Abstract

We propose an approach to achieve high-performance localization of multiple sources using an array of spatially-distributed electric and magnetic component sensors. The array comprises subarrays that are well calibrated individually but not with each other. Numerical examples demonstrate the efficacy of the proposed method.

1 Introduction

The problem of estimating electromagnetic wave parameters using antenna arrays with diversely polarized elements is important in several applications [2] [3] [4]. Examples include wireless communications, radar and remote sensing. In addition to direction of arrival, the polarization parameters of the signals can also be estimated and can be used to discriminate, identify and separate the signals impinging the antenna array.

In a recent development, Nehorai and Paldi [1] introduced the concept of vector-sensor array processing where the complete electromagnetic information of the signal is measured and processed. They applied the Poynting relationship between the electric and magnetic measurements to enable estimation of the DOA and spatial parameters of multiple signal sources using a single vector-sensor. However, an array of vector sensors is needed to further improve in parameter estimation accuracy, resolution and to uniquely identify a larger number of signal sources. In [6], See and Nehorai proposed the concept of distributed electromagnetic component array (DEMCA), an array of spatially distributed scalar magnetic and electric sensors. It is assumed that the array of scalar magnetic and electric sensors should, in aggregate, measure at least all the 3D electric and magnetic components of the electromagnetic wave. In essence, DEMCA encapsulates and generalizes the vector-sensor array concept. The proposed approach will have the following three advantages: Firstly, the full electric and magnetic field components measure by the magnetic and electric sensors; thereby effectuating derivation of the sources’ directional information. Secondly, their spatial distribution will allow extraction of additional sources directional information by way of the differential-delay measurements. Finally, the DEMCA’s structure will significantly economize the number of receivers needed to simultaneously utilize the time-delay and complete electromagnetic information for DOA estimation.

In spite of these advantages, large aperture is still needed to achieve good resolution and accuracy performance. While large aperture array offers these advantages, they are usually not well suited for highly mobile applications. Calibration errors, such as those arise from antenna positioning error, departure from test (calibration) range environment due to local scattering and uneven ground, can significant degrade the array performance. Furthermore, it can be hard to find suitable deployment ground to site large array with rigid geometry. Hence, small aperture arrays are used in such applications. Similarly, effects from calibration errors will dominate in applications where array is formed from a distributed network of sensor arrays. This problem was addressed in [8] by See and Gershman for scalar arrays. They proposed a MUSIC-like algorithm called G-RARE that estimates the DOA from an array comprising subarrays that are well-calibrated by themselves but calibration errors exist among them. A notable feature is its capability to handle very general calibration errors. They include imprecise knowledge of the inter-subarray displacements, receiver mismatch and sampling offsets among the subarrays and a combination of the aforementioned errors. Clearly, with an array of well-calibrated small aperture subarray, G-RARE will enable them to operate coherently as a large antenna array while offering deployment flexibility and ease.

In this paper, we extend the application of G-RARE from a scalar-sensor array to DEMCA. We present the DEMCA signal model with the aforementioned calibration errors. Using this signal model, we propose a MUSIC-like algorithm to estimate the DOA with an...
2 Measurement Model

Adopting the conventions in [1], the measurement model of the vector sensor is given by

$$
\begin{bmatrix}
y_E(t) \\
y_H(t)
\end{bmatrix} = \begin{bmatrix}
\mathbf{I}_3
\end{bmatrix} \mathbf{V} \mathbf{Q} \mathbf{w}s(t) + \begin{bmatrix}
e_E(t) \\
e_H(t)
\end{bmatrix},
$$

where

$$
\begin{bmatrix}
(u \times)
\end{bmatrix} = \begin{bmatrix}
0 & -u_z & u_y \\
u_z & 0 & -u_x \\
-u_y & u_x & 0
\end{bmatrix},
$$

$u$ is the unit direction vector from sensor to source and \(u_x, u_y, u_z\) are the \(x, y, z\) components. The matrices \(\mathbf{V}, \mathbf{Q}\) and vector \(\mathbf{w}\) are given by

$$
\mathbf{V} = \begin{bmatrix}
-\sin \theta & -\cos \theta \sin \phi & \cos \phi \\
\cos \theta & -\sin \theta \sin \phi & \cos \phi \\
0 & -\sin \alpha & \cos \alpha
\end{bmatrix},
\mathbf{Q} = \begin{bmatrix}
\cos \alpha & \sin \alpha & 0 \\
0 & \cos \phi & \sin \phi \\
\cos \beta & j \sin \beta
\end{bmatrix}
$$

and

$$
\mathbf{w} = \begin{bmatrix}
\cos \beta \\
j \sin \beta
\end{bmatrix}
$$

and where \(\theta, \phi, \alpha\) and \(\beta\) are the azimuth, elevation, ellipse’s orientation and eccentricity angle.

Extending from (1) and assuming that the signal sources are narrowband, we can write the measurement model of the distributed component sensor array in a multiple source environment as [6]

$$
\begin{bmatrix}
y_E(t) \\
y_H(t)
\end{bmatrix} = \sum_{k=1}^{d} \mathbf{a}^{(k)} \mathbf{s}_k(t) + \begin{bmatrix}
e_E(t) \\
e_H(t)
\end{bmatrix},
$$

where \(\mathbf{a}^{(k)} = [\theta^{(k)}, \phi^{(k)}, \alpha^{(k)}, \beta^{(k)}]\) denotes the directional and polarization parameters of the \(k^{th}\) source signal. \(\mathbf{I}_3\) is a diagonal matrix whose \(n^{th}\) diagonal entry is given by \(I_{3n} = \omega_n\), where \(\omega_n\) is the differential delay of the signal source between the \(n^{th}\) component and the phase center and \(a_n(\theta, \phi)\) is the response of the \(n^{th}\) component sensor; \(\omega_c\) is the carrier frequency and \(\mathbf{Q}\) is a \(N\) (number of component sensors) by 6 selection matrix elements of 1 and 0 to pick out from a choice of 6 types of component sensors that form the array. Clearly from (2), DEMCA generalizes the vector-sensor array [6] and allows the differential delay measurements resulting from diverse placement of the component sensors and electromagnetic field measurements to be jointly exploited in estimating the source parameters. Given both the complete electromagnetic and spatial information, better parameter estimation with a smaller aperture array can be expected over a wide frequency range as compared to vector sensor and scalar array.

Let’s denote the \(i^{th}\) DEMCA subarray as \(\mathbf{a}_i(\mathbf{\Theta}^{(k)})\), the steering vector of the array comprising \(M\) partly calibrated DEMCA is given by

$$
\mathbf{a}_F(\mathbf{\Theta}^{(k)}) = \mathbf{C}(\theta^k, \phi^k, \mathbf{w}_k) \mathbf{h}^{(k)}
$$

where \(\mathbf{h}^{(k)} = [h_1^{(k)}, \ldots, h_M^{(k)}]^T\). As pointed out in [8], \(h_1\) captures the calibration errors due to inter-subarray displacement error, receiver channel mismatch and sampling offsets among the sub-arrays. Alternatively, we can rewrite (3) as

$$
\mathbf{a}_F(\mathbf{\Theta}^{(k)}) = \mathbf{\Phi}(\theta^k, \phi^k, \mathbf{h}^k) \mathbf{w}_k,
$$

3 A DOA Estimation Algorithm

The array covariance matrix is given by

$$
\mathbf{R} = \mathbf{A}(\mathbf{\Theta}) \mathbf{P} \mathbf{A}(\mathbf{\Theta})^H + \sigma^2 \mathbf{I}
$$

where \(\mathbf{A}(\mathbf{\Theta}) = \begin{bmatrix}
\mathbf{a}_1(\mathbf{\Theta}^{(1)}) & \cdots & \mathbf{a}_M(\mathbf{\Theta}^{(d)})
\end{bmatrix},\)

\(\mathbf{\Theta} = \begin{bmatrix}
\mathbf{\Theta}^{(1)} & \cdots & \mathbf{\Theta}^{(d)}
\end{bmatrix}, \mathbf{P}\) is the source covariance matrix and \(\sigma^2\) is the noise power. Assuming that \(\mathbf{P}\) is full rank and \(\mathbf{E}_N\) is the noise subspace of \(\mathbf{R}\), we have

$$
\mathbf{a}_F^H(\mathbf{\Theta}) \mathbf{E}_N \mathbf{E}_N \mathbf{a}_F(\mathbf{\Theta}) = 0.
$$

when \(\theta = \theta^k, \phi = \phi^k, h = h^k\) and \(w = w_k\) for \(k = 1 \cdots d\). Hence, we can write the estimation cost function as

$$
\{\hat{\theta}, \hat{\phi}, \hat{h}, \hat{w}\} = \text{arg min } J
$$
Where

\[ J = w^H \Phi^H (\theta, \phi, h) E_N \Phi (\theta, \phi, h) w \quad (5) \]
\[ = h^H C^H (\theta, \phi, w_k) E_N E_N C (\theta, \phi, w_k) h \quad (6) \]

Since \( w \) and \( h \) are linear in parameters in (4) [c.f. (5) and (6)], respectively, we can minimized \( J \) at \( \theta \) and \( \phi \) (denoted by by \( J(\theta, \phi) \)) with respect to \( w \) and \( h \), respectively, using

\[ J_{\theta, \phi} (\hat{w}) = \rho \{ \Phi^H (\theta, \phi, h) E_N E_N \Phi (\theta, \phi, h) \} \]
\[ \hat{w} = v \{ \Phi^H (\theta, \phi, h) E_N E_N \Phi (\theta, \phi, h) \} \]

and

\[ J_{\theta, \phi} (\hat{h}) = \rho \{ C^H (\theta, \phi, w) (\theta) E_N E_N C (\theta, \phi, w) \} \]
\[ \hat{h} = v \{ C^H (\theta, \phi, w) (\theta) E_N E_N C (\theta, \phi, w) \} \]

where the operation \( \rho \{ A \} \) and \( v \{ A \} \) return the minimum eigenvalue and eigenvector of \( A \), respectively. Note that \( J_{\theta, \phi} (\hat{w}) \) assumes a fixed \( h \), while \( J_{\theta, \phi} (\hat{h}) \) assumes a fixed \( w \). Since \( J_{\theta, \phi} (\hat{w}) \) and \( J_{\theta, \phi} (\hat{h}) \) minimize the same cost function, they can be used in an interactive manner to find \( \hat{w} \) and \( \hat{h} \) that minimizes \( J(\theta, \phi) \).

Based on these observations, we proposed the following method to estimate the DOA’s of a partly calibrated DEMCA:

- for \( \theta = -180 : 180 \)
- for \( \phi = -180 : 180 \)
- Initialize \( i = 1, \hat{h}^{(0)} = [1 \cdots 1]^T \)
- Repeat
- \( \hat{w}^{(i)} \) using
  \[ v \{ \Phi^H (\theta, \phi, \hat{h}^{(i-1)}) E_N E_N \Phi (\theta, \phi, \hat{h}^{(i-1)}) \} \]
- \( \hat{h}^{(i)} \) using
  \[ v \{ C^H (\theta, \phi, w^{(i)}) (\theta) E_N E_N C (\theta, \phi, w^{(i)}) \} \]
- \( i = i + 1 \)
- Until Convergence
- Update \( J(\theta, \phi) \)
- end
- end
- Determine the DOA’s by selecting the first \( d \) lowest local minima.

Figure 1: MUSIC-like cost function using the proposed approach.

4 Numerical Example

We consider an array comprises of two DEMCA subarrays. Each subarray is made up of x, y and z electric and magnetic component sensors arranged as an uniformly spaced circular array of half wavelength intersensor spacing. The second subarray is displaced from the first array by \([5\lambda, 5\lambda] \), where \( \lambda \) is the signal’s wavelength. The signals impinge the array from \( \theta^{(1)} = [20^\circ, 40^\circ, 45^\circ, -50^\circ]^T \) and \( \theta^{(2)} = [50^\circ, -30^\circ, 60^\circ, -60^\circ]^T \). The SNR is fixed at 30dB and 100 independent snapshots are used to estimate the array covariance matrix. In this example, we evaluate the MUSIC-like cost function using the proposed method and in [2]. As shown in Figure 1 and 2, our proposed method was able to resolve and estimate the DOA of the signals while the method proposed in [2] was unable to do so. In particular, our proposed method determined the DOA accurately as indicated by the contour plot in Figure 1.

5 Concluding Remarks

We have presented a new approach for the localization of electromagnetic sources using an array comprising partially calibrated DEMCA. A numerical example was presented to illustrate the proposed method has significant resolution and accuracy advantage over conventional methods, such as the ones proposed in [2].

References

Figure 2: MUSIC-like cost function with the method proposed in [2]


