Performance of the Electroencephalography Inverse Problem using Electric Potential Gradient Measurements

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Abstract—The EEG inverse problem is traditionally solved with measurements from an array of electric potential sensors. In this work we analyze the expected performance if electric potential gradient measurements were used to solve the inverse problem. We use the Cramér-Rao Bound (CRB) to analyze and compare the performance of the two cases. The spatial covariance of the noise is estimated from real EEG measurements.

Keywords—Electroencephalography, Cramér-Rao Bound, gradient sensor.

I. INTRODUCTION

Electroencephalography (EEG) is used to study the electric activity generated by active neurons in the brain, which may be seen as electric current density sources. This active neurons generate an electric potential distribution that can be measured on the scalp with adequate sensors. The determination of the electric potential distribution generated by a known source is known as the forward problem, whereas the estimation of source parameters based on electric potential measurements is the inverse problem.

The EEG inverse problem is traditionally solved with measurements from an array of electric potential sensors or electrodes. In this work we analyze the performance that could be expected if electric potential gradient measurements were used to solve the inverse problem. We use the Cramér-Rao Bound (CRB) to analyze the performance of both cases.

Gradient sensors present certain a-priori advantages over conventional electrodes. They are implemented by subtracting the measurements of two closely spaced electrodes. Hence, there is no need to choose a reference electrode. This is important since the reference potential was and still is the a controversial matter in EEG [1]. Gradient sensors have a reduced sensitivity to brain spontaneous noise, especially from well separated sources, since their contributions tend to cancel out. This is important for estimation accuracy since precise values of the spatial noise covariance matrix are seldom known. In addition, the wires connecting the electrodes are naturally closely spaced and the formed loops have small area augmenting noise immunity to external magnetic fields, an interesting feature when EEG is combined with MRI. Finally, gradient sensors allow vector sensor measurements, i.e. allow us using directional information of the gradient from which we expect to improve the estimation accuracy [2]. A disadvantage of the gradient sensors is that their practical implementation is more complex than required by single electrodes but more importantly, the gradient signal has small amplitude compared with the potential, hence the signal-to-noise ratio is limited especially by the electrode-skin impedance and amplifier noise. The present paper compares both types of sensors in terms of performance.

Notice the similarities of the gradient sensors with the measurement of the surface Laplacian, see [3] and [4]. However, gradient sensors processing makes use of both orthogonal gradient components on the surface.

In the next section we present the gradient sensors, as well as models for the source of electric activity and head, and the method for solving the forward problem. Then we introduce the noise model, sensitivity to the source parameters and analyze the CRB. In section IV we show the results obtained for a noise model derived from real EEG measurements. In the last section we discuss the results of this work.

II. METHODS

A. Gradient Sensors

The proposed electric potential gradient sensors consists of an array of three electrodes placed on the vertices of an equilateral triangle with a 1 cm side length, see Fig. 1. Such a configuration is mechanically stable, and the measurement of the electric potential difference between the electrodes provides an approximation for both surface components of the gradient as well as some redundancy that can be used to reduce the noise variance.

We analyzed the error of approximating the gradient with a difference and found it to be negligible compared to the
A. Noise Model

We estimated the parameters of the spatial covariance of the spontaneous electric activity from real measurements of the electric potential generated by the basal activity of one subject. The electric potential measurements consist of 32 channels sampled at 200 samples per second, and the empirical spatial correlation matrix was obtained from a subset of 7000 samples free of artifacts and impulsive noise.

The parameters $\alpha$ and $\sigma_i$ were obtained by minimizing the Frobenius norm of the difference between the empirical covariance matrix and that predicted by the model. The fitted value of the parameters are $\alpha = 2.4$ and $\sigma_i$ between 1.8 and 3.1 $\mu$V.

B. Forward Problem

In many cases of interest a group of active neurons is concentrated in a small region of the brain. Such is the case in most epileptic seizures, and in responses to sensorial stimuli and some cognitive experiments. In these cases the source of electric activity can be modelled as an electric potential dipole [5]. We adopted this model, then our source model has six parameters, three parameters determine the position and three the orientation and intensity of the dipole.

The head is modelled as three concentric spherical shells of different conductivities. The layers represent different tissues, brain, skull and scalp in this work. The electric conductivities of the layers where chosen as 0.33 S, 0.0042 S and 0.33 S and the radii of the spherical shells as 8.7 cm, 9.2 cm and 10 cm respectively.

For this simple geometry there exists an analytical approximation to the solution of the forward problem [6], [7]. We adopted this solution for computing the electric potential, and its gradient is computed differentiating the analytic approximation.

C. Cramér-Rao Bound

To determine the expected performance of the inverse problem solutions using the electric potential gradient sensors we compute the CRB for the variance of the estimation error of the source parameters and compare the results against a similar analysis performed with electric potential measurements. The CRB is a theoretical lower bound for the variance of any unbiased estimator of the source parameters.

We will assume in this work that the noise of the measurements has a Gaussian distribution, then the CRB has a very simple expression

$$E \{ (\theta - \hat{\theta})(\theta - \hat{\theta})^T \} = \left[ \frac{\partial \Phi}{\partial \theta} \bar{C}^{-1} \frac{\partial \Phi^T}{\partial \theta} \right]^{-1}$$

where $\theta$ is a vector formed by the source parameters, $\Phi$ is the vector of measurements (the electric potential at the sensor positions or its gradient) and $\bar{C}$ is the spatial covariance matrix of the noise in the measurements.

For the simple head geometry model used in this work the sensitivity of the measured variables, i.e. electric potential or its gradient, to the source parameters can be computed differentiating the analytic approximation of the forward problem solution.

Two sources of noise in the measurements are considered: the electric noise of the sensors and the spontaneous electric activity of the brain. A normal distribution is assumed for their amplitude. Whereas the electric noise of the sensors, that comprises electrode-skin impedance and electronic amplifier noise, is independent between sensors, the spontaneous electric activity is spatially correlated [8]. We propose a covariance model for the correlation coefficient

$$\rho_{ij} = e^{-d_{ij}/\alpha}$$

where $\rho_{ij}$ is the correlation coefficient between electrodes $i$ and $j$, $d_{ij}$ is the distance between the electrodes normalized to the radius of the head, and $\alpha$ is a parameter of the noise model. The other parameters of the model are the standard deviations $\sigma_i$ of the spontaneous brain activity noise at each electrode. Once the parameters are chosen the spatial covariance matrix $\bar{C}$ for the spontaneous electric activity can be computed. It is important to note that the entries $c_{ij}$ of the matrix depend on the reference electrode position as well as the positions of electrodes $i$ and $j$. This is a complication for the electric potential measurements because different choices of the reference will affect the inverse problem solution [1]. The gradient measurements on the other hand, are differential and no common reference is needed.

III. RESULTS

In this section we present the results obtained for a particular example. First, we fit the spatial correlation model of the noise to empiric real data, and then we compute the CRB for the estimation of source parameter. The example is an array of sensors placed according to the Referential 32 Montage [9].

A. Noise Model

We estimated the parameters of the spatial covariance of the spontaneous electric activity from real measurements of the electric potential generated by the basal activity of one subject. The electric potential measurements consist of 32 channels sampled at 200 samples per second, and the empiric spatial correlation matrix was obtained from a subset of 7000 samples free of artifacts and impulsive noise.

The parameters $\alpha$ and $\sigma_i$ were obtained by minimizing the Frobenius norm of the difference between the empiric covariance matrix and that predicted by the model. The fitted value of the parameters are $\alpha = 2.4$ and $\sigma_i$ between 1.8 and 3.1 $\mu$V.
Fig. 2. Spontaneous activity noise cov. matrix. Potential measurements.

Fig. 3. Spontaneous activity noise cov. matrix. Gradient measurements.

With the estimated parameters the theoretical spatial covariance matrix of the electric potential gradient measurements was computed for an array of gradient sensors in the same positions than the potential sensors. The effect of the spatially independent electric noise of the sensors was taken into account adding a diagonal matrix to the spatial covariance matrix. An electric noise with standard deviation $\sigma_e = 0.7\mu V$ was considered for every gradient component. The predicted noise variance is in agreement with the variance of measurements taken with the gradient sensor prototype.

In Figs. 2 and 3 we plotted the entries of the spatial covariance matrix predicted by the model for the electric potential and for its gradient. We can see that the gradient measurements are almost uncorrelated, this means that the results of the inverse problem using gradient measurements will be independent of the spatial correlation model used for the spontaneous brain activity.

B. CRB

In this section we present results obtained with the example of the previous section. The CRB of different parameters was computed for the potential and gradient measurements.

Figs. 4 to 6 show the minimum standard deviation with which the depth of the source can be estimated as a function of the source position. In all cases the dipoles are tangentially oriented and 100nA intensity. In Figs. 4 and 5 the results are shown for sources placed on a sphere 5mm under the brain surface. Fig. 4 corresponds to electric potential measurements and Fig. 5 to gradient measurements. In Fig. 6 we see the
same results for oriented sources placed at different depths. We can see for sources at 5mm under the surface brain the CRB for depth estimators is under 1mm for potential measurements and under 3mm for gradient measurements. The highly localized sensitivity of the gradient sensors can be noted in Figs. 5 and 6, showing a fast degradation of the CRB when the distance from the source to the sensor increases.

Figs. 7 to 9 show the volume of the 90% concentration ellipsoid, i.e. an ellipsoid that contains the source with 90% probability, as a function of source position. The results are shown for the same sources as in the previous paragraph. The volume of the 90% concentration ellipsoid is under 1cm$^3$ for potential measurements and under 3cm$^3$ for gradient measurements. This corresponds to spheres of 4.3mm and 6.2mm radii respectively.

From these results it seems that for cortical sources electric potential sensors and gradient sensors have a similar performance. It appears clearly that gradient sensors need less noisy amplifiers and less electrode-skin impedance to take advantage of the directional information. The highly localized range of the gradient sensors is not a disadvantage, for it can be useful in cases where the neuronal activity is also localized.

IV. DISCUSSION

We have discussed the motivation to use gradient sensors in the Introduction. Although our results were derived for only one kind of sensor array and one subject, they seem to indicate that the use of electric potential gradient measurements could make a contribution to the accuracy of the EEG inverse problem. This is especially true for concentrated cortical sources.

The use of time information could further reduce the CRB. A comparison between the two kinds of sensors could be made as presented in this paper.

This line of work will continue with the study of more subjects, and with an improved head model representing a realistic geometry of the head. The design of the electric potential gradient sensors is also a key point under development, for its electronic noise is the limiting factor on the source parameter estimation accuracy.

REFERENCES