Frequency-Hopping Code Design for Colocated MIMO Radar Using Sparse Modeling[⋆]

Sandeep Gogineni, *Student Member, IEEE*, and Arye Nehorai[∗], *Fellow, IEEE*
Department of Electrical and Systems Engineering
Washington University in St. Louis
One Brookings Drive, St. Louis, MO 63130, USA

Email: {sgogineni,nehorai}@ese.wustl.edu
Phone: 314-935-7520 Fax: 314-935-7500

***Abstract*—We consider the problem of multiple target estimation using a colocated Multiple Input Multiple Output (MIMO) radar system. We employ sparse modeling to estimate the unknown target parameters (delay, Doppler) using a MIMO radar system that transmits frequency-hopping waveforms. We derive analytical expressions for the correlations between the different blocks of columns of the sensing matrix. Using these expressions, we compute the block coherence measure of the dictionary. Next, we use this measure to optimally design the sensing matrix by selecting the hopping-frequencies for all the transmitters. We demonstrate the performance improvement using numerical simulations.**

***Index Terms*—Frequency-hopping codes, MIMO radar, colocated, sparse modeling, optimal design, multiple targets**

I. Introduction

Multiple Input Multiple Output (MIMO) radar systems are commonly used in two different antenna configurations; widely-separated (distributed) and colocated. Distributed MIMO radar exploits spatial diversity by utilizing multiple uncorrelated looks of the target [1], [2]. Colocated MIMO radar systems offer performance improvement by exploiting waveform diversity [3]. Each antenna has the freedom of transmitting a waveform different from the waveforms of the other transmitters. In this paper, whenever we mention MIMO radar, we are referring to colocated MIMO radar. In [4] and [5], the authors show that frequency-hopping codes can be used to exploit waveform diversity for colocated MIMO radar. There has been recent interest in applying sparse modeling and compressive sensing to the field of radar by exploiting the sparsity in the target delay-Doppler space [6], [7]. In this paper, we will employ sparse modeling to estimate the unknown target parameters using a MIMO radar system that transmits frequency-hopping waveforms (see also [8]). Further, we will derive analytical expressions for the correlations between the different columns of the sensing matrix. Next, we use this to optimally design the sensing matrix by selecting the hopping-frequencies of all the transmitters.

⋆This work was supported by ONR Grant N000140810849, NSF Grant CCF-1014908, and AFOSR Grant FA9550-11-1-0210.
∗Corresponding author

II. Signal Model

We consider the problem of target estimation using a colocated MIMO radar system operating in a monostatic configuration. We assume there are M_{T} transmit antennas and M_{R} receive antennas arranged in linear arrays. The components of the transmit and receive arrays are separated by a distance of d_{T} and d_{R}, respectively. Further, we assume that these arrays make an angle θ with the target. The i^{th} transmitter emits constant modulus frequency-hopping waveform $u_i(t)$. Assuming L pulses comprise a waveform, the signal from the i^{th} transmitter is $u_i(t) = \sum_{l=0}^{L-1} \phi_i(t-T_l)$, where $\phi_i(t) = \sum_{q=0}^{Q-1} e^{j2\pi c_{i,q}\Delta f t} s(t-q\Delta t)$ and

$$s(t) = \begin{cases} 1, & \text{if} \quad 0 < t < \Delta t, \\ 0, & \text{otherwise.} \end{cases}$$

T_l and Δt denote the pulse repetition interval and hopping-interval duration, respectively. The design of transmit waveforms amounts to choosing the frequency of the transmitted signal during each hopping-interval $c_{i,q}$. We assume that each $c_{i,q}$ takes a value from the set $\{1,\ldots,G\}$, where G is a positive integer. We assume $\Delta f \Delta t = 1$. Further, to ensure orthogonality of the waveforms for zero-lag, we assume that for every hopping-interval q,

$$c_{i,q} \neq c_{i',q}, \forall i \neq i'. \tag{1}$$

We can arrange $c_{i,q}$ into an $M_{\mathrm{T}} \times Q$ dimensional code matrix $\boldsymbol{C}$. This code matrix describes all the transmitted frequencies.

Define $f = \frac{d_{\mathrm{R}} \sin(\theta)}{\lambda}$ and $\gamma = \frac{d_{\mathrm{T}}}{d_{\mathrm{R}}}$, where λ is the wavelength of the carrier. We assume that different scattering centers of the target resonate at different frequencies [9]. Therefore, the target has a frequency dependent RCS. The received signal at each receiver is a linear combination of the target reflected waveforms from all the transmitters. Consider R targets in the scene illuminated by the radar. Then, the received signal at the k^{th} receiver after sampling is

$$\begin{aligned} y_k(n) &= \sum_{i=1}^{M_{\mathrm{T}}} \sum_{l=0}^{L-1} \sum_{q=0}^{Q-1} \sum_{r=1}^{R} a_{i,q}^r e^{j2\pi c_{i,q}\Delta f (nT_{\mathrm{S}} - T_l - \tau^r)} \\ &\times \; s(nT_{\mathrm{S}} - q\Delta t - T_l - \tau^r) e^{j2\pi \nu^r nT_{\mathrm{S}}} e^{j2\pi f(\gamma i + k)} \\ &+ e_k(n), \end{aligned}$$

$\forall n = 1, \ldots, N$, where N denotes the total number of samples at each receiver during one processing interval and T_{S} denotes the corresponding sampling interval. $e_k(n)$ denotes the additive noise at the k^{th} receiver. τ^r, ν^r, and $a^r_{i,q}$ represent the delay, Doppler, and RCS of the r^{th} target, respectively.

III. Sparse Representation

For each of the R targets, the unknown parameters are the attenuation, delay and Doppler. We shall discretize the delay-Doppler space into V uniformly spaced grid points. Only R of these grid points correspond to the true target parameters. The goal is to estimate the correct grid points. Let τ_v and ν_v represent the delay and Doppler corresponding to the v^{th} grid point. For each grid point $v \in \{1, \ldots, V\}$, we define

$$\psi_{i,k,q}(n,v) = \sum_{l=0}^{L-1} e^{j2\pi c_{i,q}\Delta f(nT_{\mathrm{S}}-T_l-\tau_v)} \times s(nT_{\mathrm{S}} - q\Delta t - T_l - \tau_v) e^{j2\pi\nu_v nT_{\mathrm{S}}} e^{j2\pi f(\gamma i+k)}.$$

We stack $\{\psi_{i,k,q}(n,v)\}_{n=1}^{N}$ into an N dimensional column vector $\boldsymbol{\psi}_{i,k,q}(v) = [\psi_{i,k,q}(1,v), \ldots, \psi_{i,k,q}(N,v)]^T$. Similarly, we stack $\{\boldsymbol{\psi}_{i,k,q}(v)\}_{k=1}^{M_{\mathrm{R}}}$ into an NM_{R} dimensional column vector $\boldsymbol{\psi}_{i,q}(v)$. Each of these column vectors corresponds to a different transmitter and hopping-interval. While performing this, we stack the columns corresponding to the same hopping-interval together. Now, for each grid point v, we stack the column vectors into an $NM_{\mathrm{R}} \times M_{\mathrm{T}}Q$ dimensional matrix $\boldsymbol{\Psi}(v)$. Further, we arrange $\{\boldsymbol{\Psi}(v)\}_{v=1}^{V}$ into an $NM_{\mathrm{R}} \times VM_{\mathrm{T}}Q$ dimensional matrix $\boldsymbol{\Psi}$. This is the dictionary matrix that defines the basis elements of our sparse representation. Stacking $a^r_{i,q}$ corresponding to different transmitters and hopping-intervals, we obtain an $M_{\mathrm{T}}Q$ dimensional column vector $\boldsymbol{x}^r$. Further, we define a sparse vector

$$\boldsymbol{x}(v) = \begin{cases} \boldsymbol{x}^r, & \text{if} \quad (\tau_v, \nu_v) = (\tau^r, \nu^r), \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

Finally, we stack the vectors $\boldsymbol{x}(v)$ corresponding to all the grid points to obtain a $VM_{\mathrm{T}}Q$ dimensional block-sparse vector $\boldsymbol{x} = \left[\boldsymbol{x}(1)^T, \ldots, \boldsymbol{x}(V)^T\right]^T$. This sparse vector contains only R non-zero blocks, each corresponding to a different target. Further, each block contains $M_{\mathrm{T}}Q$ entries. We stack the measurements and the additive noise samples at each receiver to obtain the vectors $\boldsymbol{y}_k = [y_k(1), \ldots, y_k(N)]^T$ and $\boldsymbol{e}_k = [e_k(1), \ldots, e_k(N)]^T$. Additionally, stacking the measurement and noise vectors at all the receivers, we obtain $\boldsymbol{y} = \left[\boldsymbol{y}_1^T, \ldots, \boldsymbol{y}_{M_{\mathrm{R}}}^T\right]^T$ and $\boldsymbol{e} = \left[\boldsymbol{e}_1^T, \ldots, \boldsymbol{e}_{M_{\mathrm{R}}}^T\right]^T$. Then, our measurement model reduces to

$$\boldsymbol{y} = \boldsymbol{\Psi}\boldsymbol{x} + \boldsymbol{e}.$$

The estimation of attenuation, delay and Doppler for all the targets reduces to recovering the non-zero entries and the support set of the sparse vector $\boldsymbol{x}$ from the measurement vector $\boldsymbol{y}$. We will use Block Matching Pursuit (BMP) [[10]] to perform the sparse recovery.

IV. Block Coherence Measure

A major factor affecting the performance of the system is the block coherence measure [10]. Let $\boldsymbol{\Psi}(v)$ and $\boldsymbol{\Psi}(v')$ denote the v^{th} and v'^{th} blocks of the dictionary, respectively. Each of them contains $M_{\mathrm{T}}Q$ columns. Each column corresponds to a different transmitter and hopping-interval. Since the columns corresponding to different hopping-intervals do not overlap and further, we imposed the condition in (1) to ensure orthogonality across all the transmitters for zero-lag, all the columns within a block are orthogonal. If any columns of $\boldsymbol{\Psi}(v)$ are exactly the same as the corresponding columns in $\boldsymbol{\Psi}(v')$, we can remove them since they will not contribute to the sparse recovery problem while comparing these two blocks. Therefore, we define

$$D_{v,v'} = M_{\mathrm{T}}Q - d_{v,v'},$$

where $d_{v,v'}$ denotes the number of columns of $\boldsymbol{\Psi}(v)$ that are exactly the same as the corresponding columns of $\boldsymbol{\Psi}(v')$. Let us define the correlation matrix $\boldsymbol{M}[v,v']$ for each pair of blocks of the dictionary matrix $\boldsymbol{\Psi}$ as given below

$$\boldsymbol{M}[v,v'] = \boldsymbol{\Psi}^H(v)\boldsymbol{\Psi}(v').$$

Each entry of this matrix contains the auto-correlation between the different columns of the selected blocks. Using these notations, the authors in [10] defined the block coherence measure of the basis matrix as

$$\mu_{\mathrm{B}} = \max_{v,v' \neq v} \frac{1}{D_{v,v'}} \rho\left(\boldsymbol{M}[v,v']\right),$$

where $\rho\left(\boldsymbol{M}[v,v']\right)$ denotes the spectral norm of $\boldsymbol{M}[v,v']$

$$\rho\left(\boldsymbol{M}[v,v']\right) = \frac{1}{D_{v,v'}} \lambda_{\max}^{\frac{1}{2}} \left(\boldsymbol{M}[v,v']^H \boldsymbol{M}[v,v']\right),$$

where $\lambda_{\max}(.)$ denotes the largest eigenvalue of $(.)$. The block coherence measure should be small in order to obtain good sparse recovery performance.

V. Optimal Hopping-Frequency Design

In order to compute the optimal code matrix, we need to minimize the block coherence measure by solving the following optimization problem

$$\boldsymbol{C}_{\mathrm{opt}} = \underset{\boldsymbol{C}}{\operatorname{argmin}} \left(\max_{v,v' \neq v} \frac{1}{D_{v,v'}} \lambda_{\max}^{\frac{1}{2}} \left(\boldsymbol{M}[v,v']^H \boldsymbol{M}[v,v']\right)\right).$$

Since directly computing the block coherence measure is difficult, we will first compute the entries of the correlation matrix $\boldsymbol{M}[v,v']$. Let $M_{rc}[v,v']$ represent the $(r,c)^{th}$ element of $\boldsymbol{M}[v,v']$ such that $r = qQ + i$ and $c = q'Q + i'$, where $q, q' \in \{0, \ldots, Q-1\}$ and $i, i' \in \{1, \ldots, M_{\mathrm{T}}\}$. Note that there is always a unique mapping between r and (i,q); similarly between c and (i',q'). Therefore, we will alternatively use the notation $M_{iq,i'q'}[v,v']$ instead of $M_{rc}[v,v']$. Let grid point v correspond to the delay-Doppler pair (τ_v, ν_v). Further, let grid point v' correspond to the delay-Doppler pair $(\tau_{v'}, \nu_{v'})$. Then, we can express $M_{rc}[v, v'] = M_{iq,i'q'}[v, v']$ as

$$\sum_{k=1}^{M_{\mathrm{R}}} \sum_{n=1}^{N} \sum_{l=0}^{L-1} e^{j2\pi\left(c_{i',q'}\Delta f(nT_{\mathrm{S}}-T_l-\tau_{v'})-c_{i,q}\Delta f(nT_{\mathrm{S}}-T_l-\tau_v)\right)}$$
$$\times \; s(nT_{\mathrm{S}} - q'\Delta t - T_l - \tau_{v'}) s(nT_{\mathrm{S}} - q\Delta t - T_l - \tau_v)$$
$$\times \; e^{j2\pi(\nu_{v'}nT_{\mathrm{S}}-\nu_v nT_{\mathrm{S}})} e^{j2\pi\left(f(\gamma i'+k)-f(\gamma i+k)\right)}.$$

Each column of the dictionary contains delay-Doppler shifted versions of the transmitted waveforms. Since we chose radar waveforms that have a bounded temporal support (rectangular pulses multiplied by sinusoids), the columns have only a few non-zero samples. All the other column entries are zero. The expression inside the summation will be non-zero only when the corresponding entries of both the columns are non-zero, i.e.,

$$q\Delta t < nT_{\mathrm{S}} - T_l - \tau_v < (q+1)\Delta t, \tag{2}$$

and

$$q'\Delta t < nT_{\mathrm{S}} - T_l - \tau_{v'} < (q'+1)\Delta t. \tag{3}$$

All other entries of the summation will be zero. These conditions ensure that the rectangular pulses corresponding to both the columns overlap at the given temporal index. Further, we assume that the difference between the delays of any two grid points $(\tau_v - \tau_{v'})$ is always a multiple of the duration of the hopping-interval Δt. Additionally, we also assume that Δt is also the size of the delay grid. Therefore, it gives us the range resolution of the sparsity-based radar estimation. For a given q, q', v, v' let us denote $\mathcal{L}$ and $\mathcal{N}(\mathcal{L})$ as the sets containing all l and n satisfying the above conditions. Note that for each pulse index, the sample indices that give non zero entries are different. Then, we can express $M_{iq,i'q'}[v, v']$ as

$$M_{\mathrm{R}} e^{j2\pi f\gamma(i'-i)} \sum_{(l,n)\in\mathcal{L}\times\mathcal{N}(\mathcal{L})} e^{j2\pi nT_{\mathrm{S}}(\nu_{v'}-\nu_v)}$$
$$\times \; e^{j2\pi\left(c_{i',q'}\Delta f(nT_{\mathrm{S}}-T_l-\tau_{v'})-c_{i,q}\Delta f(nT_{\mathrm{S}}-T_l-\tau_v)\right)}.$$

Let N_{c} be the number of samples per each hopping-interval. In other words, $N_{\mathrm{c}}T_{\mathrm{S}} = \Delta t$. Further, we assume that the radial speeds of the targets are much smaller when compared with the speed of wave propagation in the medium. Under this assumption, the Doppler shift is measurable only in between pulses and negligible within the pulse duration. Therefore, we can express the correlation terms as a product of separate terms $M_{\mathrm{R}}e^{j2\pi f\gamma(i'-i)}$, $\sum_{l\in\{0,\ldots,L-1\}} e^{j2\pi T_l(\nu_{v'}-\nu_v)}$, and $\sum_{\tilde{n}\in\{0,\ldots,N_{\mathrm{c}}-1\}} e^{j2\pi\Delta f(c_{i',q'}-c_{i,q})\tilde{n}}$. The first term is independent of the temporal index. The second term represents the contribution in between the L different pulses and the third term represents the contribution from within a hopping-interval. The dependence of the third term on the code matrix is evident from the exponential. Note that the second term depends on the Doppler shift which in turn depends on the frequency of the complex exponential. This frequency is a sum of the carrier frequency and the hopping-frequency. Therefore, we conclude that the second and third terms depend on the code matrix (hopping-frequencies).

Now, we give expressions for these terms as a function of the code matrix. Define f_{c} as the carrier frequency, η as the speed of wave propagation in the medium, and $(v^r, v^{r\prime})$ as the radial speeds corresponding to the grid points v and v', respectively. Then, $\sum_l e^{j2\pi T_l(\nu_{v'}-\nu_v)}$ reduces to $\sum_l e^{j2\pi T_l \frac{1}{\eta}\left((f_{\mathrm{c}}+\Delta f c_{i',q'})v^{r\prime}-(f_{\mathrm{c}}+\Delta f c_{i,q})v^r\right)}$. Even though the above term depends on the code matrix, the dependence is negligible since it is absorbed by the carrier frequency term that is much larger when compared with the baseband code frequencies $f_{\mathrm{c}} \gg G\Delta f$, where $G\Delta f$ denotes the maximum hopping-frequency. We assume that the sampling rate is at least as big as the Nyquist rate corresponding to the largest possible hopping-frequency $\frac{1}{T_{\mathrm{S}}} \geq 2G\Delta f$. Therefore, for all choices of coding matrices, we meet the Nyquist sampling criterion. The summation of the samples of a complex exponential is zero for all T_{S} satisfying Nyquist criterion. Hence, the other summation can be expressed

$$\sum_{\tilde{n}\in\{0,\ldots,N_{\mathrm{c}}-1\}} e^{j2\pi\Delta f(c_{i',q'}-c_{i,q})\tilde{n}} = \begin{cases} N_{\mathrm{c}}, & \text{if} \quad c_{i',q'} = c_{i,q}, \\ 0, & \text{otherwise.} \end{cases}$$

Finally we express the entries $M_{iq,i'q'}[v, v']$ of the correlation matrix corresponding to the blocks v and v' as $M_{\mathrm{R}}N_{\mathrm{c}}e^{j2\pi f\gamma(i'-i)}\sum_l e^{j2\pi T_l(\nu_{v'}-\nu_v)}$ when $c_{i',q'} = c_{i,q}$. $M_{iq,i'q'}[v, v'] = 0$, otherwise. Note that the auto-correlation matrix $\boldsymbol{M}[v, v']$ need not be a Hermitian matrix since $M_{iq,i'q'}[v, v']$ need not be equal to $M^*_{i'q',iq}[v, v']$ for all i, q, i', q'. Therefore, the spectral norm and spectral radius of $\boldsymbol{M}[v, v']$ are not the same. Thus, we need to compute the eigenvalues of $\boldsymbol{M}[v, v']^H\boldsymbol{M}[v, v']$ to evaluate the spectral norm of $\boldsymbol{M}[v, v']$.

We now partition $\boldsymbol{M}[v, v']$ into Q^2 sub-matrices $\left\{\boldsymbol{M}^{qq'}[v, v']\right\}_{q,q'=0}^{Q-1}$ each of dimensions $M_{\mathrm{T}} \times M_{\mathrm{T}}$ such that

$$M_{ii'}^{qq'}[v, v'] = M_{iq,i'q'}[v, v'],$$

where $M_{ii'}^{qq'}[v, v']$ is the $(i, i')^{th}$ element of $\boldsymbol{M}^{qq'}[v, v']$. We use this notation to study the structure of $\boldsymbol{M}[v, v']$ for different pairs of grid points.

Without loss of generality, assume $\tau_{v'} \geq \tau_v$. Then, we combine the conditions in (2) and (3) to obtain the following conditions $q - q' < \frac{(\tau_{v'}-\tau_v)}{\Delta t} + 1$ and $q - q' > \frac{(\tau_{v'}-\tau_v)}{\Delta t} - 1$. Under the assumption that $(\tau_{v'} - \tau_v)$ is a multiple of Δt, the above conditions yield only a maximum of one possible positive integer value for q such that $\boldsymbol{M}^{qq'}[v, v']$ is a non-zero matrix

$$q = q' + \frac{(\tau_{v'} - \tau_v)}{\Delta t}. \tag{4}$$

When $q' + \frac{(\tau_{v'}-\tau_v)}{\Delta t} > Q - 1$, then $\boldsymbol{M}^{qq'}[v, v'] = \mathbf{0}$ for every choice of valid q. In such a scenario the entire q'^{th} column of blocks is filled with zero sub-matrices. Therefore, $\boldsymbol{M}[v, v']$ can be partitioned into a special structure of sub-matrices. It is a block-lower-triangular matrix whose non-zero blocks appear in a single diagonal line parallel to the principal diagonal.

The distance between this line and the principal diagonal is given by $\frac{(\tau_{v'}-\tau_v)}{\Delta t}$. Computing $\boldsymbol{M}[v,v']^H\boldsymbol{M}[v,v']$ for matrices following this structure yields block diagonal matrices whose non-zero diagonal blocks are given by the non-zero blocks in the diagonal line of the original matrix $\boldsymbol{M}[v,v']$. Only when $\tau_{v'}=\tau_v$, all the diagonal blocks of $\boldsymbol{M}[v,v']^H\boldsymbol{M}[v,v']$ will be non-zero. All the diagonal blocks of $\boldsymbol{M}[v,v']^H\boldsymbol{M}[v,v']$ will be zero when $(\tau_{v'}-\tau_v)>(Q-1)\Delta t$.

The largest eigenvalue of a block diagonal matrix is the largest of the eigenvalues of each of the individual blocks. Using this property, we can express $\lambda_{\max}\left(\boldsymbol{M}[v,v']^H\boldsymbol{M}[v,v']\right)$ as

$$\max_{q,q'}\left(\lambda_{\max}\left(\boldsymbol{M}^{qq'}[v,v']^H\boldsymbol{M}^{qq'}[v,v']\right)\right).$$

Let us define

$$\widetilde{\boldsymbol{M}}^{qq'}[v,v']=\boldsymbol{M}^{qq'}[v,v']^H\boldsymbol{M}^{qq'}[v,v'].$$

Using the definition of $\boldsymbol{M}^{qq'}[v,v']$, we compute the $(i,i')^{th}$ element of the Hermitian matrix $\widetilde{\boldsymbol{M}}^{qq'}[v,v']$ as

$$\widetilde{M}^{qq'}_{i,i'}[v,v']=\sum_{k'=1}^{M_{\mathrm{T}}}M^*_{k'q,iq'}[v,v']M_{k'q,i'q'}[v,v'].$$

Therefore,

$$\widetilde{M}^{qq'}_{i,i'}[v,v']=\begin{cases}|\alpha|^2\xi_{iqi'q'}e^{j2\pi f\gamma(i'-i)}, & \text{if} \quad c_{i,q'}=c_{i',q'},\\ 0, & \text{otherwise},\end{cases} \tag{5}$$

where

$$\alpha=M_{\mathrm{R}}N_{\mathrm{c}}\sum_{l\in\{0,\ldots,L-1\}}e^{j2\pi T_l(\nu_{v'}-\nu_v)},$$

and $\xi_{iqi'q'}$ denotes the number of elements in the q^{th} column of the code matrix $\boldsymbol{C}$ that have the same value as $c_{i,q'}=c_{i',q'}$.

Using the condition in (1), equation (5) reduces to

$$\widetilde{M}^{qq'}_{i,i'}[v,v']=\begin{cases}|\alpha|^2\xi_{iqi'q'}, & \text{if} \quad i=i',\\ 0, & \text{otherwise},\end{cases}$$

Therefore, $\widetilde{\boldsymbol{M}}^{qq'}[v,v']$ is a diagonal matrix. Further orthogonality for zero-lag also implies that $\xi_{iqiq'}$ can only take values from the set $\{0,1\}$. Therefore,

$$\lambda^{\frac{1}{2}}_{\max}\left(\boldsymbol{M}^{qq'}[v,v']^H\boldsymbol{M}^{qq'}[v,v']\right)=|\alpha|.$$

$|\alpha|$ depends only on the difference in the Doppler shifts corresponding to the grid points v and v', i.e., $(\nu_{v'}-\nu_v)$. Since it does not depend on the entries of the code matrix, it does not affect the code selection problem. Also,

$$d_{v,v'}=\sum_{i,q,q'=q-\frac{(\tau_{v'}-\tau_v)}{\Delta t}}\xi_{iqiq'}.$$

Note that the summation is carried out only among columns that satisfy the condition in (4). Therefore, this summation varies with respect to the difference of delays $\tau_v-\tau_{v'}$. Finally, the optimal code selection simplifies to

$$\boldsymbol{C}_{\mathrm{opt}}=\underset{\boldsymbol{C}}{\operatorname{argmin}}\left(\beta(\boldsymbol{C})\right), \tag{6}$$

where

$$\beta(\boldsymbol{C})=\left(\max_{v,v'\neq v}\sum_{i,q,q'=q-\frac{(\tau_{v'}-\tau_v)}{\Delta t}}\xi_{iqiq'}\right).$$

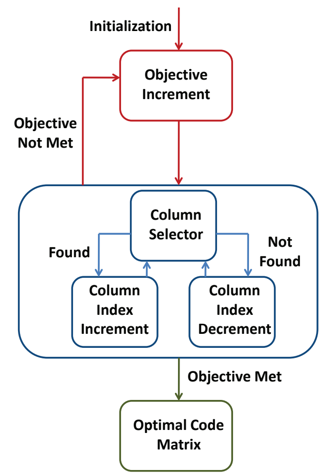

Fig. 1. Flow chart of code selection algorithm.

We will use an iterative approach (see Fig. 1) to obtain an optimal code matrix. First, we notice from (6) that for any code matrix, the objective function is a non-negative integer. Therefore, we start with a desired objective function value of 0 (corresponding to no overlaps between the columns satisfying (4) for all differences in delays) and search for availability of codes satisfying this objective. If no such codes exist, we increment the objective function and follow the same procedure iteratively. This algorithm is implemented in two major loops. The outer loop corresponds to the desired objective value and the inner loop corresponds to the code column. Let $\mathcal{G}^{M_{\mathrm{T}}}$ denote a set containing all column vectors of size M_{T} whose entries taken from $\{1,\ldots,G\}$. Further, we avoid no repetition of entries within these column to ensure orthogonality at zero-lag. Let l_o and l_i denote the iteration indices of the outer and inner loops, respectively. For the first outer iteration, $l_o=1$ and the corresponding objective is $d^{(1)}=0$. For the inner loop, we initialize by selecting any arbitrary column from the set of columns $\mathcal{G}^{M_{\mathrm{T}}}$ as the first column of our code matrix. In every subsequent iteration, we increment the column index and add a column from $\mathcal{G}^{M_{\mathrm{T}}}$ that satisfies the following condition with the already existing columns.

$$\sum_{i,l_i,q'=l_i-\frac{(\tau_{v'}-\tau_v)}{\Delta t}}\xi_{il_iiq'}\leq d^{(l_o)},\quad\forall v,v'. \tag{7}$$

If no such column exists, we decrement the column index and replace the existing column of the previous iteration with another alternative satisfying (7). If we exhaust the inner loop without obtaining sufficient columns to complete the code matrix, it means that an objective of $d^{(l_o)}$ cannot be attained by any code matrix. Therefore, we increment the objective $d^{(l_o+1)} = d^{(l_o)} + 1$ for the next outer iteration and reset the inner loop index to $l_i = 0$. We will terminate the algorithm when we obtain a full (Q columns) code matrix from the inner loop satisfying the objective given by the outer loop index. The code matrix obtained using this algorithm will always have the optimal objective function.

VI. Numerical Simulations

We choose $M_{\mathrm{T}} = 3$, $Q = 5$, and $G = 7$. We ran the iterative algorithm for code selection and obtained

$$\boldsymbol{C}_{\text{opt}} = \begin{bmatrix} 1 & 3 & 2 & 1 & 1 \\ 2 & 4 & 6 & 4 & 2 \\ 3 & 5 & 7 & 6 & 3 \end{bmatrix}.$$

Now, we compare $\beta(\boldsymbol{C})$ (a multiple of the block coherence measure) for the optimal code matrix and a random code matrix whose columns are chosen uniformly from the set of possible columns. We average across 10000 Monte Carlo runs to obtain the curve for the random code matrix. From Fig. 2, we observe that the optimal code matrix has much lower block coherence when compared with the random code matrix.

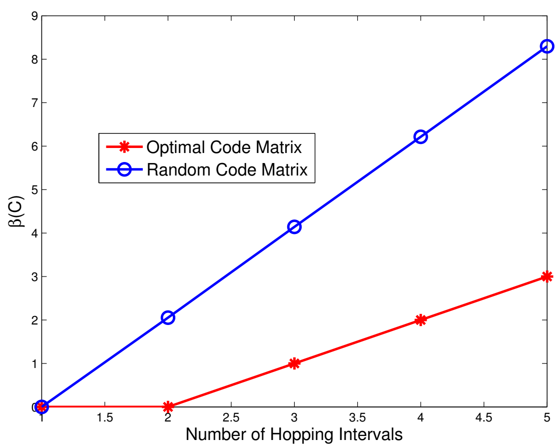

Fig. 2. $\beta(\boldsymbol{C})$ as a function of the number of hopping-intervals.

Let $M_{\mathrm{R}} = 3$, $\theta = 30^\circ$ and $d_{\mathrm{T}} = d_{\mathrm{R}} = \frac{\lambda}{2}$. Choose $L = 10$ and the time interval between pulses as 3mS. Further, let $\Delta t = 1\mu\text{S}$. The maximum hopping-frequency is $G\Delta f = 7\text{MHz}$. Therefore, we sampled at a Nyquist rate of 14×10^6 samples per second. We assume 3 targets in the illuminated space. The attenuations of each target corresponding to $G = 7$ hopping frequencies are $\boldsymbol{a}^1 = [0.4, 0.2, 0.5, 0.8, 0.1, 0.4, 0.3]$, $\boldsymbol{a}^2 = [0.6, 0.2, 0.8, 0.9, 0.1, 0.3, 0.5]$, and $\boldsymbol{a}^3 = [0.2, 0.4, 0.3, 0.7, 0.4, 0.1, 0.9]$. Now, we discretize the target delay-Doppler space. The grid sizes are $\Delta t = 1\mu\text{S}$ and 25Hz. The grid points lie uniformly in the intervals $[0, 10]$ μS and $[800, 1300]$ Hz, respectively. We assume the true delays and Doppler shifts of the targets are $[\tau^1, \tau^2, \tau^3] = [4, 9, 1]$ μS and $[\nu^1, \nu^2, \nu^3] = [1200, 1075, 1025]$ Hz. We define $\text{SNR} = 10 \log \left(\frac{\|\boldsymbol{\Psi x}\|^2}{\text{E}(\|\boldsymbol{e}\|^2)} \right) \text{dB}$. We show the reconstructed target parameters in Fig. 3 at an SNR of 2.6574dB. We observe that the delays and Doppler shifts of all three targets are exactly reconstructed. We used 30 iterations for BMP.

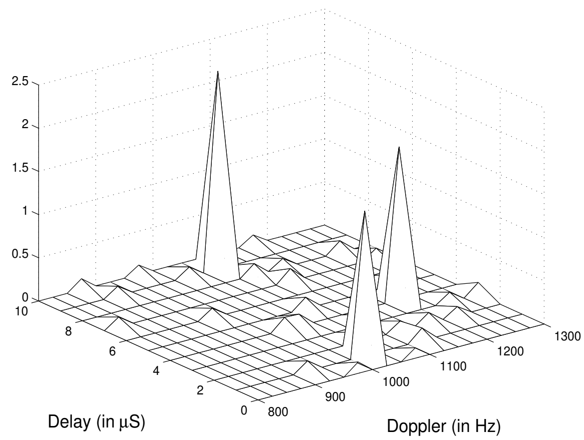

Fig. 3. Target estimates using BMP at an SNR of 2.6574dB.

VII. Concluding Remarks

We proposed a sparsity-based MIMO radar using frequency-hopping waveforms and estimated the target parameters using BMP. We derived the analytical expression for the block coherence measure and used it to select the optimal hopping-frequencies. In future work, we will develop more efficient algorithms for code design using the theory of combinatorial optimization. We will validate our results using real radar data.

References

[1] A. M. Haimovich, R. S. Blum, and L. J. Cimini, "MIMO radar with widely separated antennas," *IEEE Signal Process. Mag.*, vol. 25, pp. 116–129, Jan. 2008.

[2] S. Gogineni and A. Nehorai, "Polarimetric MIMO radar with distributed antennas for target detection," *IEEE Trans. Signal Process.*, vol. 58, pp. 1689–1697, Mar. 2010.

[3] J. Li and P. Stoica, "MIMO radar with colocated antennas," *IEEE Signal Process. Mag.*, vol. 24, pp. 106–114, Sep. 2007.

[4] C. Y. Chen and P. P. Vaidyanathan, "MIMO radar ambiguity properties and optimization using frequency-hopping waveforms," *IEEE Trans. Signal Process.*, vol. 56, pp. 5926–5936, Dec. 2008.

[5] A. Srinivas, S. Badrinath, and V. U. Reddy, "Frequency-hopping code optimization for MIMO radar using the hit-matrix formalism," in *IEEE Radar Conference*, Washington, DC, May 2010, pp. 631–636.

[6] Y. Yao, A. P. Petropulu, and H. V. Poor, "Compressive sensing for MIMO radar," in *IEEE International Conference on Acoustics, Speech and Signal Processing*, Taipei, Apr. 2009, pp. 3017–3020.

[7] S. Gogineni and A. Nehorai, "Target estimation using sparse modeling for distributed MIMO radar," *IEEE Trans. Signal Process.*, vol. 59, pp. 5315–5325, Nov. 2011.

[8] ——, "Frequency-hopping code design for MIMO radar estimation using sparse modeling," *IEEE Trans. Signal Process.*, in revision for.

[9] S. Sen and A. Nehorai, "Adaptive design of OFDM signal with improved wideband ambiguity function," *IEEE Trans. Signal Process.*, vol. 58, pp. 928–933, Feb. 2010.

[10] Y. C. Eldar, P. Kuppinger, and H. Bolcskei, "Block-sparse signals: Uncertainty relations and efficient recovery," *IEEE Trans. Signal Process.*, vol. 58, pp. 3042–3054, Jun. 2010.