t^2 \) is chosen to be \( N_t / \gamma_5 \), where the parameter \( \gamma_5 \) is proportional to the level of the training that the scout undergoes and the quality of the equipment that he uses. The variance also depends on the actual number of the targets in the scene. The more the number of targets, the higher is the probability that the scout incorrectly counts them. A similar density function was used in [1] for the number of targets counted by a human scout. Let \( p_c \) denote the probability that the scout counts at least one target incorrectly. We model the probability \( p_c \) to be inversely proportional to \( \gamma_5 \), i.e., \( p_c = b / \gamma_5 \), where \( b \) is a known constant chosen such that \( p_c \in [0, 1] \). From the expression for \( p_c \), it can be seen that well-trained scouts and scouts with better equipment have a lower probability of incorrectly identifying the targets. We model the scout’s measurements \( y_{5,t} \) to follow a multinomial distribution, whenever at least one target is identified incorrectly. The likelihood for scout’s measurement report can then be expressed as

\[
p(y_{5,t} | \theta_t) = \begin{cases} 
  p_c \sum_{k=1}^{N_{\text{max}}} g(k) \prod_{m=1}^{M} \frac{y_{5,t,m}^{k_m}}{y_{5,t,m}^{k_m}} q_{m,t}^{y_{5,t,m}} & \text{if at least one target is incorrectly identified,} \\
  (1 - p_c) + p_c \left[ g(N_{\text{hs},t}) \prod_{m=1}^{M} \frac{y_{5,t,m}^{N_{\text{hs},t,m}}}{y_{5,t,m}^{N_{\text{hs},t,m}}} q_{m,t}^{y_{5,t,m}} \right] & \text{otherwise.}
\end{cases}
\]

The probabilities \( q_{1,t}, q_{2,t}, \ldots, q_{M,t} \) are obtained based on the actual values of the state vector \( \theta_t \). We first evaluate a \( N_t \times M \) matrix \( Q_t \), such that

\[
[Q_t]_{n,t,m} = \begin{cases} 
  1 - q_c & \text{if the } n_{t}^{th} \text{ target is of category } m, \\
  \frac{q_c}{M-1} & \text{otherwise}
\end{cases}
\]

where \( 0 \leq q_c \leq 1 \) is a known constant. We then normalize \( Q_t \) to obtain the probability vector \( q_{m,t} \) as \( q_{m,t} = \frac{1}{N_t} \sum_{n=1}^{N_t} [Q_t]_{n,t,m} \).

\[ \text{D. Sensor Management} \]

In this section, we jointly address the SS, RA, and DF using a price theory framework. Our approach is based on modeling the interaction between the SNs and the SM as an interaction between the agents in an economic market.
Each agent acts as a self interested unit, with the goal of maximizing its utility. As the utilities of the agents keep changing, they re-evaluate their preferences and reach an equilibrium. At the equilibrium, none of the agents will deviate from their respective preferences due to the Pareto-optimality at the equilibrium.

We assume that all the data processing is done by the SM. We model the SM as a consumer in the market that purchases measurements from each of the sensors and sells the resources it has to the sensors. In the multiple target tracking example, the constants $\gamma_1, \ldots, \gamma_5$ in the measurement models of the radar, the infrared camera and the human scout, respectively, determine the quality of the measurements that these sensors obtain. These constants can be thought of as resources that the SM provides the SNs. However, in practice, not all SNs can obtain good data at same time, since the SM has only a finite amount of resources. For example, if the SM spends some amount of resource in training the scout, it might not have enough resources left to obtain a good camera, or to transmit the waveforms with higher energy. Hence, we enforce a constraint of the form $\sum_{p=1}^{5} \gamma_p \leq \gamma$, where $\gamma$ is the constraint on the available resource. We call these constants $\gamma_1, \ldots, \gamma_5$ as the power allocated to the respective sensors and $\gamma$ as the total available power. Let $x_t = [x_{1,t}, \ldots, x_{6,t}]^T$ denote the demand vector of the SM at time $t$. Note that since there is only one consumer in the market, we dropped the subscript for consumer index. The demand vector $x_t$ comprises two parts: the number of measurements that the SM seeks from each SN and the power that the SM distributes to the SNs. Let the number of measurements that the SM seeks from the $p^{th}$ sensor at time $t$ be denoted $x_{p,t}, p = 1, \ldots, 5$. In general, $x_{p,t} = \{0, 1\}, p = 1, \ldots, 5$, which means that the $p^{th}$ sensor is either inactive or it collects a single measurement. However, using diversity techniques, such as time, space and frequency, independent realizations of the measurement vectors can be obtained. Therefore, we consider that $x_{p,t} \in \mathbb{N}$. Denote the total power that the SM distributes be denoted using $x_{6,t}$. Note that $x_{p,t} \geq 0$, for $p = 1, \ldots, 5$, and $x_{6,t} < 0$. The number of measurements that the SM seeks correspond to the quantity of goods that the SM is willing to purchase, and the power allocated corresponds to the quantity of the good that the SM is willing sell. The optimal demand vector of the SM can be obtained by solving

$$x_t^* = \arg \max_{x_t \in B_t(p_t, e_t)} u_t(x_t),$$

where $B_t(p_t, e_t) = \{x_t \in \mathbb{N}_K : p_t^T x_t \leq p_t^T e_t\}$,

(31)

where $e_{p,t} = 0, p = 1, \ldots, 5$, and $e_{6,t} = \gamma$ is the total available power. We consider an information theoretic utility
function to characterize the preference of the SM to the various choices of the measurements that it can obtain from the sensors. Since the overall goal of the SM is to estimate the unknown target state, we chose a utility function that reduces the uncertainty about the unknown state vector $\theta_t$.

$$u_t(x_t) = -\sum_{p=1}^{5} \frac{1}{d_p} x_{p,t} H(\theta_t | y_{p,1:t-1}),$$

(32)

where $d_p$ is the dimension of the subspace that the $p^{th}$ sensor observes, and $H(z)$ is the entropy [45] of the random variable $z$ defined as $H(z) = -\sum_{z \in Z} p(z) \log(p(z))$. For the utility function defined in Eq. (32), the posterior distribution, $p(\theta_t | y_{p,1:t-1})$ can be approximated using a set of $N_p$ particles $\left\{ \theta_t^{(k)} \right\}_{k=1}^{N_p}$ and associated weights $\left\{ w_{p,t-1}^{(k)} \right\}_{k=1}^{N_p}$. This method of approximating the posterior distribution is employed in particle filtering, and we will describe the particle filtering in detail in the next subsection. With this approximation, the utility function can be simplified as

$$u_t(x_t) = \sum_{p=1}^{5} \frac{1}{d_p} x_{p,t} \sum_{k=1}^{N_p} w_{p,t-1}^{(k)} \log(w_{p,t-1}^{(k)}).$$

(33)

We model the SNs as producers in the market that obtain measurements by using the resources that the SM provides them. In the multiple target tracking example, there are five producers: the three radar antennas, the infrared camera and the human scout. Let $v_{j,t}(y_{j,t}^-)$ denote the production function of the $j^{th}$ producer. The production function defines the number of measurements that the producer can obtain, as a function of the resources allocated to it. We consider the following linear production function for each producer at time $t$

$$v_{j,t}(y_{j,t}^-) = c_j y_{j,t}^-,$$

(34)

where $v_{j,t}(y_{j,t}^-)$ represents the maximum number of measurements that $j^{th}$ sensor can obtain at time $t$, $y_{j,t}^-$ represents the resource allocated to the $j^{th}$ sensor at time $t$, and $c_j$ is a known constant. In this paper, we chose $c_j = 1, j = 1, \ldots, 5, \forall t \in \mathbb{N}$. The optimal production plan vector of each producer is obtained by solving

$$y_{j,t}^* = \arg \max_{y_{j,t} \in \mathbb{R}} p_t^T y_{j,t},$$

subject to $y_{j,t}^+ \leq v_{j,t}(y_{j,t}^-)$ and $(p_{j,t}^-)^T y_{j,t}^- \leq w_j$ $j = 1, \ldots, 5$.

(35)

In this manner, we create an artificial market for the sensor information and the resources. The SNs and the SM act as agents in the market, and these agents interact in the market to reach an equilibrium. We use the auction mechanism described in Table I to obtain an $\epsilon$-approximate equilibrium point $\left( \{x^*\}, \{y_{j,t}^*\}_{j=1}^{5}, p^* \right)$ for this market.
We use a centralized implementation of the auction mechanism in this paper for two important reasons. First, the SNs in the network collect the data, but they do not have any processing capabilities. Hence, they cannot compute their optimal production vectors. Second, bandwidth is scarce in any sensor network and exchanging the bids and prices will increase the overhead of the system in terms of communication bandwidth. Therefore, our application is suited for a centralized implementation than a distributed implementation. We also assume that processing power is available within the SM. As a result, since their production functions do not change with time, the SNs can communicate their production functions to the SM in an offline fashion at the beginning of the tracking process. As a result, there is no information exchange during the sensor management.

In the equilibrium solution thus obtained, \( x^* \), which corresponds to the number of the measurements obtained from each sensor, is the solution to the SS problem; \( (y_j^*)^+ \), which corresponds to the power allocated to each sensor, is the solution to the RA problem; and \( p^* \), which corresponds to the price of the measurements obtained by each sensor, is the solution to the DF problem.

E. Target Tracking Algorithm

We use a standard particle filter [46] to compute an estimate of the state vector. First, we compute the global posterior distribution as a linear sum of the local posterior distributions, given the measurements collected by the individual sensors. This method of data fusion is called the linear opinion pool [16].

\[
p(\theta_t | Y_{1:t}) = \sum_{p=1}^{P} \mu_p p(\theta_t | y_{p,1:t}),
\]

where each weight \( \mu_p \) represents a subjective measure of the reliability of the information from the \( p^{th} \) sensor and \( \sum_{p=1}^{P} \mu_p = 1 \). In our example, \( P = 5 \). The particle filter computes a discrete weighted approximation to the true posterior density using a set of particles that characterize the local posterior probability distributions.

\[
p(\theta_t | Y_{1:t}) \approx \sum_{p=1}^{P} \mu_p \sum_{k=1}^{N_p} u_{p,t}^{(k)} \delta(\theta_t - \theta_t^{(k)}),
\]

where \( \left\{ \theta_t^{(k)} \right\}_{k=1}^{N_p} \) are the support points (or samples) that characterize the probability distribution \( p(\theta_t | y_{p,1:t}) \), and \( \left\{ u_{p,t}^{(k)} \right\}_{k=1}^{N_p} \) are the associated weights. The samples \( \left\{ \theta_t^{(k)} \right\}_{k=1}^{N_p} \) are drawn from a known proposal distribution, and the weights are derived using the principle of importance sampling [47]. In general, the proposal distribution is
chosen to be the transitional prior. This choice results in a simple weight update equation given as

$$\tilde{w}_{p,t}^{(k)} \propto w_{p,t-1}^{(k)} p(y_p,t|\theta_t^{(k)})$$

(38)

where $\tilde{w}_{p,t}^{(k)}$ is the un-normalized weight of the $k^{th}$ particle at time $t$. We generate $N_p$ particles, $\{\theta_t^{(k)}\}_{k=1}^{N_p}$, of the state vector $\theta_t$ using importance sampling, where the importance function is chosen to be the transitional prior described in Sec. IV-B. The associated weights are then updated using Eq. (38). After updating the weights, we also resample the particle set, if the following condition holds [46]:

$$N_{eff} = \sum_{p=1}^{P} \frac{\mu_p}{\sum_{k=1}^{N_p} (\tilde{w}_{p,t}^{(k)})^2} < N_p/3.$$  

(39)

We use residual systematic resampling technique that is described in [48]. The minimum mean-squared error (MMSE) estimate of the state vector is then obtained as the mean of the global posterior distribution as

$$\hat{\theta}_t = \sum_{p=1}^{P} \mu_p \sum_{k=1}^{N_p} \tilde{w}_{p,t}^{(k)} \theta_t^{(k)}.$$  

(40)

V. NUMERICAL EXAMPLES

In this section, we use numerical examples to demonstrate the performance improvement obtained due to sensor management using the proposed price theory framework. In order to quantify the performance of the multiple target tracking system, we define four performance metrics. We describe the simulation setup first and then discuss the examples.

Target parameters: We consider surveillance of a region for a period of 20 tracking intervals. The duration of each tracking interval was 0.1 s ($\Delta t = 0.1s$). We consider tracking under two scenarios. In the first scenario, during the first 0.8 s, which corresponds to 8 intervals, there were 3 targets in the scene, during the next 0.8 s, i.e., between the 9th and the 15th interval there were 4 targets, and thereafter there were 3 targets again. The number of target categories was chosen to be 5 ($M = 5$), and the initial positions and initial velocities of the targets belonging to various categories were chosen as shown in the Table II. During the entire tracking duration, the target categories were chosen as

$$\alpha_t = \begin{cases} 
\{1, 2, 5\} & t = 1, 2, \ldots, 8 \\
\{1, 2, 3\} & t = 9, 10, \ldots, 15 \\
\{1, 2, 3\} & t = 16, \ldots, 20 
\end{cases}$$

In the second scenario, there were three targets during the entire duration and the categories of these targets were chosen to be $\alpha_t = \{1, 2, 4\}$ for $t = 1, \ldots, 20$. The initial positions and velocities of these targets were again chosen according to Table II. The probabilities of the birth and the death of the targets were chosen to be 0.01, i.e., $p_d = p_b = 0.01$, respectively.

**Signal and Sensor Parameters:** We transmit OFDM [49] waveforms with eight ($G = 8$) subcarriers loaded with same symbol in all the subcarriers from the radar antennas. The total bandwidth was 100Mhz ($B = 100$) and the carrier frequency, $f_c$, of the transmitted waveforms was 1Ghz. We used four ($L = 4$) pulses in each tracking interval. The radar antennas were located at $(x_1, y_1) = (0, 0), (x_2, y_2) = (20, 0), (x_3, y_3) = (40, 0)$, respectively, and the variance of the measurement noise at each antenna was $\sigma_{w,p}^2 = 1 \times 10^{-3}, p = 1, 2, 3$. The covariance matrices of the measurement noise at the infrared camera along the row and columns, respectively, were chosen to be $\Sigma_{ir,1} = \sigma_{ir}^2 I_R, \Sigma_{ir,2} = \sigma_{ir}^2 I_C$, with $\sigma_{ir}^2 = 1 \times 10^{-2}$. The constants $b$ and $q_c$ for the human scout were chosen to be 0.2 and 0.05, respectively. We assumed that all the targets were observable by all the sensors.

We evaluated the performance of the system using four metrics: the average number of targets detected in the scene, the average number of targets identified incorrectly, the root mean-squared error (RMSE) in the position of correctly identified targets, and the root mean-squared error in the velocity of correctly identified targets. Let the estimate of the state vector at time $t$ in the $i^{th}$ Monte-Carlo iteration be $\hat{\theta}_{t,i} = [\hat{N}_t, \hat{\rho}_1^T, \hat{\rho}_2^T, \hat{\alpha}_1, \cdots, \hat{\rho}_N^T, \hat{\rho}_N^T, \hat{\alpha}_N]_i^T$. 

<table>
<thead>
<tr>
<th>Target Category</th>
<th>Initial Position</th>
<th>Initial Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(5,5)</td>
<td>(10,5)</td>
</tr>
<tr>
<td>2</td>
<td>(15,10)</td>
<td>(5,0)</td>
</tr>
<tr>
<td>3</td>
<td>(10,10)</td>
<td>(5,10)</td>
</tr>
<tr>
<td>4</td>
<td>(40,40)</td>
<td>(-10,-10)</td>
</tr>
<tr>
<td>5</td>
<td>(0,40)</td>
<td>(5,-10)</td>
</tr>
</tbody>
</table>
The four performance metrics are then defined as:

\[
PM_1 = \frac{1}{N_{mc}} \sum_{i=1}^{N_{mc}} \hat{N}_{t,i},
\]

\[
PM_2 = \frac{1}{N_{mc}} \sum_{i=1}^{N_{mc}} \text{card}\left( (\alpha_t - \hat{\alpha}_{t,i}) \cup (\hat{\alpha}_{t,i} - \alpha_t) \right),
\]

\[
PM_3 = \frac{1}{|B_t|N_{mc}} \sum_{i=1}^{N_{mc}} \sum_{n_t \in B_t} \sqrt{(\rho_{x,n_t} - \hat{\rho}_{x,n_t})^2 + (\rho_{y,n_t} - \hat{\rho}_{y,n_t})^2},
\]

\[
PM_4 = \frac{1}{|B_t|N_{mc}} \sum_{i=1}^{N_{mc}} \sum_{n_t \in B_t} \sqrt{(\hat{\rho}_{x,n_t} - \hat{\rho}_{x,n_t})^2 + (\hat{\rho}_{y,n_t} - \hat{\rho}_{y,n_t})^2},
\]

where \( B_t \) is the set of correctly identified targets. In the above, \( PM_1 \) corresponds to the actual target number, \( PM_2 \) corresponds to the number of targets whose categories have been incorrectly estimated, \( PM_3 \) and \( PM_4 \) correspond to the RMSE in the range and the velocity estimates, respectively.

In Fig. 1, we plot these metrics for two methods. In the first method, which we refer as standard approach,
there was no sensor management. We used all sensors at all times, collected one measurement from each sensor, and equally distributed the power among them. The particle weights corresponding to each sensor were updated following Eq. (38) and the global posterior density was computed following Eq. (36) by giving equal weights to the sensors, i.e., $\mu_p = 1/P, p = 1, \ldots, P$. In the second method, which we refer as price-theory approach, we used the proposed price theory framework to select the number of measurements that the each sensor should obtain, and to allocate power to the selected sensors using the demand and the production plan vectors of the SM and the SNs, respectively. Further, we computed the price of the measurements collected by the individual sensors, and used the price of data as the weight given to the corresponding sensor to evaluate the global posterior density (see Eq. (36)). During the interaction between the SNs and the SM, the SM will request for a higher number of measurements from the SN which improves the utility function of the SM. In order to meet the demand, this SN will increase the price of data it collects. As a result, the SNs which improve the utility function will give a higher price to the data they collect, and therefore, the price of the data can act as a measure of importance of the data.

In order to find the demand vector, the production plan vector, and the price, i.e., $\{x, \{y_j\}_{j=1}^5, p\}$, we computed the $\epsilon$-approximate Walrasian equilibrium for market model described in Sec. IV-D using the algorithm described in Table I. We used a branch-and-bound technique [42] to solve the constrained optimization problems given by Eqs. (31) and (35). For the auction algorithm, we chose $\epsilon = 3$ and $N_{th} = 1000$. To find the estimates of the state vector, we used a particle filter with $N_p = 1500$ particles. The simulations were averaged over $N_{mc} = 100$ Monte-carlo iterations.

We can see from Fig. 1(a) that the system was able to accurately estimate the number of targets when sensor management was used. On the other hand, not using sensor management resulted in an incorrect estimation of the number of targets. It can also been seen that employing sensor management resulted in higher correct identifications of the target categories compared to the case when sensor management was not used. However, from Figs. 1(c) and 1(d), it can be seen that the average RMSE in the range and the velocity, per target, for the correctly identified targets, using sensor management, increased after a few iterations and was comparable to the average RMSE obtained without using sensor management. This is because when the number of targets changes in a particular tracking interval, there are a very few particles that correspond to the changed target number. A price theory approach to sensor management gives higher weights to these particles and computes the state estimates based on these fewer
particles. As a result, the RMSE in the range and the velocity estimates increases. When sensor management is not used, the actual target number is not estimated correctly. As a result, there are several particles, that do not correspond to the actual target state, that get a higher weight which results in a lower RMSE in this case. Hence, the RMSE using the price theory approach for sensor management increases at the intervals where the number of the targets change.

In Fig. 2, we plot the performance metrics for the second scenario. For this case, it can be seen that using sensor management resulted in an accurate estimate of the number of targets, higher correct identifications, and a lower average RMSE in range and velocity estimates. As the number of targets remained same throughout the tracking period, there was no sudden increase in RMSE of the range and velocity estimates unlike the first scenario.

In Figs 3 and 4, we plot the output of the sensor selection, resource allocation and data fusion for the two scenarios that we considered. The SM seeks more measurements from the human scout compared to the other
sensors. This is expected since the scout is trained in accurately counting the number of targets and their categories compared to other sensors. The power allocated to the other sensors and the weights assigned to them change with the utility function.

VI. SUMMARY

We considered the problems of sensor selection (SS), resource allocation (RA) and data fusion (DF) that comprise the sensor management in multi-modal networks, and we developed a framework, based on economic price theory, to jointly solve SS, RA, and DF. We illustrated this framework using a scenario where the task is to track an unknown and time-varying number of targets, using the measurements obtained by three different kinds of sensors: a multistatic radar, an infrared camera and a human scout. Numerical examples showed that the proposed framework for sensor management was effective in estimating the number of targets, accurately identifying the target categories,
and producing lower root mean-squared error in the range and velocity estimates of the targets when compared to the root mean-squared error obtained when sensor management was not employed. In the future, we will analytically derive the convergence rate of the double auction mechanism proposed in this paper. We will also extend the framework to include other utility functions and production functions. We will use this framework for solving sensor management in other applications that use multi-modal networks.

REFERENCES


---

**Phani Chavali** (S’08) received his B. Tech degree in electronics and communications engineering (with Honors in signal processing and communications) from International Institute of Information Technology-Hyderabad, India in 2007 and his M.S degree in electrical engineering from Washington University in St. Louis (WUSTL), USA in 2010. He is currently working towards the Ph.D. degree in the electrical and systems engineering from WUSTL. Prior to joining WUSTL, he was employed with the Wireless Terminal Division of Samsung Electronics in Bangalore, India. His research interests are broadly in the areas of statistical signal processing, radar systems, optimization techniques, Monte-Carlo methods, learning and inference algorithms.
Arye Nehorai (S’80–M’83–SM’90–F’94) is the Eugene and Martha Lohman Professor and Chair of the Preston M. Green Department of Electrical and Systems Engineering (ESE) at Washington University in St. Louis (WUSTL). He serves as the Director of the Center for Sensor Signal and Information Processing at WUSTL. Earlier, he was a faculty member at Yale University and the University of Illinois at Chicago. He received the B.Sc. and M.Sc. degrees from the Technion, Israel and the Ph.D. from Stanford University, California. Under his leadership as ESE chair, undergraduate enrollment has more than doubled in the last three years.

Dr. Nehorai served as Editor-in-Chief of *IEEE Transactions on Signal Processing* from 2000 to 2002. From 2003 to 2005 he was the Vice President (Publications) of the IEEE Signal Processing Society (SPS), the Chair of the Publications Board, and a member of the Executive Committee of this Society. He was the founding editor of the special columns on Leadership Reflections in *IEEE Signal Processing Magazine* from 2003 to 2006.

Dr. Nehorai received the 2006 IEEE SPS Technical Achievement Award and the 2010 IEEE SPS Meritorious Service Award. He was elected Distinguished Lecturer of the IEEE SPS for a term lasting from 2004 to 2005. He was a co-recipient of the IEEE SPS 1989 Senior Award for Best Paper, a co-author of the 2003 Young Author Best Paper Award, and a co-recipient of the 2004 Magazine Paper Award. In 2001 he was named University Scholar of the University of Illinois. Dr. Nehorai was the Principal Investigator of the Multidisciplinary University Research Initiative (MURI) project titled Adaptive Waveform Diversity for Full Spectral Dominance from 2005 to 2010. He has been a Fellow of the IEEE since 1994 and of the Royal Statistical Society since 1996.